# Problem Set 17

There was a typo on Problem 4 from Problem Set 16. It should read as:

**Problem 1.** *Let $C$ be the non singular cubic curve in $\mathbb{P}^2$ determined by the equation $y^2z = x^3 + ax^2z + bxz^2 + cz^3$. Let $0 = [0 : 1 : 0]$. Let $P_i = [x_i : y_i : 1]$ for $i = 1, 2, 3$. Suppose $P_1 \oplus P_2 = P_3$. If $x_1 \neq x_2$ then let $\lambda = (y_1 - y_2)/(x_1 - x_2)$. If $P_1 = P_2$ and $y_1 \neq 0$, let $\lambda = (3x_1^2 + 2ax_1 + b)/(2y_1)$. Let $\mu = y_i - \lambda x_i$ (for $i = 1$ or 2). Show that $x_3 = \lambda^2 - a - x_1 - x_2$ and $y_3 = -\lambda x_3 - \mu$.*

Problem 5 (and problem 6 with a bit of work) of the previous problem set can be done without the aid of the problem above. It can even be done on the computer by using ideal quotients.

**Problem 2.** *Let $C$ be an irreducible cubic. Let $L$ be a line such that $L \cdot C = P_1 + P_2 + P_3$ with the $P_i$ distinct. Let $L_i$ be the tangent line to $C$ at $P_i$ (So $L_i \cdot C = 2P_i + Q_i$ for some $Q_i$). Show that $Q_1, Q_2, Q_3$ lie on a line.*

**Problem 3.** *Let $C$ be a nonsingular cubic. Let 0 be a flex point of $C$. Let $P_1, P_2, \ldots, P_{3m} \in C$. Show that $P_1 \oplus P_2 \oplus \cdots \oplus P_{3m} = 0$ if and only if there is a curve $F$ of degree $m$ such that $F \cdot C = \sigma_{i=1}^{3m} P_i$. See the bottom of the page if you want a hint. If you don't want a hint then you can probably figure out that you shouldn't look at the bottom of the page.*

A set of points is said to be in $d$-**general position** if *every* subset of the points impose independent conditions on polynomials of degree $d$ (or if there are no degree $d$ polynomials passing through the subset of points). A set of points is said to be in **general position** if it is in $d$-general position for every $d$.

**Problem 4.** *Show that the following set of points is not in 3-general position:* $[0 : 0 : 1], [0 : 1 : 1], [1 : 0 : 1], [1 : 1 : 1], [0 : 2 : 1], [2 : 0 : 1], [1 : 2 : 1], [2 : 1 : 1], [2 : 2 : 1]$.

*Hint: Use induction. Let $L \cdot C = P_1 + P_2 + Q$, $L' \cdot C = P_3 + P_4 + R$, $L'' \cdot C = Q + R + S$. Apply induction to $S, P_5, \ldots, P_{3m}$.*