An Approximate F test example

In our TV show example, the factor year is a random effect and the factors network and type are fixed effects. The data are not completely balanced, but we will use it to illustrate the calculations for an approximate F test. To correspond to our earlier notation, we will use factor A = network, factor B = type, and factor C = year for these data. The output from a standard three-way ANOVA shows $MS_C = 278.06$, $MS_{AC} = 15.97$, $MS_{BC} = 11.65$, and $MS_{ABC} = 6.91$, with degrees of freedom of $7, 14, 7$, and 14 respectively. The linear combination of mean squares equals:

$$M = MS_{AC} + MS_{BC} - MS_{ABC} = 15.97 + 11.65 - 6.91 = 20.71,$$

and its degrees of freedom is:

$$\nu = \frac{M^2}{\sum \frac{(a_i MS_i)^2}{\nu_i}} = \frac{A}{B}$$

where

$$A = 20.71^2$$

and

$$B = \sum \frac{(a_i MS_i)^2}{\nu_i} = \frac{(1 * 15.97)^2}{14} + \frac{(1 * 11.65)^2}{7} + \frac{(-1 * 6.91)^2}{14} = 41.02.$$

So our F statistic is

$$F = \frac{MS_C}{MS_{AC} + MS_{BC} - MS_{ABC}} = \frac{278.06}{20.71} = 13.43$$

on degrees of freedom of 7 and $\nu = A/B = 20.71^2/41.02 = 428.90/41.02 = 10.46$.

The output from the SAS program that uses a RANDOM /TEST statement includes a table of expected mean squares and results for the denominator mean square, the degrees of freedom for the denominator mean square, and the F value that nearly match those shown above. The small difference is due to the fact that our data were not completely balanced, so the coefficients that are actually used are slightly different than those in our EMS table.