Homework 7 (Math462)

Problem 1 (3 points)

Let $f(x) = x^3 + 2x^2 - 3x + 4 \in \mathbb{Z}_5[x]$. Determine whether or not $f(x)$ is irreducible over $\mathbb{Z}_5$. Justify your answer.

Problem 2 (4 points)

Find all monic irreducible polynomials of degree 2 over $\mathbb{Z}_3$.

Hint. A polynomial $f(x)$ is said to be *monic* if its leading coefficient is 1.

Problem 3 (3 points)

Let $f(x)$ and $g(x)$ be two polynomials over a field F, both of degree n or less. Prove that if $m > n$ and if there exist m distinct elements $c_1, \ldots, c_m$ of F such that $f(c_i) = g(c_i)$ for every $i \in \{1, \ldots, m\}$, then $f(x) = g(x)$.

Hint. Use Corollary 1.9 (or Corollary 3 in the textbook page 297).