Newton’s law of cooling/heating

**PROBLEM**
A can of coca cola takes 1 hr to cool from 30°C to 20°C in a refrigerator at 10°C.

(a) Find the object’s temperature 30 minutes after it started to cool.

\[ y(t) = T + (y_0 - T) e^{kt} = 10 + (30 - 10) e^{kt} = 10 + 20 e^{kt} \]

measure \( t \) in minutes

\[ y(60) = 10 + 20 e^{60k} = 20 \quad \Rightarrow \quad 20 e^{60k} = 10 \quad \Rightarrow \quad e^{60k} = \frac{1}{2} \]

\[ \Rightarrow \quad 60k = \ln\left(\frac{1}{2}\right) = -\ln 2 \quad \Rightarrow \quad k = -\frac{\ln 2}{60} \]

\[ y(t) = 10 + 20 e^{-\frac{\ln 2}{60} t} = 10 + 20 \cdot 2^{-t/60} \]

\[ y(30) = 10 + 20 \cdot 2^{-30/60} = 10 + 20 \cdot 2^{-1/2} = 10 + \frac{20}{\sqrt{2}} \sim 24°C \]

(b) How long does it take for the soda to cool an additional 6°C (after \( y = 24°C \))?

\[ y(t) = 10 + 20 \cdot 2^{-t/60} = 18 \]

\[ \Rightarrow \quad 20 \cdot 2^{-t/60} = 8 \quad \Rightarrow \quad 2^{-t/60} = 0.4 = e^{-\frac{\ln 2}{60} t} \]

\[ \Rightarrow \quad -\frac{\ln 2}{60} \cdot t = \ln 0.4 \quad \Rightarrow \quad t = 60 \cdot \frac{\ln 0.4}{\ln 0.5} \sim 80 \text{ minutes} \]

**Note**

As \( y(t) \to T \), the rate of cooling decreases.