1 More about contrasts and interactions

Refer to the computer code for today’s lectures to see how to perform these calculations in SAS.

1.1 Doing 1 way ANOVA as a set of contrasts

The ANOVA null hypothesis for 4 groups $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ is equivalent to the following set of contrasts:

\[
L_1 : \mu_1 - \mu_2 = 0 \\
L_2 : \mu_1 - \mu_3 = 0 \\
L_3 : \mu_1 - \mu_4 = 0
\]

For these three contrasts to be simultaneously equal to zero is equivalent to having all four means equal to each other.

1.2 Contrasts with interactions

For an experiment with factorial treatment structure, with 2 factors each having three levels, we can look at the treatment-combination means in the following arrangement:

\[
\begin{array}{cccc}
\text{B} & 1 & 2 & 3 \\
1 & \mu_{11} & \mu_{12} & \mu_{13} \\
2 & \mu_{21} & \mu_{22} & \mu_{23} \\
3 & \mu_{31} & \mu_{32} & \mu_{33}
\end{array}
\]

1.3 Simple main effects tests

We can express questions of interest regarding an AxB interaction in the form of contrasts. The first example is simple-main-effect tests. If we wish to test the equality of factor A when factor B is held at the first level (in our police training example, we are testing the three patrols when training time is fixed at 5 hours) we wish to test $H_0 : \mu_{11} = \mu_{21} = \mu_{31}$, which is equivalent to the following set of contrasts:

\[
L_1 : \mu_{11} - \mu_{21} = 0 \\
L_2 : \mu_{11} - \mu_{31} = 0
\]

1.4 Treatment-contrast interactions

The problem with simple-main-effect tests is that they involve a combination of main effects and interaction effects. If you repeat the above question for when training time is held at 10 and then 15 hours, the sum of these three simple-main-effect test sum of squares equals SSA + SSAB from the factorial model. Thus these tests are mixing main effects and interaction effects together. An alternative approach is to take a contrast in one factor, and see if it interacts with the other factor. In the police example, it may be of interest to see if the upper-class patrol - middle-class patrol contrast (call it $\psi_{1(A)} = \mu_{11} - \mu_{21} - \mu_{11} - \mu_{31}$) interacts with the training
time. The null hypothesis of no interaction between $\psi_1(A)$ and training is equivalent to the following set of contrasts.

\[
L_1 : (\mu_{11} - \mu_{21}) - (\mu_{12} - \mu_{22}) = 0 \\
L_2 : (\mu_{11} - \mu_{21}) - (\mu_{13} - \mu_{23}) = 0
\]

If we find that this interaction is non-significant, then we may proceed to test the main effect of $\psi_1(A) = \text{upper-class patrol - middle-class globally}$. Several of the ways that the author of our text discusses interaction can addressed in these ways.