Homework 10 (Math462)

Problem 1 (3 points)

Let F be a field and let $p(x) \in F[x]$ be irreducible over F. Suppose that E is an extension of F that contains a zero α of $p(x)$. Consider the ring homomorphism $\varphi : F[x] \to E$ defined by $\varphi(f(x)) = f(\alpha)$ for every $f(x) \in F[x]$. Prove that $\ker(\varphi) = (p(x))$.

Problem 2 (4 points)

Find the minimal polynomial for $\sqrt{2}+\sqrt{3}$ over $\mathbb{Q}$. What is $\left[\mathbb{Q}(\sqrt{2}+\sqrt{3}) : \mathbb{Q}\right]$?
Hint. $(\sqrt{2}+\sqrt{3})^2 = \sqrt{5+2\sqrt{6}}$, $(5+2\sqrt{6})(5-2\sqrt{6}) = 1$ and $5+2\sqrt{6}+(5-2\sqrt{6}) = 10$.

Problem 3 (3 points)

Let F be a field, let E be an extension of F and let α, β be elements of E. Prove that $F(\alpha, \beta) = (F(\alpha))(\beta)$.