Statistics 550  Homework assignment 2

Problems 1-5 are problems 7.4, 8.10, 9.6, 9.10, and 9.15* in the text, respectively.

6. For the simple linear regression model, \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \), verify that the vector partial derivatives of the sum of squares function \( S(b) \) are correct. In other words, write out the scalar expression for \( y' y - (2 y' X b + b' (X' X) b) \) and then verify that the partial derivative equations can be rewritten in the form \( -2X' y + 2X' X b \).

7. For the small data set from lecture 6 in which two values (12 and 36) are recorded for group 1 and two values (54 and 72) are recorded for group 2, consider the model

\[
y_{ij} = \mu + \alpha_j + \epsilon_{ij},
\]

where deviation coding is used for the \( \alpha_j \) parameter (use a full rank approach, so there is just one \( \alpha_j \) parameter). Write down the \( X \) matrix for the model and then use the matrix-based approach to calculate the least-squares estimates \( \hat{\beta} \) and their sampling variances \( \hat{V}(\beta) \) by hand (for estimating \( \hat{V}(\beta) \), use the fact that \( \hat{\sigma}^2 = 225 \)). Specify a matrix \( L \) and a vector \( c \) so that the null hypothesis of no difference between groups can be tested with the \( F \) statistic:

\[
F_0 = \frac{(L b - c)' [L (X' X)^{-1} L']^{-1} (L b - c)}{q S_E^2},
\]

then compute the \( F_0 \) statistic by hand. Verify your results by comparing them to the output from a computer regression analysis.

8. Calculate the moment generating function \( M_X(t) = E e^{tX} \) for the normal distribution with mean \( \mu \) and variance \( \sigma^2 \). Hint: find \( M_X(t) \) for the standard normal distribution and then use the linear transformation result \( M_{aX+b}(t) = e^{bt} M_X(at) \) to obtain the result for the general normal distribution.

* Note in exercise 9.15 that the interaction constraints should be: \( \gamma_{21} = \gamma_{22} = \gamma_{31} = \gamma_{32} = 0 \).