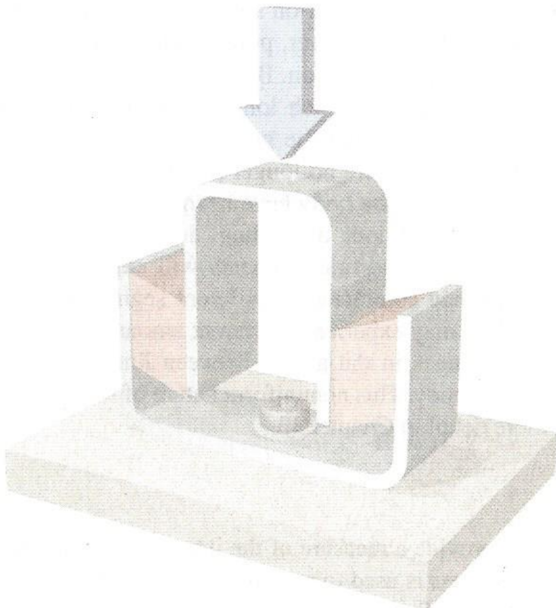
CHAPTER 2

Strain



2.1 Displacement, Deformation, and the Concept of Strain

In the design of structural elements or machine components, the deformations sustained by the body because of applied loads often represent a design consideration equally as important as stress. For this reason, the nature of the deformations sustained by a real deformable body as a result of internal stress will be studied, and methods for calculating deformations will be established.

Displacement

When a system of loads is applied to a machine component or structural element, individual points of the body generally move. This movement of a point with respect to some convenient reference system of axes is a vector quantity known as a **displacement**. In some instances, displacements are associated with a translation and/or rotation of the body as a whole. The size and shape of the body are not changed by this type of displacement, which is termed a **rigid-body displacement**. In Figure 2.1*a*, consider points H and K on a solid body. If the body is displaced (both translated and rotated), points H and K will move to new locations H' and K'. The position vector between H' and K', however, has the same length as the

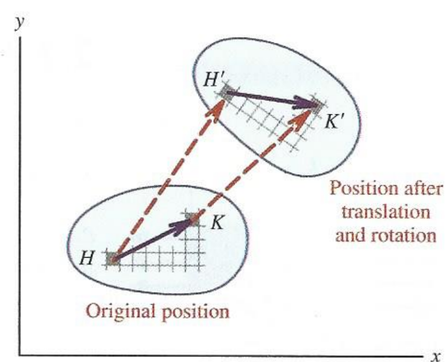


FIGURE 2.1a Rigid-body displacement.

position vector between H and K. In other words, the orientation of H and K relative to each other does not change when a body undergoes a displacement.

Deformation

When displacements are caused by an applied load or a change in temperature, individual points of the body move relative to each other. The change in any dimension associated with these load- or temperature-induced displacements is known as **deformation**. Figure 2.1b shows a body both before and after a deformation. For simplicity, the deformation shown in the figure is such that point H does not change location; however, point K on the undeformed body moves to location K' after the deformation. Because of the deformation, the position vector between H and K' is much longer than the HK vector in the undeformed body. Also, notice that the grid squares shown on the body before deformation (Figure 2.1a) are no longer squares after the deformation: Both the size and the shape of the body have been altered by the deformation.

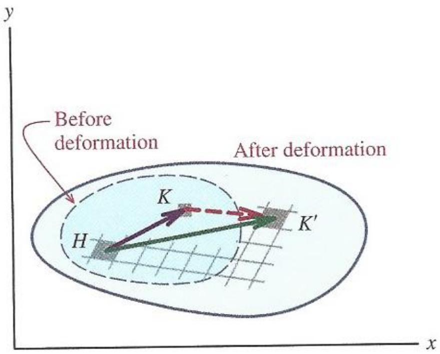


FIGURE 2.1*b* Deformation of a body.

Under general conditions of loading, deformations will not be uniform throughout the body. Some line segments will experience extensions, while others will experience contractions. Different segments (of the same length) along the same line may experience different amounts of extension or contraction. Similarly, changes in the angles between line segments may vary with position and orientation in the body. This nonuniform nature of load-induced deformations will be investigated in more detail in Chapter 13.

Strain

Strain is a quantity used to provide a measure of the intensity of a deformation (deformation per unit length), just as stress is used to provide a measure of the intensity of an internal force (force per unit area). In Sections 1.2 and 1.3, two types of stresses were defined: normal stresses and shear stresses. The same classification is used for strains. **Normal strain**, designated by the Greek letter ε (epsilon), is used to provide a measure of the elongation or contraction of an arbitrary line segment in a body after deformation. **Shear strain**, designated by the Greek letter γ (gamma), is used to provide a measure of angular distortion (change in the angle between two lines that are orthogonal in the undeformed state). The deformation, or strain, may be the result of a change in temperature, of a stress, or of some other physical phenomenon, such as grain growth or shrinkage. In this book, only strains resulting from changes in stress or temperature are considered.

2.2 Normal Strain

Average Normal Strain

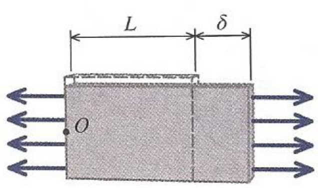


FIGURE 2.2 Normal strain.

The deformation (change in length and width) of a simple bar under an axial load (see Figure 2.2) can be used to illustrate the idea of a normal strain. The average normal strain ε_{avg} over the length of the bar is obtained by dividing the axial deformation δ of the bar by its initial length L; thus,

$$\varepsilon_{avg} = \frac{\delta}{L} \tag{2.1}$$

Accordingly, a positive value of δ indicates that the axial member gets longer, and a negative value of δ indicates that the axial member gets shorter (termed *contraction*).

A normal strain in an axial member is also termed an **axial strain**.

Normal Strain at a Point

In those cases in which the deformation is nonuniform along the length of the bar (e.g., a long bar hanging under its own weight), the average normal strain given by Equation (2.1) may be significantly different from the normal strain at an arbitrary point O along the bar. The normal strain at a point can be determined by decreasing the length over which the actual deformation is measured. In the limit, a quantity defined as the normal strain at the point $\varepsilon(O)$ is obtained. This limit process is indicated by the expression

$$\varepsilon(O) = \lim_{\Delta L \to 0} \frac{\Delta\delta}{\Delta L} = \frac{d\delta}{dL} \tag{2.2}$$

Strain Units

Equations (2.1) and (2.2) indicate that normal strain is a dimensionless quantity; however, normal strains are frequently expressed in units of in./in., mm/mm, m/m, μin./in., μm/m, or με. The symbol μ in the context of strain is spoken as "micro," and it denotes a factor of 10^{-6}. The conversion from dimensionless quantities such as in./in. or m/m to units of "microstrain" (such as μin./in., μm/m, or με) is

$$1\,\mu\varepsilon = 1 \times 10^{-6}\text{ in./in.} = 1 \times 10^{-6}\text{ m/m}$$

Since normal strains are small, dimensionless numbers, it is also convenient to express strains in terms of *percent*. For most engineered objects made from metals and alloys, normal strains seldom exceed values of 0.2%, which is equivalent to 0.002 m/m.

Measuring Normal Strains Experimentally

Normal strains can be measured with a simple loop of wire called a **strain gage**. The common strain gage (Figure 2.3) consists of a thin metal-foil grid that is bonded to the surface of a machine part or a structural element. When loads or temperature changes are imposed, the object being tested elongates or contracts, creating normal strains. Since the strain gage is bonded to the object, it undergoes the same strain as the object. As the strain gage elongates or contracts, the electrical resistance of the metal-foil grid changes proportionately. The relationship between strain in the gage and its corresponding change in resistance is predetermined by the strain gage manufacturer through a calibration procedure for each type of gage. Consequently, the precise measurement of changes in resistance in the gage serves as an indirect measure of strain. Strain gages are accurate and extremely sensitive, enabling normal strains as small as 1 με to be measured. Applications involving strain gages will be discussed in more detail in Chapter 13.

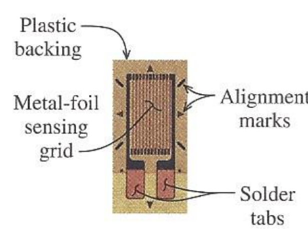


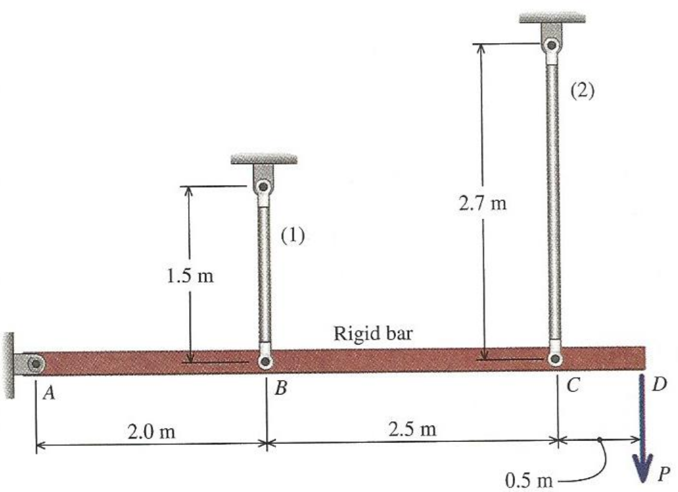
FIGURE 2.3

Sign Conventions for Normal Strains

From the definitions given by Equation (2.1) and Equation (2.2), normal strain is positive when the object elongates and negative when the object contracts. In general, elongation will occur if the axial stress in the object is tension. Therefore, positive normal strains are referred to as *tensile strains.* The opposite will be true for compressive axial stresses; therefore, negative normal strains are referred to as *compressive strains.*

In developing the concept of normal strain through example problems and exercises, it is convenient to use the notion of a **rigid bar**. A rigid bar is meant to represent an object that undergoes no deformation of any kind. Depending on how it is supported, the rigid bar may translate (i.e., move up/down or left/right) or rotate about a support location (see Example 2.1), but it does not bend or deform in any way regardless of the loads acting on it. If a rigid bar is straight before loads are applied, then it will be straight after loads are applied. The bar may translate or rotate, but it will remain straight.

EXAMPLE 2.1



A rigid bar *ABCD* is pinned at *A* and supported by two steel rods connected at *B* and *C*, as shown. There is no strain in the vertical rods before load *P* is applied. After load *P* is applied, the normal strain in rod (2) is 800 με. Determine

(a) the axial normal strain in rod (1).
(b) the axial normal strain in rod (1) if there is a 1 mm gap in the connection between the rigid bar and rod (2) before the load is applied.

Plan the Solution

For this problem, the definition of normal strain will be used to relate strain and elongation for each rod. Since the rigid bar is pinned at *A*, it will rotate about the support; however, it will remain straight. The deflections at points *B*, *C*, and *D* along the rigid bar can be determined by similar triangles. In part (b), the 1 mm gap will cause an increased deflection in the rigid bar at point *C*, and this deflection will in turn lead to increased strain in rod (1).

SOLUTION

(a) The normal strain is given for rod (2); therefore, the deformation in that rod can be computed as follows:

$$\varepsilon_2 = \frac{\delta_2}{L_2} \qquad \therefore \delta_2 = \varepsilon_2 L_2 = (800\ \mu\varepsilon)\left[\frac{1\text{ mm/mm}}{1{,}000{,}000\ \mu\varepsilon}\right](2{,}700\text{ mm}) = 2.16\text{ mm}$$

Note that the given strain value ε_2 must be converted from units of με into dimensionless units (i.e., mm/mm). Since the strain is positive, rod (2) elongates.

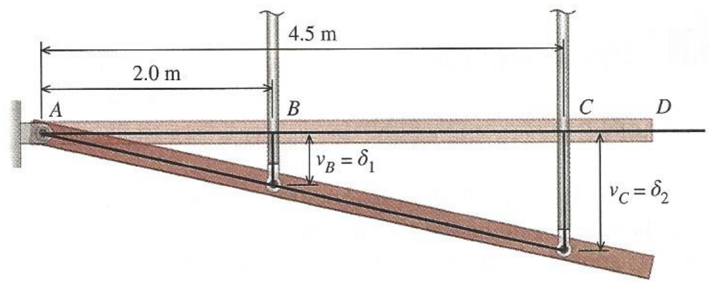
Because rod (2) is connected to the rigid bar and because rod (2) elongates, the rigid bar must deflect 2.16 mm downward at joint *C*. However, rigid bar *ABCD* is supported by a pin at joint *A*, so deflection is prevented at its left end. Therefore, rigid bar *ABCD* rotates about pin *A*. Sketch the configuration of the rotated rigid bar, showing the deflection that takes place at *C*. Sketches of this type are known as **deformation diagrams**.

Although the deflections are very small, they have been greatly exaggerated here for clarity in the sketch. For problems of this type, the small-deflection approximation

$$\sin\theta \approx \tan\theta \approx \theta$$

is used, where θ is the rotation angle of the rigid bar in radians.

To distinguish clearly between elongations that occur in the rods and deflections at locations along the rigid bar, rigid-bar *transverse deflections* (i.e., deflections up or down in this case) will be denoted by the symbol v. Therefore, the rigid-bar deflection at joint C is designated v_C.



We will assume that there is a perfect fit in the pin connection at joint C; therefore, the rigid-bar deflection at C is equal to the elongation that occurs in rod (2) ($v_C = \delta_2$).

From the deformation diagram of the rigid-bar geometry, the rigid-bar deflection v_B at joint B can be determined from **similar triangles**:

$$\frac{v_B}{2.0\text{ m}} = \frac{v_C}{4.5\text{ m}} \qquad \therefore\ v_B = \frac{2.0\text{ m}}{4.5\text{ m}}(2.16\text{ mm}) = 0.96\text{ mm}$$

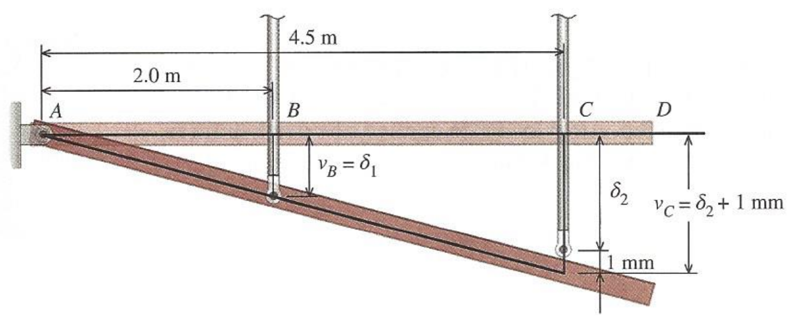
If there is a perfect fit in the connection between rod (1) and the rigid bar at joint B, then rod (1) elongates by an amount equal to the rigid-bar deflection at B; hence, $\delta_1 = v_B$. Knowing the deformation produced in rod (1), we can now compute its strain:

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{0.96\text{ mm}}{1{,}500\text{ mm}} = 0.000640\text{ mm/mm} = 640\ \mu\varepsilon$$ **Ans.**

(b) As in part (a), the deformation in the rod can be computed as

$$\varepsilon_2 = \frac{\delta_2}{L_2} \qquad \therefore\ \delta_2 = \varepsilon_2 L_2 = (800\,\mu\varepsilon)\left[\frac{1\text{ mm/mm}}{1{,}000{,}000\ \mu\varepsilon}\right](2{,}700\text{ mm}) = 2.16\text{ mm}$$

Sketch the configuration of the rotated rigid bar for case (b). In this case, there is a 1 mm gap between rod (2) and the rigid bar at C. Because of this gap, the rigid bar will deflect 1 mm downward at C before it begins to stretch rod (2). The total deflection of C is made up of the 1 mm gap plus the elongation that occurs in rod (2); hence, $v_C = 2.16\text{ mm} + 1\text{ mm} = 3.16\text{ mm}$.



As before, the rigid-bar deflection v_B at joint B can be determined from similar triangles:

$$\frac{v_B}{2.0\text{ m}} = \frac{v_C}{4.5\text{ m}} \qquad \therefore\ v_B = \frac{2.0\text{ m}}{4.5\text{ m}}(3.16\text{ mm}) = 1.404\text{ mm}$$

Since there is a perfect fit in the connection between rod (1) and the rigid bar at joint B, it follows that $\delta_1 = v_B$, and the strain in rod (1) can be computed:

$$\varepsilon_1 = \frac{\delta_1}{L_1} = \frac{1.404\text{ mm}}{1{,}500\text{ mm}} = 0.000936\text{ mm/mm} = 936\ \mu\varepsilon$$ **Ans.**

Compare the strains in rod (1) for cases (a) and (b). Notice that a very small gap at C caused the strain in rod (1) to increase markedly.

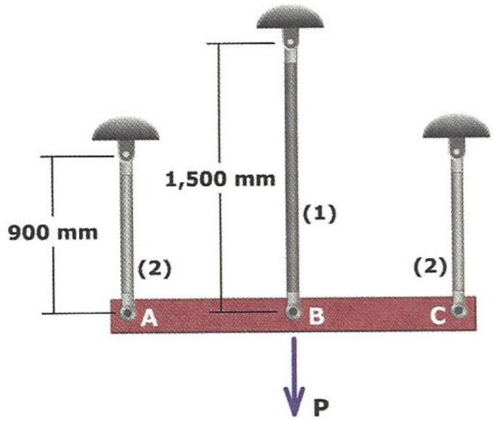
MecMovies

EXAMPLES

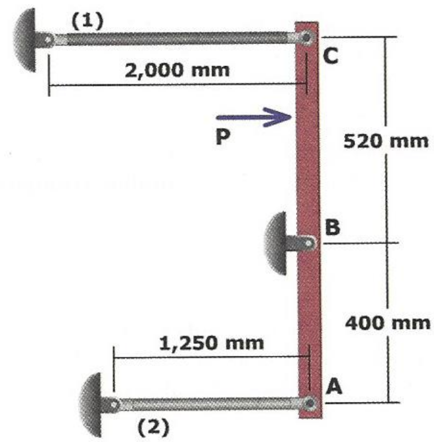
M2.1 A rigid steel bar *ABC* is supported by three rods. There is no strain in the rods before load *P* is applied. After load *P* is applied, the axial strain in rod (1) is 1,200 με.

(a) Determine the axial strain in rods (2).

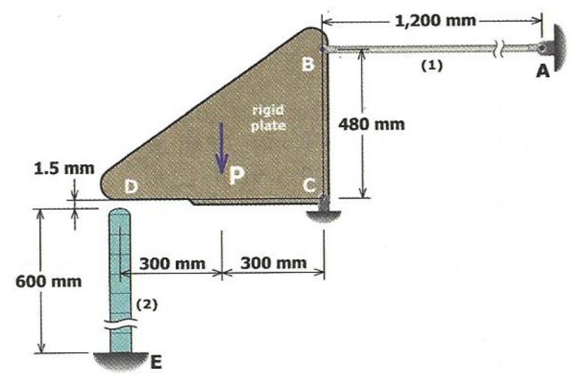
(b) Determine the axial strain in rods (2) if there is a 0.5 mm gap in the connections between rods (2) and the rigid bar before the load is applied.



M2.2 A rigid steel bar *ABC* is pinned at *B* and supported by two rods at *A* and *C*. There is no strain in the rods before load *P* is applied. After load *P* is applied, the axial strain in rod (1) is +910 με. Determine the axial strain in rod (2).



M2.4 The load *P* produces an axial strain of −1,800 με in post (2). Determine the axial strain in rod (1).



EXERCISES

M2.1 A rigid horizontal bar *ABC* is supported by three vertical rods. There is no strain in the rods before load *P* is applied. After load *P* is applied, the axial strain is a specified value. Determine the deflection of the rigid bar at *B* and the normal strain in rods (2) if there is a specified gap between rod (1) and the rigid bar before the load is applied.

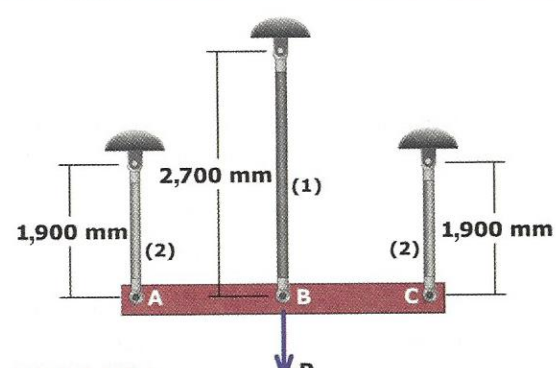


FIGURE M2.1

M2.2 A rigid steel bar AB is pinned at A and supported by two rods. There is no strain in the rods before load P is applied. After load P is applied, the axial strain in rod (1) is a specified value. Determine the axial strain in rod (2) and the downward deflection of the rigid bar at B.

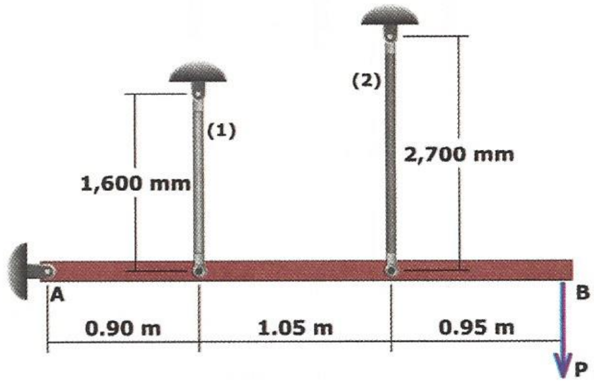


FIGURE M2.2

M2.3 Use normal-strain concepts for four introductory problems using these two structural configurations.

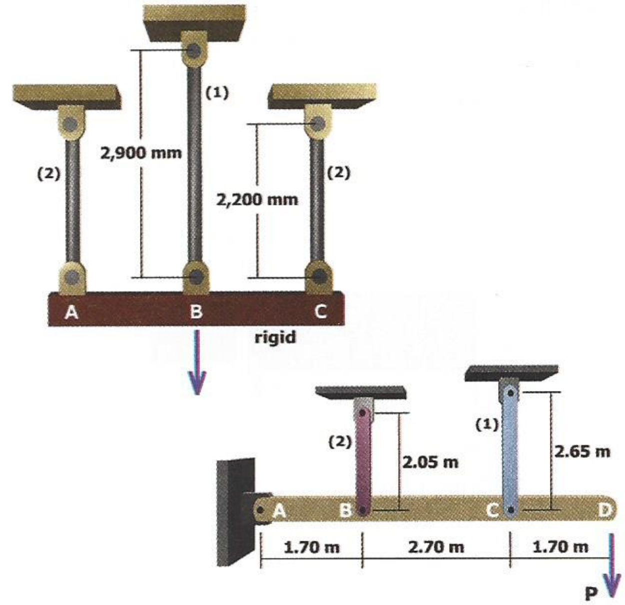


FIGURE M2.3

PROBLEMS

P2.1 When an axial load is applied to the ends of the two-segment rod shown in Figure P2.1, the total elongation between joints A and C is 7.5 mm. The segment lengths are $a = 1.2$ m and $b = 2.8$ m. In segment (2), the normal strain is measured as 2,075 μm/m. Determine

(a) the elongation of segment (2).
(b) the normal strain in segment (1) of the rod.

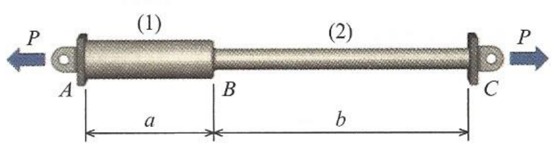


FIGURE P2.1

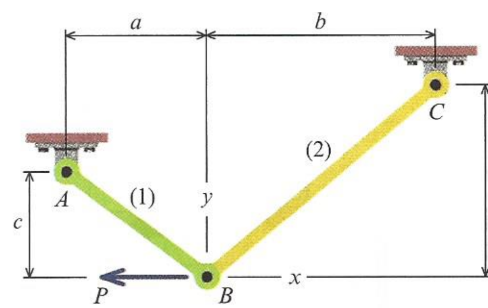


FIGURE P2.2

P2.2 The two bars shown in Figure P2.2 are used to support load P. When unloaded, joint B has coordinates (0, 0). After load P is applied, joint B moves to the coordinate position (–0.55 in., –0.15 in.). Assume that $a = 15$ ft, $b = 27$ ft, $c = 11$ ft, and $d = 21$ ft. Determine the normal strain in each bar.

P2.3 Pin-connected rigid bars AB, BC, and CD are initially held in the positions shown in Figure P2.3 by taut wires (1) and (2). The bar lengths are $a = 24$ ft and $b = 18$ ft. Joint C is given a horizontal displacement of 5 in. to the right. (Note that this displacement causes both joints B and C to move to the right and slightly downward.) What is the change in the average normal strain in wire (1) after the displacement?

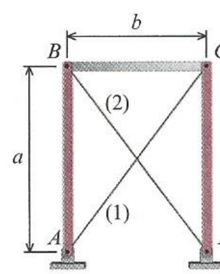


FIGURE P2.3

P2.4 Bar (1) has a length of $L_1 = 2.50$ m, and bar (2) has a length of $L_2 = 0.65$ m. Initially, there is a gap of $\Delta = 3.5$ mm between the rigid plate at B and bar (2). After application of the loads P to the rigid plate at B, the rigid plate moved to the right, stretching bar (1) and compressing bar (2). The normal strain in bar (1) was measured as 2,740 μm/m after the loads P were applied. Determine the normal strain produced in bar (2).

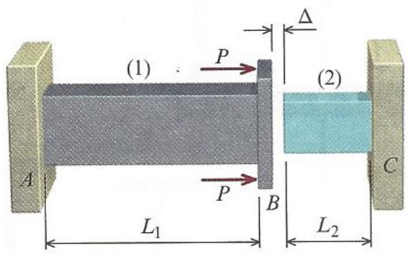


FIGURE P2.4

P2.5 In Figure P2.5, rigid bar ABC is supported by a pin at B and by post (1) at A. However, there is a gap of $\Delta = 10$ mm between the rigid bar at A and post (1). After load P is applied to the rigid bar, point C moves to the left by 8 mm. If the length of post (1) is $L_1 = 1.6$ m, what is the average normal strain that is produced in post (1)? Use dimensions of $a = 1.25$ m and $b = 0.85$ m.

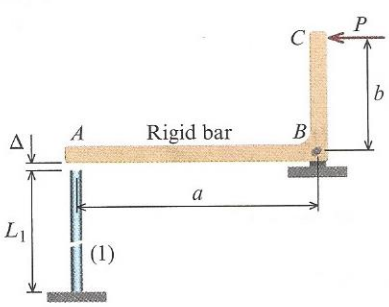


FIGURE P2.5

P2.6 The rigid bar ABC is supported by three bars as shown in Figure P2.6. Bars (1) attached at A and C are identical, each having a length $L_1 = 160$ in. Bar (2) has a length $L_2 = 110$ in.; however, there is a clearance $c = 0.25$ in. between bar (2) and the pin in the rigid bar at B. There is no strain in the bars before load P is applied, and $a = 50$ in. After application of load P, the tensile normal strain in bar (2) is measured as 960 με. What is the normal strain in bars (1)?

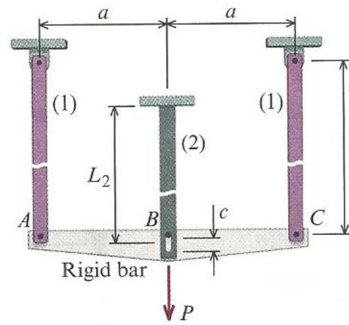


FIGURE P2.6

P2.7 Rigid bar $ABCD$ is supported by two bars as shown in Figure P2.7. There is no strain in the vertical bars before load P is applied. After load P is applied, the normal strain in bar (2) is measured as −3,300 μm/m. Use the dimensions $L_1 = 1{,}600$ mm, $L_2 = 1{,}200$ mm, $a = 240$ mm, $b = 420$ mm, and $c = 180$ mm. Determine

(a) the normal strain in bar (1).
(b) the normal strain in bar (1) if there is a 1 mm gap in the connection at pin C before the load is applied.
(c) the normal strain in bar (1) if there is a 1 mm gap in the connection at pin B before the load is applied.

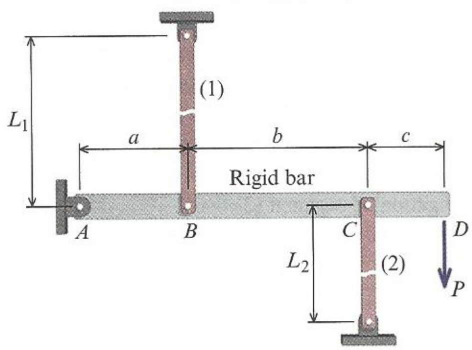


FIGURE P2.7

P2.8 The sanding-drum mandrel shown in Figure P2.8 is made for use with a hand drill. The mandrel is made from a rubberlike material that expands when the nut is tightened to secure the sanding sleeve placed over the outside surface. If the diameter D of the mandrel increases from 2.00 in. to 2.15 in. as the nut is tightened, determine

(a) the average normal strain along a diameter of the mandrel.
(b) the circumferential strain at the outside surface of the mandrel.

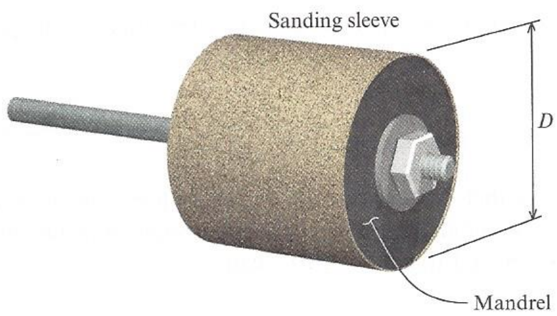


FIGURE P2.8

P2.9 The normal strain in a suspended bar of material of varying cross section due to its own weight is given by the expression $\gamma y/3E$, where γ is the specific weight of the material, y is the distance from the free (i.e., bottom) end of the bar, and E is a material constant. Determine, in terms of γ, L, and E,

(a) the change in length of the bar due to its own weight.
(b) the average normal strain over the length L of the bar.
(c) the maximum normal strain in the bar.

P2.10 A steel cable is used to support an elevator cage at the bottom of a 2,000 ft deep mine shaft. A uniform normal strain of 250 μin./in. is produced in the cable by the weight of the cage. At each point, the weight of the cable produces an additional normal strain that is proportional to the length of the cable below the point. If the total normal strain in the cable at the cable drum (upper end of the cable) is 700 μin./in., determine

(a) the strain in the cable at a depth of 500 ft.
(b) the total elongation of the cable.

2.3 Shear Strain

A deformation involving a change in shape (a distortion) can be used to illustrate a shear strain. An average shear strain γ_{avg} associated with two reference lines that are orthogonal in the undeformed state (two edges of the element shown in Figure 2.4) can be obtained by dividing the shear deformation δ_x (the displacement of the top edge of the element with respect to the bottom edge) by the perpendicular distance L between these two edges. If the deformation is small, meaning that $\sin\gamma \approx \tan\gamma \approx \gamma$ and $\cos\gamma \approx 1$, then shear strain can be defined as

$$\gamma_{avg} = \frac{\delta_x}{L} \tag{2.3}$$

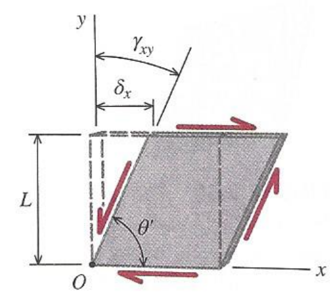


FIGURE 2.4 Shear strain.

For those cases in which the deformation is nonuniform, the shear strain at a point, $\gamma_{xy}(O)$, associated with two orthogonal reference lines x and y is obtained by measuring the shear deformation as the size of the element is made smaller and smaller. In the limit,

$$\gamma_{xy}(O) = \lim_{\Delta L \to 0} \frac{\Delta\delta_x}{\Delta L} = \frac{d\delta_x}{dL} \tag{2.4}$$

Since shear strain is defined as the tangent of the angle of distortion, and since the tangent of that angle is equal to the angle in radians for small angles, an equivalent expression for shear strain that is sometimes useful for calculations is

$$\gamma_{xy}(O) = \frac{\pi}{2} - \theta' \tag{2.5}$$

In this expression, θ' is the angle in the deformed state between two initially orthogonal reference lines.

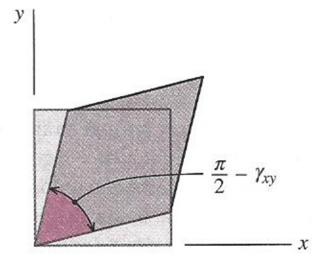


FIGURE 2.5*a* A positive value for the shear strain γ_{xy} means that the angle θ' between the x and y axes decreases in the deformed object.

Units of Strain

Equations (2.3) through (2.5) indicate that shear strains are dimensionless angular quantities, expressed in radians (rad) or microradians (μrad). The conversion from radians, a dimensionless quantity, to microradians is **1 μrad = 1 × 10⁻⁶ rad**.

Measuring Shear Strains Experimentally

Shear strain is an angular measure, and it is not possible to directly measure the extremely small angular changes typical of engineered structures. However, shear strain can be determined experimentally by using an array of three strain gages called a **strain rosette**. Strain rosettes will be discussed in more detail in Chapter 13.

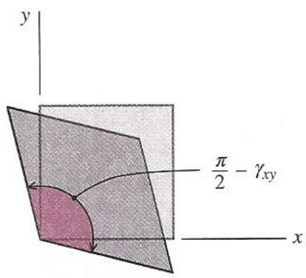


FIGURE 2.5*b* The angle between the x and y axes increases when the shear strain γ_{xy} has a negative value.

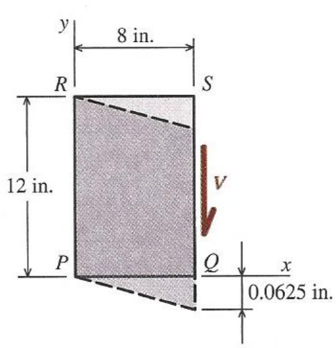
Sign Conventions for Shear Strains

Equation (2.5) shows that shear strains will be positive if the angle θ' between the x and y axes decreases. If the angle θ' increases, the shear strain is negative. To state this relationship another way, Equation (2.5) can be rearranged to give the angle θ' in the deformed state between two reference lines that are initially 90° apart:

$$\theta' = \frac{\pi}{2} - \gamma_{xy}$$

If the value of γ_{xy} is positive, then the angle θ' in the deformed state will be less than 90° (i.e., less than $\pi/2$ rad) (Figure 2.5*a*). If the value of γ_{xy} is negative, then the angle θ' in the deformed state will be greater than 90° (Figure 2.5*b*). Positive and negative shear strains are not given special or distinctive names.

EXAMPLE 2.2



The shear force V shown causes side QS of the thin rectangular plate to displace downward 0.0625 in. Determine the shear strain γ_{xy} at P.

Plan the Solution

Shear strain is an angular measure. Determine the angle between the x axis and side PQ of the deformed plate.

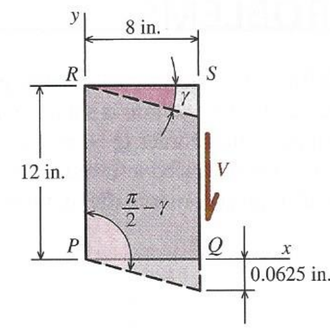
SOLUTION

Determine the angles created by the 0.0625 in. deformation. **Note:** The small-angle approximation will be used here; therefore, $\sin\gamma \approx \tan\gamma \approx \gamma$, and we have

$$\gamma = \frac{0.0625 \text{ in.}}{8 \text{ in.}} = 0.0078125 \text{ rad}$$

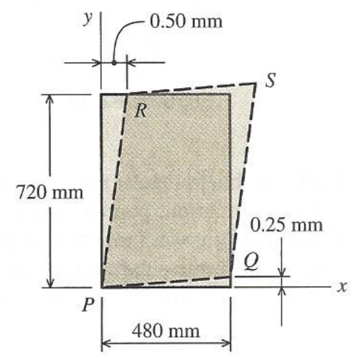
In the undeformed plate, the angle at P is $\pi/2$ rad. After the plate is deformed, the angle at P increases. Since the angle after deformation is equal to $(\pi/2) - \gamma$, the shear strain at P must be a negative value. A simple calculation shows that the shear strain at P is the shear strain at P is

$$\gamma = -0.00781 \text{ rad} \qquad \textbf{Ans.}$$



EXAMPLE 2.3

A thin rectangular plate is uniformly deformed as shown. Determine the shear strain γ_{xy} at P.



Plan the Solution

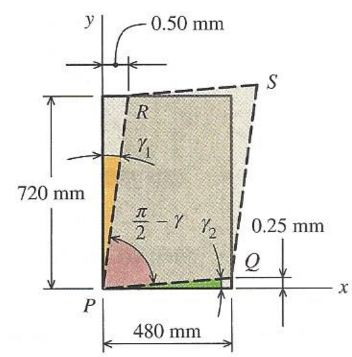
Shear strain is an angular measure. Determine the two angles created by the 0.25 mm deflection and the 0.50 mm deflection. Add these two angles together to determine the shear strain at P.

SOLUTION

Determine the angles created by each deformation. **Note:** The small-angle approximation will be used here; therefore, $\sin\gamma \approx \tan\gamma \approx \gamma$. From the given data, we obtain

$$\gamma_1 = \frac{0.50 \text{ mm}}{720 \text{ mm}} = 0.000694 \text{ rad}$$

$$\gamma_2 = \frac{0.25 \text{ mm}}{480 \text{ mm}} = 0.000521 \text{ rad}$$



The shear strain at P is simply the sum of these two angles:

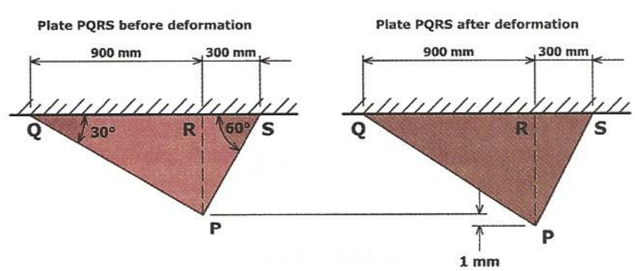
$$\gamma = \gamma_1 + \gamma_2 = 0.000694 \text{ rad} + 0.000521 \text{ rad} = 0.001215 \text{ rad}$$
$$= 1{,}215 \ \mu\text{rad} \qquad \textbf{Ans.}$$

Note: The angle at P in the deformed plate is less than $\pi/2$, as it should be for a positive shear strain. Although not requested in the problem, the shear strain at corners Q and R will be negative and will have the same magnitude as the shear strain at corner P.

MecMovies

EXAMPLE

M2.5 A thin triangular plate is uniformly deformed. Determine the shear strain at P after point P has been displaced 1 mm downward.



PROBLEMS

P2.11 A thin rectangular polymer plate *PQRS* of width $b = 400$ mm and height $a = 180$ mm is shown in Figure P2.11. The plate is deformed so that corner *Q* is displaced upward by $c = 3.0$ mm and corner *S* is displaced leftward by the same amount. Determine the shear strain at corner *P* after deformation.

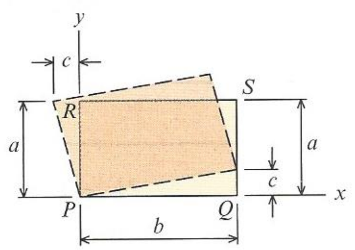


FIGURE P2.11

P2.12 A thin triangular plate *PQR* forms a right angle at point *Q*. During deformation, point *Q* moves to the right by $u = 0.8$ mm and upward by $v = 1.3$ mm to new position *Q′*, as shown in Figure P2.12. Determine the shear strain γ at corner *Q′* after deformation. Use $a = 225$ mm, $b = 455$ mm, and $d = 319.96$ mm.

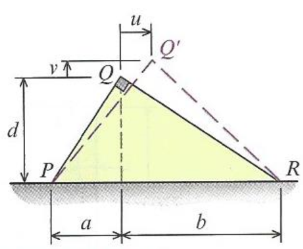


FIGURE P2.12

P2.13 A thin triangular plate *PQR* forms a right angle at point *Q*. During deformation, point *Q* moves to the left by $u = 2.0$ mm and upward by $v = 5.0$ mm to new position *Q′*, as shown in Figure P2.13. Determine the shear strain γ at corner *Q′* after deformation. Use $c = 700$ mm, $\alpha = 28°$, and $\beta = 62°$.

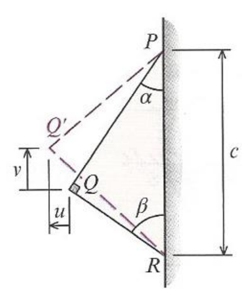


FIGURE P2.13

P2.14 A thin square polymer plate is deformed into the position shown by the dashed lines in Figure P2.14. Assume that $a = 800$ mm, $b = 85$ mm, and $c = 960$ mm. Determine the shear strain γ_{xy} (a) at corner *P* and (b) at corner *Q*, after deformation.

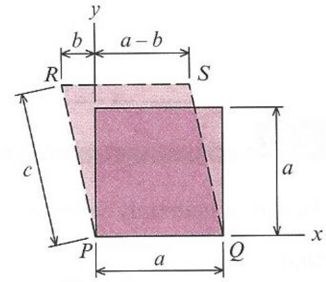


FIGURE P2.14

P2.15 A thin square plate *PQRS* is symmetrically deformed into the shape shown by the dashed lines in Figure P2.15. The initial length of diagonals *PR* and *QS* is $d = 295$ mm. After deformation, diagonal *PR* has a length of $d_{PR} = 295.3$ mm and diagonal *QS* has a length of $d_{QS} = 293.7$ mm. For the deformed plate, determine

(a) the normal strain of diagonal *QS*.
(b) the shear strain γ_{xy} at corner *P*.

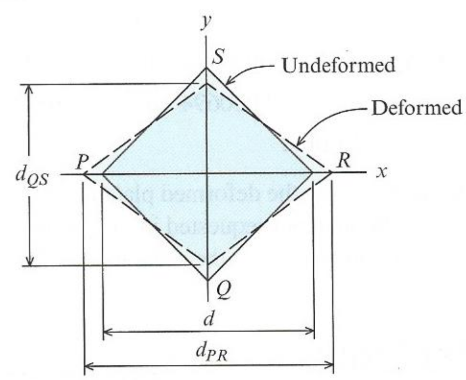


FIGURE P2.15

2.4 Thermal Strain

When unrestrained, most engineering materials expand when heated and contract when cooled. The thermal strain caused by a one-degree (1°) change in temperature is designated by the Greek letter α (alpha) and is known as the **coefficient of thermal expansion**. The strain due to a temperature change of ΔT is

$$\varepsilon_T = \alpha \Delta T \tag{2.6}$$

The coefficient of thermal expansion is approximately constant over a considerable range of temperatures. (In general, the coefficient increases with an increase in temperature.) For a uniform material (termed a **homogeneous material**) that has the same mechanical properties in every direction (termed an **isotropic material**), the coefficient applies to all dimensions (i.e., all directions). Values of the coefficient of expansion for common materials are included in Appendix D.

A material of uniform composition is called a **homogeneous material**. In materials of this type, local variations in composition can be considered negligible for engineering purposes. Furthermore, homogeneous materials cannot be mechanically separated into different materials (the way carbon fibers in a polymer matrix can). Common homogeneous materials are metals, alloys, ceramics, glass, and some types of plastics.

Total Strains

Strains caused by temperature changes and strains caused by applied loads are essentially independent. The total normal strain in a body acted on by both temperature changes and an applied load is given by

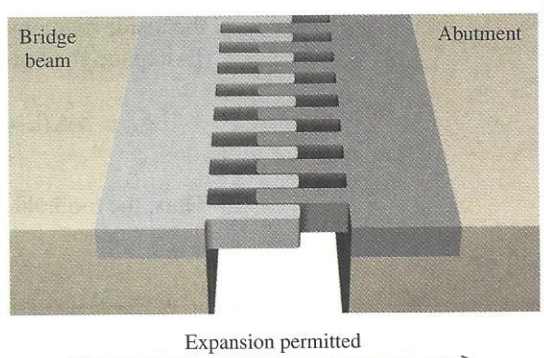
$$\varepsilon_{\text{total}} = \varepsilon_\sigma + \varepsilon_T \tag{2.7}$$

An **isotropic material** has the same mechanical properties in all directions.

Since homogeneous, isotropic materials, when unrestrained, expand uniformly in all directions when heated (and contract uniformly when cooled), neither the shape of the body nor the shear stresses and shear strains are affected by temperature changes.

EXAMPLE 2.4

A steel bridge beam has a total length of 150 m. Over the course of a year, the bridge is subjected to temperatures ranging from −40°C to +40°C, and the associated temperature changes cause the beam to expand and contract. Expansion joints between the bridge beam and the supports at the ends of the bridge (called abutments) are installed to allow this change in length to take place without restraint. Determine the change in length that must be accommodated by the expansion joints. Assume that the coefficient of thermal expansion for steel is $11.9 \times 10^{-6}/°\text{C}$.



Typical "finger-type" expansion joint for bridges.

Plan the Solution

Determine the thermal strain from Equation (2.6) for the total temperature variation. The change in length is the product of the thermal strain and the beam length.

SOLUTION

The thermal strain for a temperature variation of 80°C (40°C – (–40°C) = 80°C) is

$$\varepsilon_T = \alpha\,\Delta T = (11.9 \times 10^{-6}/°\text{C})(80°\text{C}) = 0.000952 \text{ m/m}$$

The total change in the beam length is, therefore,

$$\delta_T = \varepsilon L = (0.000952 \text{ m/m})(150 \text{ m}) = 0.1428 \text{ m} = 142.8 \text{ mm} \qquad \textbf{Ans.}$$

Thus, the expansion joint must accommodate at least 142.8 mm of horizontal movement.

EXAMPLE 2.5

Cutting tools such as mills and drills are connected to machining equipment by means of toolholders. The cutting tool must be firmly clamped by the toolholder to achieve precise machining, and shrink-fit toolholders take advantage of thermal expansion properties to achieve this strong, concentric clamping force. To insert a cutting tool, the shrink-fit holder is rapidly heated while the cutting tool remains at room temperature. When the holder has expanded sufficiently, the cutting tool drops into the holder. The holder is then cooled, clamping the cutting tool with a very large force exerted directly on the tool shank.

At 20°C, the cutting tool shank has an outside diameter of 18.000 ± 0.005 mm and the toolholder has an inside diameter of 17.950 ± 0.005 mm. If the tool shank is held at 20°C, what is the minimum temperature to which the toolholder must be heated in order to insert the cutting tool shank? Assume that the coefficient of thermal expansion of the toolholder is $11.9 \times 10^{-6}/°\text{C}$.

Plan the Solution

Use the diameters and tolerances to compute the maximum outside diameter of the shank and the minimum inside diameter of the holder. The difference between these two diameters is the amount of expansion that must occur in the holder. For the tool shank to drop into the holder, the inside diameter of the holder must equal or exceed the shank diameter.

SOLUTION

The maximum outside diameter of the shank is 18.000 + 0.005 mm = 18.005 mm. The minimum inside diameter of the holder is 17.950 – 0.005 mm = 17.945 mm. Therefore, the inside diameter of the holder must be increased by 18.005 – 17.945 mm = 0.060 mm. To expand the holder by this amount requires a temperature increase

$$\delta_T = \alpha\,\Delta T d = 0.060 \text{ mm} \qquad \therefore \Delta T = \frac{0.060 \text{ mm}}{(11.9 \times 10^{-6}/°\text{C})(17.945 \text{ mm})} = 281°\text{C}$$

Thus, the toolholder must attain a minimum temperature of

$$20°\text{C} + 281°\text{C} = 301°\text{C} \qquad \textbf{Ans.}$$

PROBLEMS

P2.16 An airplane has a half-wingspan of 96 ft. Determine the change in length of the aluminum alloy [$\alpha = 13.1 \times 10^{-6}/°F$] wing spar if the plane leaves the ground at a temperature of 59°F and climbs to an altitude where the temperature is −70°F.

P2.17 A square high-density polyethylene [$\alpha = 158 \times 10^{-6}/°C$] plate has a width of 300 mm. A 180 mm diameter circular hole is located at the center of the plate. If the temperature of the plate increases by 40°C, determine

(a) the change in width of the plate.
(b) the change in diameter of the hole.

P2.18 A circular steel [$\alpha = 6.5 \times 10^{-6}/°F$] band is to be mounted on a circular steel drum. The outside diameter of the drum is 50 in. The inside diameter of the circular band is 49.95 in. The band will be heated and then slipped over the drum. After the band cools, it will grip the drum tightly. This process is called *shrink fitting*. If the temperature of the band is 72°F before heating, compute the minimum temperature to which the band must be heated so that it can be slipped over the drum. Assume that an extra 0.05 in. in diameter is needed for clearance so that the band can be easily slipped over the drum. Assume that the drum diameter remains constant.

P2.19 At a temperature of 60°F, a gap of $a = 0.125$ in. exists between the two polymer bars shown in Figure P2.19. Bar (1) has a length $L_1 = 40$ in. and a coefficient of thermal expansion of $\alpha_1 = 47 \times 10^{-6}/°F$. Bar (2) has a length $L_2 = 24$ in. and a coefficient of thermal expansion of $\alpha_2 = 66 \times 10^{-6}/°F$. The supports at A and D are rigid. What is the lowest temperature at which the gap is closed?

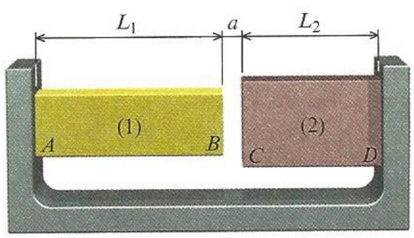


FIGURE P2.19

P2.20 An aluminum pipe has a length of 60 m at a temperature of 10°C. An adjacent steel pipe at the same temperature is 5 mm longer. At what temperature will the aluminum pipe be 15 mm longer than the steel pipe? Assume that the coefficient of thermal expansion of the aluminum is $22.5 \times 10^{-6}/°C$ and that the coefficient of thermal expansion of the steel is $12.5 \times 10^{-6}/°C$.

P2.21 The simple mechanism shown in Figure P2.21 can be calibrated to measure temperature change. Use dimensions of $a = 25$ mm, $b = 90$ mm, and $L_1 = 180$ mm. The coefficient of thermal expansion of member (1) is $23.0 \times 10^{-6}/°C$. Determine the horizontal displacement of pointer tip D for the mechanism shown in response to a temperature increase of 35°C. Assume that pointer BCD is not affected significantly by temperature change.

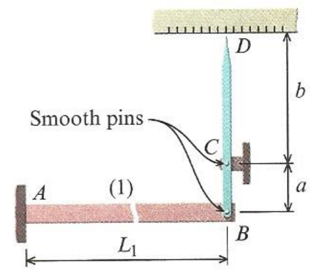


FIGURE P2.21

P2.22 For the assembly shown in Figure P2.22, high-density polyethylene bars (1) and (2) each have coefficients of thermal expansion of $\alpha = 88 \times 10^{-6}/°F$. If the temperature of the assembly is decreased by 50°F from its initial temperature, determine the resulting displacement of pin B. Assume that $b = 32$ in. and $\theta = 55°$.

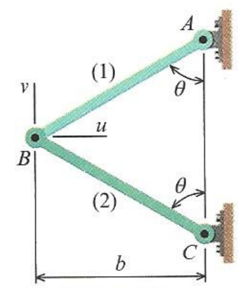


FIGURE P2.22