15) \( y'' - 2y' + y = te^t + 4 \)
\( y(0) = 1, \ y'(0) = 1 \)

1. \( y'' - 2y' + y = 0 \)
\( r^2 - 2r + 1 = 0 \)
\( (r-1)^2 = 0 \)
\( r = 1, 1 \)

\( y_1(t) = e^t, \ y_2(t) = te^t \)

2. \( y'' - 2y' + y = te^t \)

Try \( y = (At + B)e^t \)

Note: \( \beta e^t \) is not a solution to the homogeneous equation, but \( \beta e^t \) is not a solution to the non-homogeneous equation.

\( y = \beta e^t \) is not a solution to the non-homogeneous equation.

Try \( y = t(At + B)e^t \)

Note: \( \beta e^t \) is not a solution to the homogeneous equation, but \( \beta e^t \) is not a solution to the non-homogeneous equation.

\( y = (At + B)e^t \)

\( y = At e^t + Bte^t \)

\( y = At e^t + 3Ate^t + Bte^t + 2Bte^t \)

\( y = At e^t + 3Ate^t + 3Ate^t + 6Ate^t + Bte^t + 2Bte^t + 2Bte^t \)

\( y = At e^t + (6A + B)t e^t + 6Ate^t + 4Bte^t + 2Bte^t \)

\( y = At e^t - 6Ate^t - 2Bte^t - 4Bte^t + At e^t + 3Bte^t + 4Bte^t \)

\( y = t e^t \)

\( \omega = 1 \)

\( A = \frac{1}{\omega} \)

\( B = \beta \)

\( Y = \frac{2}{\omega} \left( \frac{1}{\omega} + t \right) e^t \)
(3) \( y'' - 2y' + y = 4 \)  
\[ A = y, \quad y = 0, \quad y'' = 0 \]
\[ A = 4 \]

General solution:
\[ y = C_1 e^t + C_2 te^t + \frac{1}{6} t e^t + 4 \]
\[ y' = C_1 e^t + C_2 e^t + C_2 e^t + \frac{1}{6} e^t + \frac{1}{2} e^t \]

\( y(0) = 1 \); \( 1 = C_1 + 4 \)  \[ C_1 = -3 \]

\( y'(0) = 1 \); \( 1 = C_1 + C_2 \)  \[ C_2 = 4 \]

So:
\[ y = -3e^t + 4te^t + \frac{1}{6} t e^t + 4 \]