

## Homework 2 (Math461 EO)

### Problem 1 (4 points)

Let  $a, b \in \mathbb{Z}$  with  $b \neq 0$  and let  $r$  be the remainder in the division of  $a$  by  $b$ . Prove that  $\gcd(a, b) = \gcd(b, r)$ .

### Problem 2 (2 points)

Use the Euclidean Algorithm to find  $d = \gcd(8767, 252)$  and integers  $s, t$  such that  $d = 8767s + 252t$ .

### Problem 3 (4 points)

Find a solution  $x \in \mathbb{Z}$ ,  $0 \leq x < n$ , for each of the congruence  $ax \equiv b \pmod{n}$  in (i) and (ii). Note that in each case,  $a$  and  $n$  are relatively prime.

(i)  $2x \equiv 3 \pmod{7}$ .

(ii)  $55x \equiv 59 \pmod{42}$ .

### Bonus Problem (2 points)

If  $m$  is an integer, show that  $m^2$  is congruent module 8 to one of the integers 0, 1, or 4.