Problem Set 15

Problem 1. *Let H be a hexagon. The intersections of the opposite sides of H determine 3 points A, B, C. Prove that if A, B, C lie on a line then the vertices of the hexagon lie on a conic.*

Problem 2. *Let C be an irreducible curve of degree 2 in $\mathbb{P}^2$. Prove that the dual of C is a conic.*

Problem 3. *State the dual form of Pascal's theorem.*

Problem 4. *State the dual form of Pappus's theorem.*

Problem 5. *Give a brief explanation of why there is not a conic passing through 6 general points in $\mathbb{P}^2$.*

Problem 6. *Let $P_1, P_2, \ldots, P_6$ be 6 points on an irreducible curve, C, of degree 2 in $\mathbb{P}^2$. Let L_{ij} denote the line passing through points P_i and P_j. Let $Q_1 = L_{12} \cap L_{34}, Q_2 = L_{23} \cap L_{45}, Q_3 = L_{34} \cap L_{56}, Q_4 = L_{45} \cap L_{61}, Q_5 = L_{56} \cap L_{12}, Q_6 = L_{16} \cap L_{23}$. Prove that there is a conic passing through $Q_1, \ldots, Q_6$.*

Problem 7. *Let C and D be curves in $\mathbb{P}^2$ of degree n. Suppose C and D meet in exactly n^2 points. Suppose there is a curve of degree $m < n$ which contains mn of the n^2 points. Use Bezout's thereom to prove that there is a curve F of degree $n - m$ which contains the remaining $n^2 - nm$ points.*