Math 275 – Review Questions for Exam 1

1. Describe in words the set of points in $\mathbb{R}^3$ which satisfy the inequality $x^2 + z^2 \leq 4$.

2. Find the center and radius of the sphere $x^2 + y^2 + 4y + z^2 - 4z = 0$.

3. Find the equation of the sphere with diameter between $(3, 0, 2)$ and $(5, -4, -4)$.

4. Find the vector of length 5 with the same direction as the vector $\langle 2, -1, -2 \rangle$.

5. An airplane flies on a heading due east at an airspeed of 150 miles per hour. There is a steady 30 mile per hour wind blowing towards the direction 30 degrees west of due south (so that the wind is blowing slightly against and towards the right of the plane’s heading). Calculate the plane’s heading vector and speed relative to the ground.

6. Compute the cosine of the angle between the vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

7. Find $x$ so that the vectors $\langle x, 2, 2 \rangle$ and $\langle 4, 6, x \rangle$ are orthogonal.

8. Write the vector $\langle 2, 5, -1 \rangle$ as the sum of a vector parallel to $\langle 1, 2, 0 \rangle$ and a vector orthogonal to $\langle 1, 2, 0 \rangle$.

9. Find a unit vector orthogonal to both of the vectors $\langle 4, 2, -1 \rangle$ and $\langle 3, 0, 2 \rangle$.

10. Consider the points $(0, 1, 4)$, $(3, 0, 8)$, and $(2, 2, 2)$.

   (a) Find the area of the triangle formed by these three points.

   (b) Find the equation of the plane through these three points. Write this in the form $ax + by + cz = d$.

11. Find the parametric equation of the line through the points $(2, 1, 1)$ and $(3, 0, -2)$. At what point does this line intersect the plane $x + y + z = 2$?

12. Find the equation of the line at which the planes $x + 2y + z = 4$ and $2x - y + z = 4$ intersect.

13. Find the distance from the point $(3, -1, 2)$ to the plane $x - y + 2z = 5$.

14. Identify the type of the quadric surface $2x^2 - y^2 - 4z^2 + 4 = 0$ and draw a rough sketch of it.

15. Sketch the graph of $\mathbf{r}(t) = \langle t \cos t, t \sin t, t \rangle$ for $t > 0$.

16. Find a tangent vector to the curve $\mathbf{r}(t) = \langle t^2 - 10, 3 - t, \sqrt{t} \rangle$ at the point $(6, -1, 2)$.

17. Compute $\int_1^3 (3t\mathbf{i} + t^{-2}\mathbf{j} - 2\mathbf{k})dt$

18. Find the length of the curve $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j} + (\frac{1}{3}t^3 + t^{-1})\mathbf{k}$, $1 \leq t \leq 3$. 