How to start Macaulay2

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Macaulay2 is a software system for research in algebraic geometry and in commutative algebra. This program is available at Macaulay2 home page:

http://www.math.uiuc.edu/Macaulay2

For the course "Algebraic Geometry" in SMI, Macaulay2 has been already installed, but only in Linux, so take the following steps:

(i) Open a shell window in Windows by clicking the icon SSH Linux on your desktop.

(ii) Then you'll be asked to enter your account name and password and do so (you must give your password twice).

(iii) Start Macaulay2 with the command M2 in the shell window:

```
$ M2
Macaulay 2, version
--Copyright 1993-2001, D. R. Grayson and M. E. Stillman
--Singular-Factory 1.3b, copyright 1993-2001, G.-M. Greuel, et al.
--Singular-Libfac 0.3.2, copyright 1996-2001, M. Messollen
```

Another way to run Macaulay2 is with emacs. This is maybe the best way, because some answers can be very wide. emacs does not wrap output lines and allows you to scroll horizontally to see the rest of the output. The emacs tutorial can be started up with the keystroke C-x H (type Control-H by holding down the control key and press H). Before running Macaulay2 in emacs, you have to do the following two things:

(1) Copy the file .emacs to your home directory as follows:

```
$ cp /usr/share/Macaulay2-0.9.2/emacs/.emacs .
```

(2) Start up emacs with the command emacs. Load the file .emacs with C-x f. At the bottom of this file, you'll find the following sentence:

```
(defvar M2HOME "u/CE/YOUR_HOME_DIR/Macaulay2")
```

Replace

```
YOUR_HOME_DIR
```

by your account name. Then save this file with `C-x s`.

Exit `emacs` with `C-x C-c` and restart `emacs`. Frequently, we start emacs with your file:

```
$ emacs your_file_name
```

The file should have the form `*.m2`, in order to enter a special mode for editing `Macaulay2` code. If you are reading your file with `emacs`, then use the keystrokes `C-x 2`. This divides the buffer into two windows. Then press the `F12` functions key to start up `Macaulay2` in one of these windows. You can switch from one window to the other with `C-x o`. To present each line to `Macaulay2`, position the cursor on this line and press the `F11` function key.

Example: Consider the following two polynomials in $\mathbf{Q}[x]$:

$$f = x^3 + 3x^2 + 5x + 7 \text{ and } g = x^4 + 11x^3 + 2x^2 + x + 5.$$

Question: Do f and g share a root?

The polynomials have been randomly chosen. So we can expect that the set

$$\{f, xf, x^2f, x^3f, g, xg, x^2g\}$$

is a basis for the vector space spanned by all the polynomials whose degrees are less than or equal to $3 + 4 - 1 = 6$. In this case, f and g share no root by Problem 2 in Problem Set 1 . We would like to check whether or not this is correct.

One way is to compute the resultant of these polynomial with respect to x and show that this is not equal to 0. Another way uses the following proposition:

Proposition. Let $\mathbf{k}$ be a field and let $f, g \in \mathbf{k}[x]$. Then f and g have no common factor if and only if $1 \in \mathbf{k}[x]$ can be written as a $\mathbf{k}[x]$-linear combination of f and g.

Thus it suffices to show that the ideal $(1) \subset \mathbf{k}[x]$ generated by 1 is contained in the ideal $I \subset \mathbf{k}[x]$ generated by f and g. Needless to say, (1) contains I, and thus $I = (1)$. Let's check this with `Macaulay2`! We start by setting up the ring of polynomials with rational coefficients:

```
i1 : S=QQ[x]

o1 = S

o1 : PolynomialRing
```

Then we define the ideal `I` generated by f and g and the ideal `J` generated by 1:

```
i2 : I=ideal(x^3+3*x^2+5*x+7,x^4+11*x^3+2*x^2+x+5)

             3     2              4      3     2
o2 = ideal (x  + 3x  + 5x + 7, x  + 11x  + 2x  + x + 5)

o2 : Ideal of S

i3 : J=ideal(1_S)

o3 = ideal 1

o3 : Ideal of S
```

The equality will be tested with the command `==` by checking whether two sets of generators produce the same ideal (this involves a computation with Gröbner bases):

```
i4 : I==J

o4 = true
```

So $I = (1)$. The answer to the question is, therefore, No, as we expected.