1 More on analysis of factorial experiments

Some additional topics covered in Chapter 8 of the text include parameter constraints for the effects model, analysis of experiments with only a single replicate per treatment combination, the use of pooling mean squares to add to the error mean square, and the concept of hierarchy. The concept of hierarchy is very important in understanding the analysis of unbalanced factorial data. A model satisfies the principle of hierarchy if whenever an interaction occurs, all lower-order terms in the interaction are also in the model.

2 Interpreting interactions in factorial experiments

There are a variety of approaches to understanding interactions in factorial experiments. An essential starting point is the examination of profile plots to try to visualize the interactions. I will summarize three approaches in order of complexity: 1) Description using profile plots, 2) Simple main-effect tests, and 3) Decomposition of interaction sum of squares by contrasts. We will use Example 9.2 from the text to illustrate these approaches. The original design has four factors, A, B, C, and D, each at two levels. In showing the profile plot in Figure 9.1 the author converts the design into two factors, each having four levels. We will use this form of the data, using E as the factor combining factors A and B, and using F as the factor combining factors C and D.

2.1 Description using profile plots

The simplest approach here is just to show the profile plot to describe the interaction. Figure 9.1 in the text gives a good illustration of the interaction. In Figure 9.1 the leftmost column of means (when A and B are both 'low') shows a distinctly different ordering of means. Some researchers distinguish between orderly interactions, where lines are not parallel but do not cross, and disorderly interactions, where lines in the profile plot cross. Aside from pointing out the pattern in the profile plot, we could informally document it by running a separate analysis without the 'low-low' combinations of A and B and showing that the resulting combinations do not have interaction between the factors.

2.2 Simple main-effect tests

A more computationally intensive approach is to do essentially one-way ANOVAs across one factor for each separate level of the other factor. If the results for the first factor change as we change the second factor, that can provide some detail about the nature of the interaction. An advantage of this approach is that it provides more detail about the interaction. Disadvantages are that the tests are combining main-effect and interaction effects, and the possible need to conduct multiple test adjustments.

2.3 Decomposing interaction SS using contrasts

Earlier in the course we saw that for one-way ANOVA we could find a set of orthogonal contrasts and they would decompose the treatment sum of squares. Decomposing the treatment sum of squares means that they would precisely divide the treatment SS into separate (independent) parts that add up to the treatment SS. We can do the same for an interaction sum of squares, so that we can very precisely partition the interaction effect into smaller parts. The data from Example 9.2 in the text provide a dramatic example. The advantage of this approach is that we can very precisely pinpoint reasons for interaction, including the 'one-cell interaction' example from the text. The disadvantage of this approach is that is takes more effort to correctly specify these contrasts, and the possible need to use multiple test adjustments.