1 The Split-plot design (with CRD for whole plot)

Suppose you are studying the effect of two factors on the tenderness of a salmon fillet, 1) the fish’s diet and 2) post-harvest storage method. You obtain a sample of salmon, then for each fish you randomize it to one of three diets. The fish receive this diet for a predetermined period of time. After harvesting the fish, you separate the fillet into four similar-size parts, and randomize them to four different storage methods. At some time later, you obtain a measurement of the tenderness of each part of each fish. This design differs from previous ones that we have studied in that 1) there were two separate randomizations and 2) there are two separate sizes of experimental units. The whole salmon is the experimental unit for the diets, while a part of a salmon is the experimental unit for the storage method. This is an example of a split-plot design, with the fish as the whole plot (diet unit) and the part of the fish as the split plot (storage unit). In some social science applications, the whole-plot factor is called the between-subjects factor, and the split-plot factor is called the within-subjects factor.

As shown in the text, the model equation for the split-plot design is:

\[ y_{ijk} = \mu + \alpha_i + \eta_{k(i)} + \beta_j + \alpha\beta_{ij} + \varepsilon_{k(ij)}, \]

where \( \eta_{k(i)} \) is the whole-plot random error and \( \varepsilon_{k(ij)} \) is the split-plot random error. Note that \( k \) indexes the whole-plot unit, so in the example above \( k \) would be the fish index.

Viewed on its own, the test for the whole-plot effect is just like analyzing a completely randomized design (CRD). In fact, if we average over the split-plot factor (in our example, average the four tenderness measurements for each fish) we can use a CRD analysis to test the whole-plot effect.

Viewed on its own, the split-plot analysis is like doing separate randomized block (RB) analyses in each whole plot and then pooling them. We also obtain a test of the AB (whole-plot \( \times \) split-plot) interaction.

The different nature of these tests shows how the tests for B and AB have more power than the test for A. Another way to think about the split-plot design is as an alternative to the randomized block factorial, so that we do not have to use all of the AB combinations in each block.

It is also possible to have a split-plot design with other whole-plot design structures besides CRD; for example we could use the age of the fish as a blocking factor in the example above.

Although Proc GLM can be used to obtain correct tests for A, B, and AB in the split-plot design, the standard errors for the B and AB treatment means are incorrect and cannot be corrected in Proc GLM. Here we have an example where Proc MIXED is clearly better for data analysis.