Homework 6 (Math461 EO)

Problem 1 (2.5 points)

Find all the subgroups of the cyclic group $G = \langle a \rangle$ of order 45.

Problem 2 (2.5 points)

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 2 & 6 & 7 & 4 & 9 & 1 & 5 \end{pmatrix} \in S_9$.

(i) Express σ as a product of disjoint cycles.

(ii) Express σ^{-1} as a product of disjoint cycles.

Problem 3 (2.5 points)

Let $\sigma = (123)(145) \in S_5$. Write the cycle decomposition of σ^{99}.

Problem 4 (2.5 points)

(i) Let S_A be the group of permutations on a non-empty set A. Let $a \in A$ and define

$$\text{stab}(a) = \{\sigma \in S_A \mid \sigma(a) = a\}.$$

Prove that $\text{stab}(a) \leq S_A$.

(ii) Let $H = \{\sigma \in S_5 \mid \sigma(1) = 1 \text{ and } \sigma(3) = 3\}$. Prove or disprove that $H \leq S_5$.