1. State the following theorems. Pay attention to the detail conditions.
   (a). Weierstrass $M$-Test for series of functions.
   (b). Taylor’s Theorem for functions of two variables.
   (c). Fubini’s Theorem.
   (d). Fundamental Theorem of Calculus for line integrals.
   (e). Green’s Theorem – circulation/curl form.

2. Prove or disprove (if true, prove it; if false, find a counterexample.)
   (a). If $\{f_n(x)\}$ is uniformly bounded on $D$, and $\sum_{n=1}^{\infty} |g_n(x)|$ converges uniformly on $D$, then $\sum_{n=1}^{\infty} f_n(x)g_n(x)$ converges uniformly on $D$.
   (b). If $\nabla f(a, b)$ exists, then $\frac{\partial f}{\partial \vec{u}}(a, b) = \nabla f(a, b) \cdot \vec{u}$.
   (c). If $f(x, y)$ is continuous on $(0, 1) \times (0, 1)$, and $g(t) = f(1 - t, t^2)$, then $g(t)$ is continuous on $(0, 1)$.
   (d). Let $\vec{u}$ and $\vec{v}$ be two linear independent vectors. Suppose $D_{\vec{u}} f$ and $D_{\vec{v}} f$ are continuous, then $f$ is differentiable.
   (e). Series $\sum_{n=1}^{\infty} x(1 - x)^n$ does not converge uniformly on $[0, 1]$.

3. Use Taylor series to compute $\lim_{x \to 1} \frac{x - 1}{\ln x}$.

4. If $f(x, y) = x^2 + xy^3$, and $g(x, y) = \ln(x^2 + 1) + e^{xy}$. Find $\nabla f(1, 0) \times \nabla g(1, 0)$.

5. Find the coefficient of the term $(x - 1)^2(y - 2)$ in the third degree Taylor polynomial of $e^{x-xy}$ at $(1, 2)$.

6. Suppose $f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ Find $\lim_{(x,y)\to(0,0)}(x,y), f_x(0,0)$ and $f_y(0,0)$ if they exist.

7. Suppose $f(x, y)$ satisfies $f_{xx} + f_{yy} = 0$. If $x = 2t, y = s^2 - t^2$. Show that $f_{ss} + f_{tt} = 0$.

8. Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin y \frac{y}{y} dy dx$.

9(a). Suppose $\vec{F}(x, y) = \langle y^2 + 3x^2 y - \cos x + 4, x^3 - e^y \rangle$, and $C$: $\gamma(t) = (\cos t, -\sin t), 0 \leq t \leq 2\pi$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.
9(b). Evaluate $\int_{C_1} \vec{F} \cdot d\vec{r}$ if $C_1$: $\gamma(t) = (\cos t, -\sin t), 0 \leq t \leq \pi$.

10. Evaluate $\int_C 3x^2 y dx - 2xy^2 dy$, where $C$ is a trapezoid with corners $(-2, 2), (-1, 1), (1, 1)$ and $(2, 2)$. 