Homework 1 (Math461 EO)

Notation:

- $\mathbb{N}$ = the set of positive integers.
- $\mathbb{R}$ = the set of real numbers.
- $|x|$ = the absolute value of $x \in \mathbb{R}$.

Problem 1 (2 points)

Let $A = \{\ 4n \mid n \in \mathbb{N}\ \}$ and let $B = \{\ 6m \mid m \in \mathbb{N}\ \}$. Find $A \cap B$.

Problem 2 (2 points)

Let f be the map from $\mathbb{R}$ to $(-1, 1) = \{\ x \in \mathbb{R} \mid -1 < x < 1\ \}$ defined by

$$f(x) = \frac{x}{1 + |x|} \text{ for evey } x \in \mathbb{R}.$$

(a) Prove that f is one-to-one.

(b) Determine whether or not f is onto. Justify your answer.

Problem 3 (2 points)

For any set A, the *power set* of A, denoted $\mathcal{P}(A)$, is the set of all subsets of A (i.e., $\mathcal{P}(A) = \{\ X \mid X \subseteq A\ \}$). Let $A = \{1, 2, 3\}$. Find $\mathcal{P}(A)$.

Note. The empty set $\emptyset$ can be thought of as a subset of any set.

Problem 4 (2 points)

Let A, B and C be non-empty sets and let $f : A \to B$, $g : B \to C$ be maps. Prove that if the composite $g \circ f$ is one-to-one, then f is one-to-one.

Problem 5 (2 points)

Let $A = \{1, 2, 3, 4\}$ and let $\mathcal{P}(A)$ be its power set. Define a relation R on $\mathcal{P}(A) \setminus \{\emptyset\}$ by xRy if and only if $x \cap y \neq \emptyset$. Determine whether R is reflexive, symmetric, or transitive.