Math 571 - Functional Analysis I - Fall 2017

Homework 3
Due: Wednesday, September 20, 2017

1. (# 4, Section 1.4) Show that a Cauchy sequence is bounded.

2. (# 5, Section 1.4) Is boundedness of a sequence in a metric space sufficient for
the sequence to be Cauchy? Convergent?

3. (# 5, Section 1.5) Show that the set $X$ of all integers with metric $d$ defined by
$d(m, n) = |m - n|$ is a complete metric space.

4. (# 6, Section 1.5) Show that the set of all real numbers constitutes an incomplete
metric space if we choose

$$d(x, y) = |\arctan x - \arctan y|.$$

5. (# 7, Section 1.5) Let $X$ be the set of all positive integers and $d(m, n) = |m^{-1} - n^{-1}|$. Show that $(X, d)$ is not complete.

6. (# 8, Section 1.5) Show that the subspace $Y \subset C[a, b]$ consisting of all $x \in C[a, b]$ such that $x(a) = x(b)$ is complete.

7. (# 13-14, Section 1.5) Show that in example 1.5-9, another Cauchy sequence is $(x_n)$, where

$$x_n(t) = \begin{cases} 
    n, & \text{if } 0 \leq t \leq n^{-2} \\
    t^{-1/2}, & \text{if } n^{-2} \leq t \leq 1.
\end{cases}$$

Show that this Cauchy sequence does not converge.