Problem Set 7

The picture below lies in $\mathbb{A}^2_{\mathbb{R}}$. Each of the 3 darker line segments in the picture represents a steel bar of length 1. The other two line segments have length 2. Let $A = (0,0), B = (0,2), C = (x_1, y_1)$ and $D = (x_2, y_2)$. Points A and B cannot move but points C and D can move. The steel bars are connected with hinges. Similar to the last problem set, as the three bars move into every allowable position, the point M sweeps out a curve. This curve is an irreducible affine variety given as $V(F)$ for some polynomial $F \in \mathbb{R}[X,Y]$. In the problems following the picture, you will compute F (almost) using Macaulay 2 and elimination theory.

Problem 1. *Try to give a rough sketch of the curve traced out by M.*

Now we will construct an ideal, I, in $\mathbb{R}[X, Y, x_1, x_2, y_1, y_2]$ which represents all allowable configurations and the corresponding positions of M. Each bar yields a constraint on the variables x_1, x_2, y_1, y_2, this yields 3 quadratic polynomials. Let $M = (X, Y)$ and write down the coordinates of M in terms of x_1, x_2, y_1, y_2, this yields 2 more quadratic polynomials. Let I be the ideal generated by the five quadratic.

Problem 2. *Write out the equations for I.*

We would like to know all of the allowable values of X and Y. This corresponds to computing $J = I \cap \mathbb{R}[X, Y]$. If you carry out this computation in Macaulay 2 you will obtain an ideal with a single generator, F. F is not quite the equation of the curve, $F = GH$ with G, H irreducible. One of the factors is the equation of the curve.

Problem 3. *What is the other factor of F? Can you find the equation of F?*