Deviations from exponential model predicted values due solely to variations in $r$.

Growth rate can be estimated by averaging $r$’s from smaller intervals (each time step).

Some final thoughts on exponential growth:

• Exponential growth does not always mean ‘rapid’ growth

• Be clear on the implications of the models you use / assumptions you make about where stochasticity occurs.
Population growth models

Carrying capacity (k)

Classic growth curve, unlimited resources

Classic growth curve, limited resources (k)

Population growth

Reproduction, births, natality (B)

Immigration (I)

Emigration (E)

Mortality, death (D)

\[ n_{t+1} = n_t + (B + I) - (D + E) \]

\( r \) or lambda

Gains

Losses
How Birth rate and/or Mortality could change with density:

Logistic Population Growth (Density dependent growth)

- At higher densities these factors limit population growth b/c they influence one or some combination of BIDE.

- This shows up as our measure of growth rate (lambda, r) changing as a function of population size (density). Growth rate is negatively affected by higher populations.

- And results in a sigmoidal shaped function of population growth over time....most typically described as logistic growth.

- Growth slows as pop size approaches the maximum number/density that a given area can sustain—the carrying capacity (K).

Fig. 6.2 Four possible ways that density dependence could affect per-capita mortality and reproduction rates. All graphs show only negative density dependence except (b), which at low density shows a small range of positive density dependence. The bands embrace the uncertainty and changes in the relationships. The region where the bands cross represents the carrying capacity (K), where births and deaths are equal.

Fig. 6.3 The contrast between exponential and logistic growth. (a) Exponential growth. The ruled per-capita population growth rate measured from a time series (ΔN/Δt) is equal to the intrinsic exponential growth rate (r), no matter what the population size is. This lack of density dependence leads to exponential growth of the population over time. (b) Logistic growth. With a linear decline in realized per-capita population growth rate as the population size increases, the population increases exponentially at first, then slows its growth as it approaches carrying capacity.
Logistic Population Growth (Density dependent growth)

There are various models that describe different ways that $r$ changes with increasing density. We will cover 3.

**Ricker (logistic) Model:**

Assumes a constant, linear decrease of $r$ as population size increases.

\[
\ln(n_{t+1}) = \ln(n_t) + r_{\text{max}} + b(n_t) + F
\]

Where:
- $b$ is a parameter measuring the strength of intraspecific competition
- $r_{\text{max}}$ is the populations maximum growth rate in the absence of density dependent competition (what we’ve dealt with up to now)

**Rickers (logistic) Growth:**

Constant linear decrease in $r$ as $n_t$ increases

\[
\ln(n_{t+1})/\ln(n_t) = + r_{\text{max}} + b(n_t) + F
\]

$r_{\text{max}}$ is the $y$-intercept of this line
- $b$ is the slope of this line
- $K$ (carrying capacity) is the $x$-intercept of this line
Sometimes useful to think about recruitment, or yield.

What is added to the population at each time step (product of rate and $n$)

Logistic Population Growth (Density dependent growth)

**Gompertz Model:**
Similar to Ricker's except for underlying assumption of how $r$ changes with density.

$$\ln(n_{t+1}) = \ln(n_t) + r_{\text{max}} + b(\ln(n_t)) + F$$

Where:
- $b$ is a parameter measuring the strength of intraspecific competition
- $r_{\text{max}}$ is the populations maximum growth rate in the absence of density dependent competition (what we've dealt with up to now)

Also produces sigmoidal curve
Gompertz model of DD growth:

Constant linear decrease in $r$ as $\ln(n_i)$ increases

\[
\ln(n_{t+1}) / \ln(n_t) = r_{max} + b(\ln(n_t)) + F
\]

$r_{max}$ is the $y$-intercept of this line

$b$ is the slope of this line

$K$ (carrying capacity) is the $x$-intercept of this line

Concave relationship between $r$ and abundance

Where is dd strongest?
Logistic Population Growth (Density dependent growth)

**Theta-logistic model:**
DD growth rate is a function of population size raised to the power of $\theta$

$$\ln(n_{t+1})/\ln(n_t) = r_{\text{max}} + b(n_t^\theta) + F$$

Where:
- $b$ is a parameter measuring the strength of intraspecific competition
- $r_{\text{max}}$ is the population's maximum growth rate in the absence of density dependent competition (what we've dealt with up to now)
- $\theta$ is a parameter describing the curvature of the $r$ and $n$ relationship

The shape of the relationship between growth rate and population size varies such that when $\theta$ is:

- $\theta = 1$
- $0 < \theta < 1$
- $\theta > 1$

**Rickers**
Lots of DD at small numbers
Lots of DD at large numbers