The Delta Method

The delta method is a way of estimating approximate asymptotic standard errors for differentiable nonlinear functions of parameters. For example, in a generalized linear model, suppose we are interested in a nonlinear function $f(\beta)$ of the regression vector β. Then a first-order Taylor approximation is:

$$f(\widehat{\beta}) \approx f(\beta) + (\widehat{\beta}_0 - \beta_0)\frac{\partial f}{\partial \widehat{\beta}_0} + \cdots + (\widehat{\beta}_k - \beta_k)\frac{\partial f}{\partial \widehat{\beta}_k}.$$

From this we can see that the approximate variance of $\widehat{\gamma} \equiv f(\widehat{\beta})$ is just:

$$\widehat{V}(\widehat{\gamma}) \approx \sum_{j=0}^{k}\sum_{j'=0}^{k} v_{jj'} \frac{\partial \widehat{\gamma}}{\partial \widehat{\beta}_j} \frac{\partial \widehat{\gamma}}{\partial \widehat{\beta}_{j'}},$$

where $v_{jj'}$ is the j, j' th element of the estimated asymptotic covariance matrix of the coefficients $\widehat{V}(\widehat{\beta})$.Another way to think about this expression from the initial Taylor series is that we are adding up all the variances of $\widehat{\beta}_j$ terms (multiplied by appropriate derivatives squared) plus 2 times all of the covariance terms between each $\widehat{\beta}_j$, $\widehat{\beta}_{j'}$ pair (again, multiplied by derivative terms). We will look at one or more examples of using the Delta method.