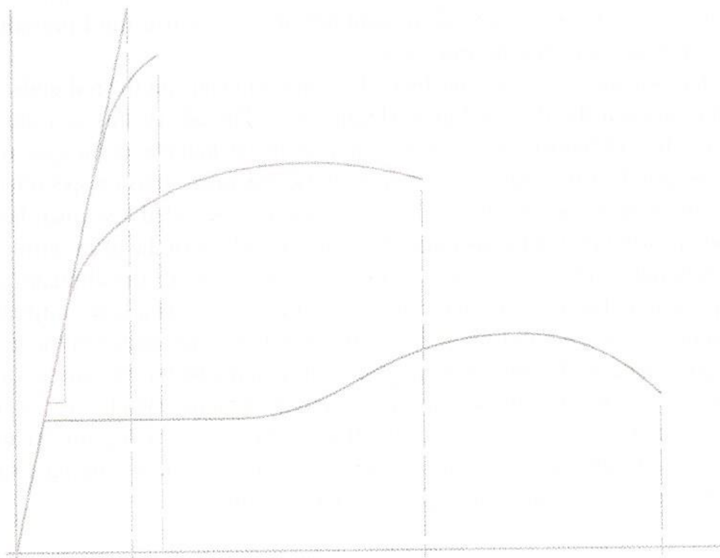
CHAPTER 3

Mechanical Properties of Materials



3.1 The Tension Test

To design a structural or mechanical component properly, the engineer must understand the characteristics of the component and work within the limitations of the material used in it. Materials such as steel, aluminum, plastic, and wood each respond uniquely to applied loads and stresses. To determine the strength and characteristics of materials such as these requires laboratory testing. One of the simplest and most effective laboratory tests for obtaining engineering design information about a material is called the **tension test**.

The tension test is very simple. A specimen of the material, usually a round rod or a flat bar, is pulled with a controlled tension force. As the force is increased, the elongation of the specimen is measured and recorded. The relationship between the applied load and the resulting deformation can be observed from a plot of the data. This load–deformation plot has limited direct usefulness, however, because it applies only to the specific specimen (meaning the specific diameter or cross-sectional dimensions) used in the test procedure.

A more useful diagram than the load–deformation plot is a plot showing the relationship between stress and strain, called the **stress–strain diagram**. The stress–strain diagram is more useful because it applies to the material in general rather than to the particular specimen used in the test. The information obtained from the stress–strain diagram can be applied to all components, regardless of their dimensions. The load and elongation data obtained in the tension test can be readily converted to stress and strain data.

Tension Test Setup

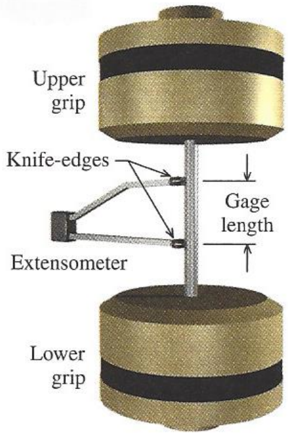


FIGURE 3.1 Tension test setup.

To conduct the tension test, the test specimen is inserted into grips that hold the specimen securely while a tension force is applied by the testing machine (Figure 3.1). Generally, the lower grip remains stationary while the upper grip moves upward, thus creating tension in the specimen.

Several types of grips are commonly used, depending on the specimen being tested. For plain round or flat specimens, wedge-type grips are often used. The wedges are used in pairs that ride in a V-shaped holder. The wedges have teeth that bite into the specimen. The tension force applied to the specimen drives the wedges closer together, increasing the clamping force on the specimen. More sophisticated grips use fluid pressure to actuate the wedges and increase their holding power.

Some tension specimens are machined by cutting threads on the rod ends and reducing the diameter between the threaded ends (Figure 3.2). Threads of this sort are called *upset threads*. Since the rod diameter at the ends is larger than the diameter of the specimen, the presence of the threads does not reduce the strength of the specimen. Tension specimens with upset threads are attached to the testing machine by means of threaded specimen holders, which eliminate any possibility that the specimen will slip or pull out of the grips during the test.

FIGURE 3.2 Tension test specimen with upset threads.

An instrument called an *extensometer* is used to measure the elongation in the tension test specimen. The extensometer has two knife-edges, which are clipped to the test specimen (clips not shown in Figure 3.1). The initial distance between the knife-edges is called the *gage length*. As tension is applied, the extensometer measures the elongation that occurs in the specimen within the gage length. Extensometers are capable of very precise measurements—elongations as small as 0.0001 in. or 0.002 mm. They are available in a range of gage lengths, with the most common models ranging from 0.3 in. to 2 in. (in U.S. units) and from 8 mm to 100 mm (in SI units).

Tension Test Measurements

Several measurements are made before, during, and after the test. Before the test, the cross-sectional area of the specimen must be determined. The area of the specimen will be used with the force data to compute the normal stress. The gage length of the extensometer should also be noted. Normal strain will be computed from the deformation of the specimen (i.e., its axial elongation) and the gage length. During the test, the force applied to the specimen is recorded and the elongation in the specimen between the extensometer knife-edges is measured. After the specimen has broken, the two halves of the specimen are fitted together so that the final gage length and the diameter of the cross section at the fracture location can be measured. The average engineering strain determined from the final and initial gage lengths provides one measure of ductility. The reduction in area (between the area of the fracture surface and the original cross-sectional area) divided by the original cross-sectional area provides a second measure of the ductility of the material. The term **ductility** describes the amount of strain that the material can withstand before fracturing.

MecMovies 3.1 shows an animated tension test.

Tension Test Results. The typical results from a tension test of a ductile metal are shown in Figure 3.3. Several characteristic features are commonly found on the load–deformation plot. As the load is applied, there is a range in which the deformation is linearly related to the load (1). At some load, the load–deformation plot will begin to curve and there will be noticeably larger deformations in response to relatively small increases in load (2). As the

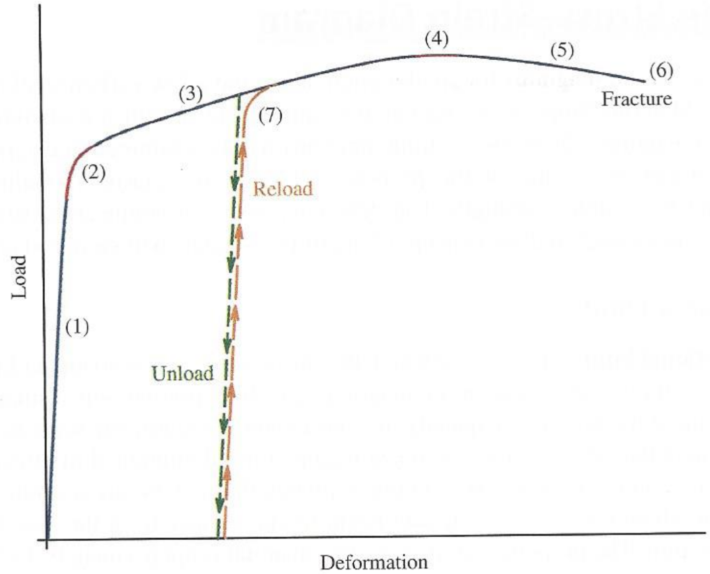


FIGURE 3.3 Load–deformation plot from tension test.

load is continually increased, stretching in the specimen will be obvious (3). At some point, a maximum load intensity will be reached (4). Immediately following this peak, the specimen will begin to narrow and elongate markedly at one specific location, causing the load acting in the specimen to decrease (5). Shortly thereafter, the specimen will fracture (6), breaking into two pieces at the narrowest cross section.

Another interesting characteristic of materials, particularly metals, can be observed if the test is interrupted at a point beyond the linear region. For the test depicted in Figure 3.3, the specimen was loaded into region (3) and then the load was removed. In that case, the specimen does not unload along the original loading curve. Rather, it unloads along a path that is parallel to the initial linear plot (1). Then, when the load is completely removed, the deformation of the specimen is not zero, as it was at the outset of the test. In other words, the specimen has been permanently and irreversibly deformed. When the test resumes and the load is increased, the reloading path follows the unloading path exactly. As it approaches the original load–deformation plot, the reloading plot begins to curve (7) in a fashion similar to region (2) on the original plot. However, the load at which the reloading plot markedly turns (7) is larger than it was in the original loading (2). The process of unloading and reloading has strengthened the material so that it can withstand a larger load before it becomes distinctly nonlinear. The unload–reload behavior seen here is a very useful characteristic, particularly for metals. One technique for increasing the strength of a material is a process of stretching and relaxing called **work hardening**.

Preparing the Stress–Strain Diagram. The load–deformation data that are obtained in the tension test provide information about only one specific size of specimen. The test results are more useful if they are generalized into a stress–strain diagram. To construct a stress–strain diagram from tension test results,

(a) divide the specimen elongation data by the extensometer gage length to obtain the normal strain,

(b) divide the load data by the initial specimen cross-sectional area to obtain the normal stress, and

(c) plot strain on the horizontal axis and stress on the vertical axis.

3.2 The Stress–Strain Diagram

MecMovies 3.1 shows an animated discussion of stress–strain diagrams.

Typical stress–strain diagrams for an aluminum alloy and a low-carbon steel are shown in Figure 3.4. Material properties essential for engineering design are obtained from the stress–strain diagram. These stress–strain diagrams will be examined to determine several important properties, including the proportional limit, the elastic modulus, the yield strength, and the ultimate strength. The difference between engineering stress and true stress will be discussed, and the concept of ductility in metals will be introduced.

Proportional Limit

The **proportional limit** is the stress at which the stress–strain plot is no longer linear. Strains in the linear portion of the stress–strain diagram typically represent only a small fraction of the total strain at fracture. Consequently, it is necessary to enlarge the scale to observe the linear portion of the curve clearly. The linear region of the aluminum alloy stress–strain diagram is enlarged in Figure 3.5. A best-fit line is plotted through the stress–strain data points. The stress at which the stress–strain data begin to curve away from this line is called the proportional limit. The proportional limit for this material is approximately 43.5 ksi.

Recall the unload–reload behavior shown in Figure 3.3. As long as the stress in the material remains below the proportional limit, no permanent damage will be caused during loading and unloading. In an engineering context, this property means that a component can be loaded and unloaded many, many times and it will still behave "just like new." The property is called **elasticity**, and it means that a material returns to its original dimensions during unloading. The material itself is said to be **elastic** in the linear region.

Most engineered components are designed to function elastically to avoid permanent deformations that occur after the proportional limit is exceeded. In addition, the size and shape of an object are not significantly changed if strains and deformations are kept small. This property can be a particularly important consideration for mechanisms and machines, which consist of many parts that must fit together to operate properly.

Elastic Modulus

Most components are designed to function elastically. Consequently, the relationship between stress and strain in the initial, linear region of the stress–strain diagram is of particular interest regarding engineering materials. In 1807, Thomas Young proposed characterizing the material's behavior in the elastic region by the ratio between normal stress

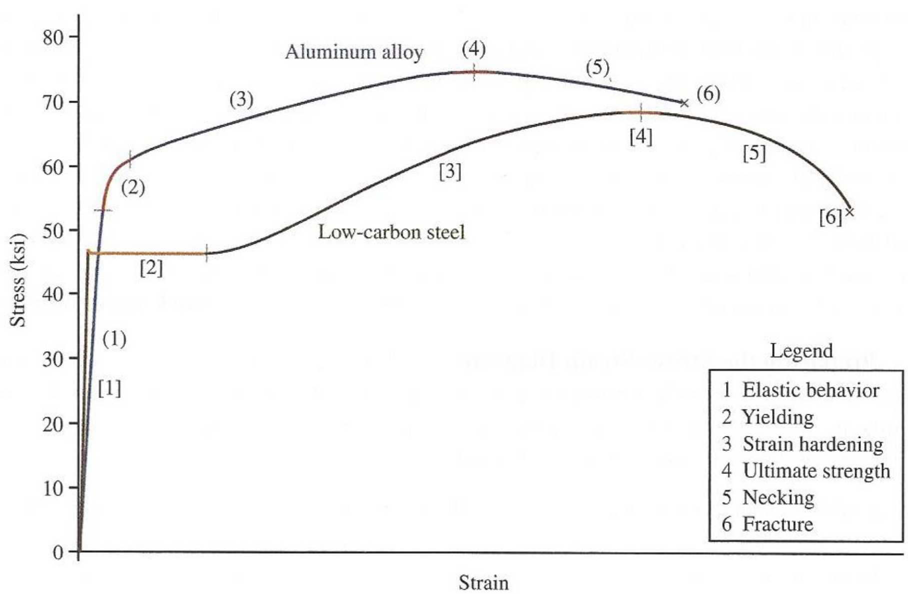


FIGURE 3.4 Typical stress–strain diagrams for two common metals.

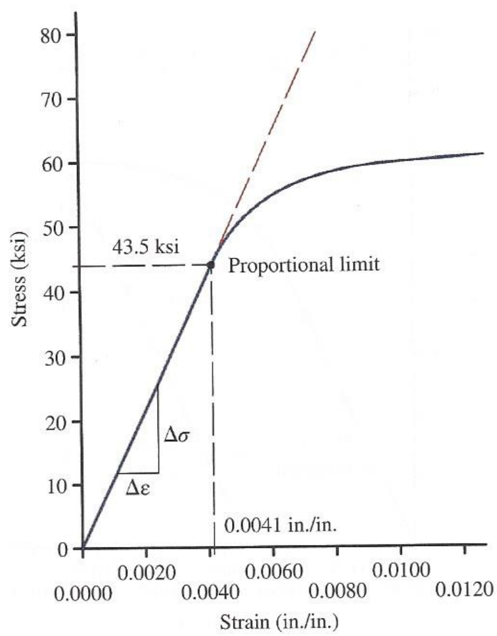


FIGURE 3.5 Proportional limit.

and normal strain. This ratio is the slope of the initial straight-line portion of the stress–strain diagram. It is called **Young's modulus**, the **elastic modulus**, or the **modulus of elasticity**, and it is denoted by the symbol E:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{3.1}$$

The elastic modulus E is a measure of the material's *stiffness*. In contrast to strength measures, which predict how much load a component can withstand, a stiffness measure such as the elastic modulus E is important because it defines how much stretching, compressing, bending, or deflecting will occur in a component in response to the loads acting on it.

In any experimental procedure, there is some amount of error associated with making a measurement. To minimize the effect of this measurement error on the computed elastic modulus value, it is better to use widely separated data points to calculate E. In the linear portion of the stress–strain diagram, the two most widely spaced data points are the proportional limit point and the origin. Using the proportional limit and the origin, we would compute the elastic modulus E as

$$E = \frac{43.5 \text{ ksi}}{0.0041 \text{ in./in.}} = 10{,}610 \text{ ksi} \tag{3.2}$$

In practice, the best value for the elastic modulus E is obtained from a least-squares fit of a line to the data between the origin and the proportional limit. Using a least-squares analysis, we find that the elastic modulus for this material is $E = 10{,}750$ ksi.

Work Hardening

The effect of unloading and reloading on the load–deformation plot was shown in Figure 3.3. The effect of unloading and reloading on the stress–strain diagram is shown

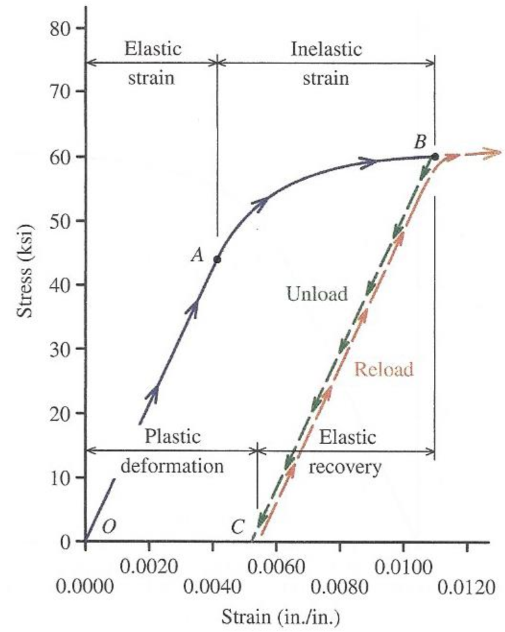


FIGURE 3.6 Work hardening.

in Figure 3.6. Suppose that the stress acting on a material is increased above the proportional limit stress to point *B*. The strain between the origin *O* and the proportional limit *A* is termed **elastic strain**. This strain will be fully recovered after the stress is removed from the material. The strain between the points *A* and *B* is termed **inelastic strain**. When the stress is removed (i.e., unloaded), only a portion of this strain will be recovered. As stress is removed from the material, it unloads on a path parallel to the elastic modulus line—that is, parallel to path *OA*. A portion of the strain at *B* is recovered elastically. However, a portion of the strain remains in the material permanently. This strain is referred to as **residual strain** or **permanent set** or **plastic deformation**. As stress is reapplied, the material reloads along path *CB*. Upon reaching point *B*, the material will resume following the original stress–strain curve. The proportional limit after reloading becomes the stress at point *B*, which is greater than the proportional limit for the original loading (i.e., point *A*). This phenomenon is called **work hardening**, because it has the effect of increasing the proportional limit for the material.

In general, a material acting in the linear portion of the stress–strain curve is said to exhibit **elastic behavior**. Strains in the material are temporary, meaning that all strain is recovered when the stress on the material is removed. Beyond the elastic region, a material is said to exhibit **plastic behavior**. Although some strain in the plastic region is temporary and can be recovered upon removal of the stress, a portion of the strain in the material is permanent. The permanent strain is termed **plastic deformation**.

Elastic Limit

Most engineered components are designed to act elastically, meaning that when loads are released, the component will return to its original, undeformed configuration. For proper design, therefore, it is important to define the stress at which the material will no longer behave elastically. With most materials, there is a gradual transition from elastic to plastic behavior, and the point at which plastic deformation begins is difficult to define with precision. One measure that has been used to establish this threshold is termed the elastic limit.

The **elastic limit** is the largest stress that a material can withstand without any measurable permanent strain remaining after complete release of the stress. The procedure required to determine the elastic limit involves cycles of loading and unloading, each time incrementally increasing the applied stress (Figure 3.7). For instance, stress is increased to point *A* and then removed, with the strain returning to the origin *O*. This process is repeated for points *B*, *C*, *D*, and *E*. In each instance, the strain returns to the origin *O* upon unloading. Eventually, a stress will be reached (point *F*) such that not all of the strain will be recovered during unloading (point *G*). The elastic limit is the stress at point *F*.

How does the elastic limit differ from the proportional limit? Although such materials are not common in engineered applications, a material can be elastic even though its stress–strain relationship is nonlinear. For a nonlinear elastic material, the elastic limit could be substantially greater than the proportional limit stress. Nevertheless, the proportional limit is generally favored in practice since the procedure required to establish the elastic limit is tedious.

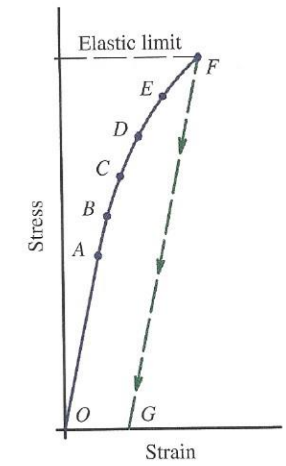


FIGURE 3.7 Elastic limit.

Yielding

For many common materials (such as the low-carbon steel shown in Figure 3.4 and enlarged in Figure 3.8), the elastic limit is indistinguishable from the proportional limit. Past the elastic limit, relatively large deformations will occur for small or almost negligible increases in stress. This behavior is termed **yielding**.

A material that behaves in the manner depicted in Figure 3.8 is said to have a **yield point**. The yield point is the stress at which there is an appreciable increase in strain with no increase in stress. Low-carbon steel, in fact, has two yield points. Upon reaching the upper yield point, the stress drops abruptly to a sustained lower yield point. When a material yields without an increase in stress, the material is often referred to as being **perfectly plastic**. Materials having a stress–strain diagram similar to Figure 3.8 are termed **elastoplastic**.

Not every material has a yield point. Materials such as the aluminum alloy shown in Figure 3.4 do not have a clearly defined yield point. While the proportional limit marks the uppermost end of the linear portion of the stress–strain curve, it is sometimes difficult in practice to determine the proportional limit stress, particularly for materials with a gradual transition from a straight line to a curve. For such materials, a yield strength is defined. The **yield strength** is the stress that will induce a specified permanent set (i.e., plastic deformation) in the material, usually 0.05% or 0.2%. (**Note:** A permanent set of 0.2% is another way of expressing a strain value of 0.002 in./in., or 0.002 mm/mm.) To determine the yield strength from the stress–strain diagram, mark a point on the strain axis at the specified permanent set (Figure 3.9). Through this point, draw a line that is parallel to the initial elastic modulus line. The stress at which the offset line intersects the stress–strain diagram is termed the yield strength.

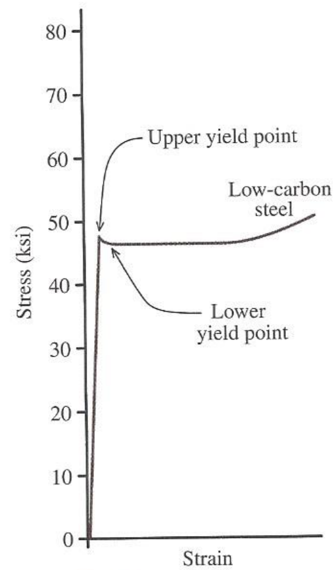


FIGURE 3.8 Yield point for low-carbon steel.

Strain Hardening and Ultimate Strength

After yielding has taken place, most materials can withstand additional stress before fracturing. The stress–strain curve rises continuously toward a peak stress value, which is termed the **ultimate strength**. The ultimate strength may also be called the tensile strength or the ultimate tensile strength (UTS). The rise in the curve is called **strain hardening**. The strain-hardening regions and the ultimate strength points for a low-carbon steel and an aluminum alloy are indicated on the stress–strain diagrams in Figure 3.4.

Necking

In the yield and strain-hardening regions, the cross-sectional area of the specimen decreases uniformly and permanently. Once the specimen reaches the ultimate strength, however, the change in the specimen cross-sectional area is no longer uniform throughout the gage length. The cross-sectional area begins to decrease in a localized

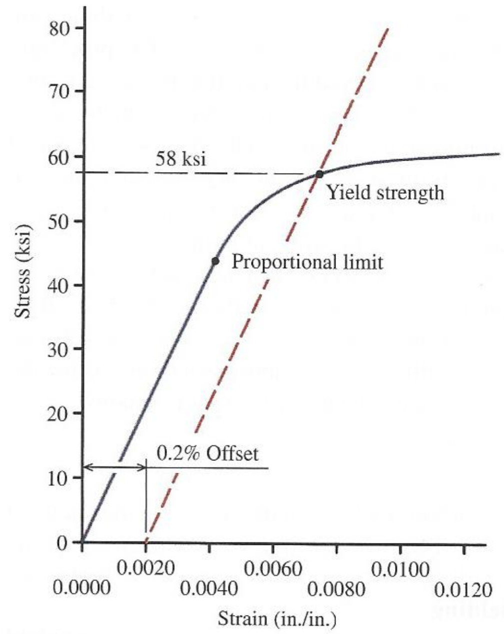


FIGURE 3.9 Yield strength determined by offset method.

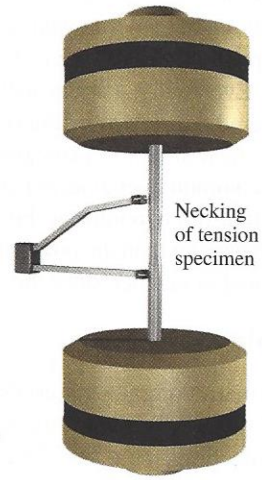


FIGURE 3.10 Necking in a tension specimen.

region of the specimen, forming a contraction, or "neck." This behavior is referred to as **necking** (Figure 3.10 and Figure 3.11). Necking occurs in ductile materials, but not in brittle materials. (See discussion of ductility, to follow.)

Fracture

Many ductile materials break in what is termed a cup-and-cone fracture (Figure 3.12). In the region of maximum necking, a circular fracture surface forms at an angle of roughly 45° with respect to the tensile axis. This failure surface appears as a cup on one portion of the broken specimen and as a cone on the other portion. In contrast, brittle materials often fracture on a flat surface that is oriented perpendicular to the tensile axis. The stress at which the specimen breaks into two pieces is called the **fracture stress**. Examine the relationship between the ultimate strength and the fracture stress in Figure 3.4. *Does it seem odd that the fracture stress is less than the ultimate strength?* If the specimen did not

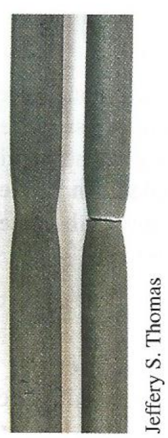


FIGURE 3.11 Necking in a ductile metal specimen.

FIGURE 3.12 Cup-and-cone failure surfaces.

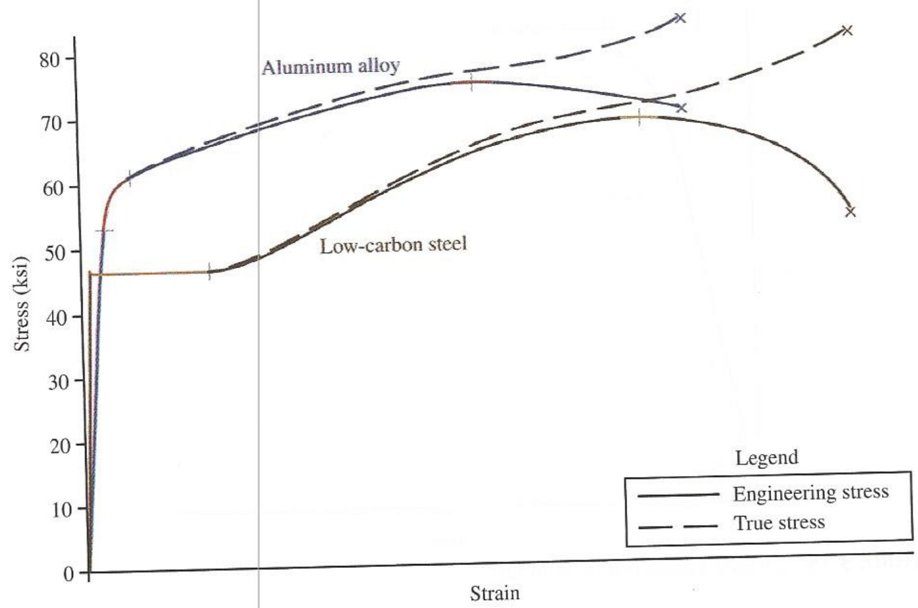


FIGURE 3.13 True stress versus engineering stress.

break at the ultimate strength, why would it break at a lower stress? Recall that the normal stress in the specimen was computed by dividing the specimen load by the original cross-sectional area. This method of calculating stresses is known as **engineering stress**. Engineering stress does not take into account any changes in the specimen's cross-sectional area during application of the load. After the ultimate strength is reached, the specimen starts to neck. As contraction within the localized neck region grows more pronounced, the cross-sectional area decreases continually. The engineering stress calculations, however, are based on the original specimen cross-sectional area. Consequently, the engineering stress computed at fracture and shown on the stress–strain diagram is not an accurate reflection of the **true stress** in the material. If one were to measure the diameter of the specimen during the tension test and compute the true stress according to the reduced diameter, one would find that the true stress continues to increase above the ultimate strength (Figure 3.13).

Ductility

Strength and stiffness are not the only properties of interest to a design engineer. Another important property is ductility. **Ductility** describes the material's capacity for plastic deformation.

A material that can withstand large strains before fracture is called a **ductile material**. Materials that exhibit little or no yielding before fracture are called **brittle materials**. Ductility is not necessarily related to strength. Two materials could have exactly the same strength, but very different strains at fracture (Figure 3.14).

Often, increased material strength is achieved at the cost of reduced ductility. In Figure 3.15, stress–strain curves for four different types of steel are compared. All four curves branch from the same elastic modulus line; therefore, each of the steels has the same stiffness. The steels range from a brittle steel (1) to a ductile steel (4). Steel (1) represents a hard tool steel, which exhibits no plastic deformation before fracture. Steel (4) is typical of low-carbon steel, which exhibits extensive plastic deformation before fracture. Of these steels, steel (1) is the strongest, but also the least ductile. Steel (4) is the weakest, but also the most ductile.

For the engineer, ductility is important in that it indicates the extent to which a metal can be deformed without fracture in metalworking operations such as bending, rolling, forming, drawing, and extruding. In fabricated structures and machine components, ductility also gives an indication of the material's ability to deform at holes, notches, fillets, grooves, and other

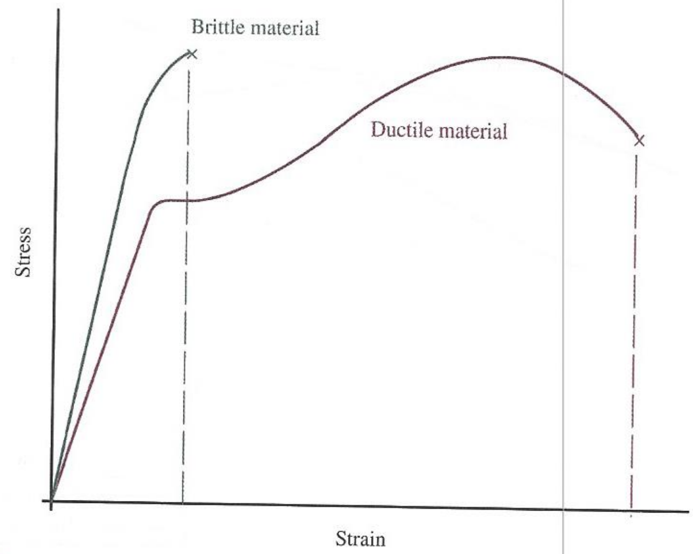


FIGURE 3.14 Ductile versus brittle materials.

discontinuities that cause stresses to intensify locally. Plastic deformation in a ductile material allows stress to flow to a larger region around discontinuities. This redistribution of stress minimizes peak stress magnitudes and helps to prevent fracture in the component. Since ductile materials stretch greatly before fracturing, excessive component deformations in buildings, bridges, and other structures can warn of impending failure, providing opportunities for safe exit from the structure and allowing for repairs. Brittle materials exhibit sudden failure with little or no warning. Ductile materials also give the structure some capacity to absorb and redistribute the effects of extreme load events such as earthquakes.

Ductility Measures. Two measures of ductility are obtained from the tension test. The first is the engineering strain at fracture. To determine this measure, the two halves of the broken specimen are fitted together, the final gage length is measured, and then the average strain is calculated from the initial and final gage lengths. This value is usually expressed as a percentage, and it is referred to as the **percent elongation**.

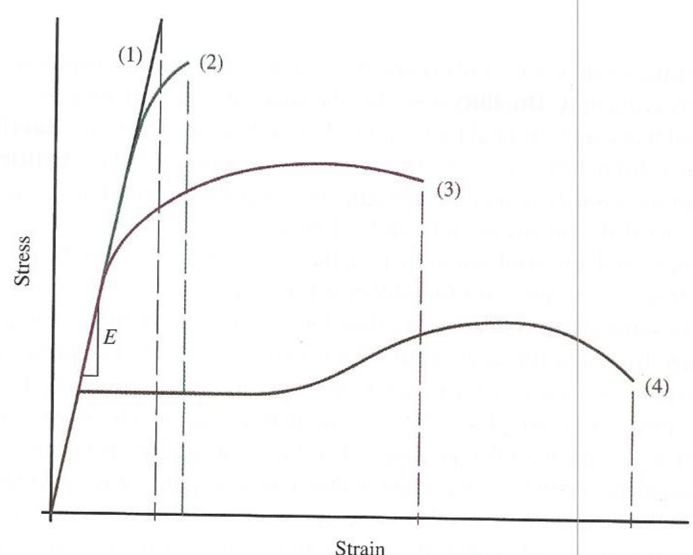


FIGURE 3.15 Trade-off between strength and ductility for steels.

The second measure is the reduction in area at the fracture surface. This value is also expressed as a percentage and is referred to as the **percent reduction of area**. It is calculated as

$$\text{Percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%) \tag{3.3}$$

where A_0 = original cross-sectional area of the specimen and A_f = cross-sectional area on the fracture surface of the specimen.

Review of Significant Features

The stress–strain diagram provides essential engineering design information that is applicable to components of any shape or size. While each material has its particular characteristics, several important features are found on stress–strain diagrams for materials commonly used in engineering applications. These features are summarized in Figure 3.16.

Strain hardening

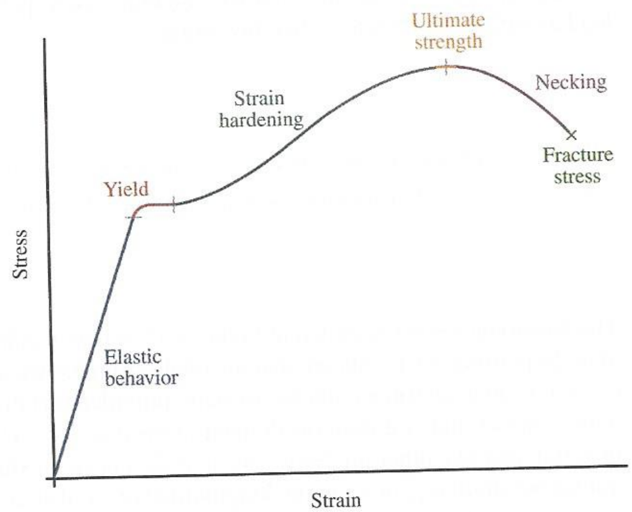
- As the material stretches, it can withstand increasing amounts of stress.

Ultimate strength

- According to the engineering definition of stress, the ultimate strength is the largest stress that the material can withstand.

Yield

- A slight increase in stress causes a marked increase in strain.
- Beginning at yield, the material is permanently altered. Only a portion of the strain will be recovered after the stress has been removed.
- Strains are termed inelastic since only a portion of the strain will be recovered upon removal of the stress.
- The yield strength is an important design parameter for the material.



Necking

- The cross-sectional area begins to decrease markedly in a localized region of the specimen.
- The tension force required to produce additional stretch in the specimen decreases as the area is reduced.
- Necking occurs in ductile materials, but not in brittle materials.

Elastic behavior

- In general, the initial relationship between stress and strain is linear.
- Elastic strain is temporary, meaning that all strain is fully recovered upon removal of the stress.
- The slope of this line is called the elastic modulus or the modulus of elasticity.

Fracture stress

- The fracture stress is the engineering stress at which the specimen breaks into two pieces.

FIGURE 3.16 Review of significant features on the stress–strain diagram.

3.3 Hooke's Law

As discussed previously, the initial portion of the stress–strain diagram for most materials used in engineering structures is a straight line. The stress–strain diagrams for some materials, such as gray cast iron and concrete, show a slight curve even at very small stresses, but it is common practice to neglect the curvature and draw a straight line in order to average the data for the first part of the diagram. The proportionality of load to deflection was first recorded by Robert Hooke, who observed in 1678, *Ut tensio sic vis* ("As the stretch, so the force"). This relationship is referred to as **Hooke's law**. For normal stress σ and normal strain ε acting in one direction (termed **uniaxial** stress and strain), Hooke's law is written as

$$\sigma = E\varepsilon \tag{3.4}$$

where E is the elastic modulus.

Hooke's law also applies to shear stress τ and shear strain γ,

$$\tau = G\gamma \tag{3.5}$$

where G is called the **shear modulus** or the **modulus of rigidity**.

3.4 Poisson's Ratio

A material loaded in one direction will undergo strains perpendicular to the direction of the load as well as parallel to it. In other words,

- If a solid body is subjected to an axial tension, it contracts in the lateral directions.
- If a solid body is compressed, it expands in the lateral directions.

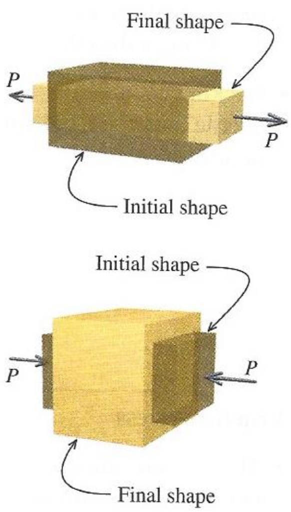


FIGURE 3.17 Lateral contraction and lateral expansion of a solid body subjected to axial forces.

This phenomenon is illustrated in Figure 3.17, where the deformations are *greatly exaggerated*. Experiments have shown that the relationship between lateral and longitudinal strains caused by an axial force remains constant, provided that the material remains *elastic* and is *homogeneous* and *isotropic* (as defined in Section 2.4). This constant is a property of the material, just like other properties, such as the elastic modulus E. The ratio of the lateral or transverse strain (ε_{lat} or ε_t) to the longitudinal or axial strain (ε_{long} or ε_a) for a uniaxial state of stress is called **Poisson's ratio**, after Siméon D. Poisson, who identified the constant in 1811. Poisson's ratio is denoted by the Greek symbol ν (nu) and is defined as follows:

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{\varepsilon_t}{\varepsilon_a} \tag{3.6}$$

The ratio $\nu = -\varepsilon_t/\varepsilon_a$ is valid only for a uniaxial state of stress (i.e., simple tension or compression). The negative sign appears in Equation (3.6) because the lateral and longitudinal strains are always of opposite signs for uniaxial stress (i.e., if one strain is elongation, the other strain is contraction).

Values vary for different materials, but for most metals, Poisson's ratio has a value between 1/4 and 1/3. Because the volume of material must remain constant, the largest possible value for Poisson's ratio is 0.5. Values approaching this upper limit are found only for materials such as rubber.

Relationship Between E, G, and ν

Poisson's ratio is related to the elastic modulus E and the shear modulus G by the formula

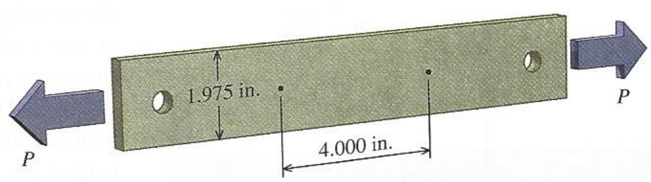
$$G = \frac{E}{2(1+\nu)} \tag{3.7}$$

The Poisson effect exhibited by materials causes no additional stresses in the lateral direction unless the transverse deformation is inhibited or prevented in some manner.

EXAMPLE 3.1

A tension test was conducted on a 1.975 in. wide by 0.375 in. thick specimen of a nylon plastic. A 4.000 in. gage length was marked on the specimen before application of the load. In the elastic portion of the stress–strain curve at an applied load of $P = 6{,}000$ lb, the elongation in the gage length was measured as 0.023 in. and the contraction in the width of the bar was measured as 0.004 in. Determine

(a) the elastic modulus E.
(b) Poisson's ratio ν.
(c) the shear modulus G.



Plan the Solution

(a) From the load and the initial measured dimensions of the bar, the normal stress can be computed. The normal strain in the longitudinal (i.e., axial) direction, ε_{long}, can be computed from the elongation in the gage length and the initial gage length. With these two quantities, the elastic modulus E can be calculated from Equation (3.4).
(b) From the contraction in the width and the initial width of the bar, the strain in the lateral (i.e., transverse) direction, ε_{lat}, can be computed. Poisson's ratio can then be found from Equation (3.6).
(c) The shear modulus can be calculated from Equation (3.7).

SOLUTION

(a) The normal stress in the plastic specimen is

$$\sigma = \frac{6{,}000 \text{ lb}}{(1.975 \text{ in.})(0.375 \text{ in.})} = 8{,}101.27 \text{ psi}$$

The longitudinal strain is

$$\varepsilon_{long} = \frac{0.023 \text{ in.}}{4.000 \text{ in.}} = 0.005750 \text{ in./in.}$$

Therefore, the elastic modulus is

$$E = \frac{\sigma}{\varepsilon} = \frac{8{,}101.27 \text{ psi}}{0.005750 \text{ in./in.}} = 1{,}408{,}916 \text{ psi} = 1{,}409{,}000 \text{ psi}$$ **Ans.**

(b) The lateral strain is

$$\varepsilon_{lat} = \frac{-0.004 \text{ in.}}{1.975 \text{ in.}} = -0.002025 \text{ in./in.}$$

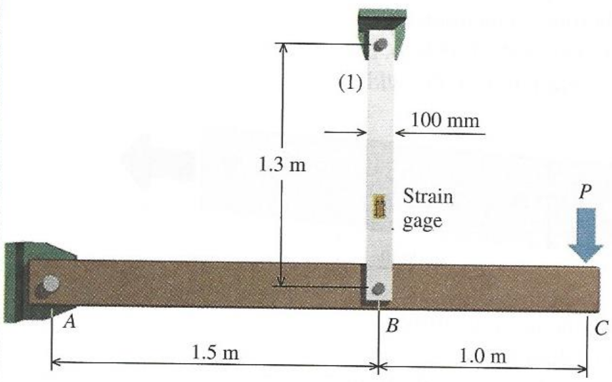
From Equation (3.6), Poisson's ratio can be computed as

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.002025 \text{ in./in.}}{0.005750 \text{ in./in.}} = 0.352 \qquad \textbf{Ans.}$$

(c) The shear modulus is then computed from Equation (3.7) as

$$G = \frac{E}{2(1+\nu)} = \frac{1{,}408{,}916 \text{ psi}}{2(1+0.352)} = 521{,}049 \text{ psi} = 521{,}000 \text{ psi} \qquad \textbf{Ans.}$$

EXAMPLE 3.2



Rigid bar *ABC* is supported by a pin at *A* and a 100 mm wide by 6 mm thick aluminum [E = 70 GPa; $\alpha = 22.5 \times 10^{-6}$/°C; $\nu = 0.33$] alloy bar at *B*. A strain gage affixed to the surface of the aluminum bar is used to measure its longitudinal strain. Before load *P* is applied to the rigid bar at *C*, the strain gage measures zero longitudinal strain at an ambient temperature of 20°C. After load *P* is applied to the rigid bar at *C* and the temperature drops to −10°C, a longitudinal strain of 2,400 με is measured in the aluminum bar. Determine

(a) the stress in member (1).
(b) the magnitude of load *P*.
(c) the change in the width of the aluminum bar (i.e., the 100 mm dimension).

Plan the Solution

This problem illustrates some misconceptions that are common in applying Hooke's law and Poisson's ratio, particularly when temperature change is a factor in the analysis.

SOLUTION

(a) Since the elastic modulus E and the longitudinal strain ε are given in the problem, one might be tempted to compute the normal stress in aluminum bar (1) from Hooke's law [Equation (3.4)]:

$$\sigma_1 = E_1\varepsilon_1 = (70 \text{ GPa})(2{,}400\ \mu\varepsilon)\left[\frac{1{,}000 \text{ MPa}}{1 \text{ GPa}}\right]\left[\frac{1 \text{ mm/mm}}{1{,}000{,}000\ \mu\varepsilon}\right] = 168 \text{ MPa}$$

This calculation is not correct for the normal stress in member (1). Why is it incorrect?

From Equation (2.7), the total strain ε_{total} in an object includes a portion ε_σ due to stress and a portion ε_T due to temperature change. The strain gage affixed to member (1) has measured the total strain in the aluminum bar as ε_{total} = 2,400 με = 0.002400 mm/mm. In this problem, however, the temperature of member (1) has dropped 30°C before the strain measurement. From Equation (2.6), the strain caused by the temperature change in the aluminum bar is

$$\varepsilon_T = \alpha\,\Delta T = (22.5 \times 10^{-6}/°\text{C})(-30°\text{C}) = -0.000675 \text{ mm/mm}$$

Hence, the strain caused by normal stress in member (1) is

$$\varepsilon_{\text{total}} = \varepsilon_\sigma + \varepsilon_T$$

$$\therefore \varepsilon_\sigma = \varepsilon_{\text{total}} - \varepsilon_T = 0.002400 \text{ mm/mm} - (-0.000675 \text{ mm/mm})$$
$$= 0.003075 \text{ mm/mm}$$

Using this strain value, we can now compute the normal stress in member (1) from Hooke's law:

$$\sigma_1 = E\varepsilon = (70 \text{ GPa})(0.003075 \text{ mm/mm}) = 215.25 \text{ MPa} = 215 \text{ MPa}$$ **Ans.**

(b) The axial force in member (1) is computed from the normal stress and the bar area:

$$F_1 = \sigma_1 A_1 = (215.25 \text{ N/mm}^2)(100 \text{ mm})(6 \text{ mm}) = 129{,}150 \text{ N}$$

Now write an equilibrium equation for the sum of moments about joint A, and solve for load P:

$$\Sigma M_A = (1.5 \text{ m})(129{,}150 \text{ N}) - (2.5 \text{ m})P = 0$$

$$\therefore P = 77{,}490 \text{ N} = 77.5 \text{ kN}$$ **Ans.**

(c) The change in the width of the bar is computed by multiplying the lateral (i.e., transverse) strain ε_{lat} by the 100 mm initial width. To determine ε_{lat}, the definition of Poisson's ratio [Equation (3.6)] is used:

$$\nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} \qquad \therefore \varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}}$$

Using the given value of Poisson's ratio and the measured strain, we could calculate ε_{lat} as

$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -(0.33)(2{,}400\ \mu\varepsilon) = -792\ \mu\varepsilon$$

This calculation is not correct for the lateral strain in member (1). Why is it incorrect?

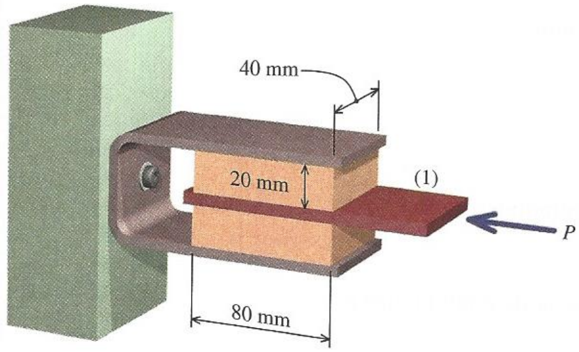
The Poisson effect applies only to strains caused by stresses (i.e., mechanical effects). When they are unrestrained, homogeneous, isotropic materials expand uniformly in all directions as they are heated (and contract uniformly as they cool). Consequently, thermal strains should not be included in the calculation of Poisson's ratio. For this problem, the lateral strain should be calculated as

$$\varepsilon_{\text{lat}} = -(0.33)(0.003075 \text{ mm/mm}) + (-0.000675 \text{ mm/mm}) = -0.0016898 \text{ mm/mm}$$

The change in the width of the aluminum bar is, therefore,

$$\delta_{\text{width}} = (-0.0016898 \text{ mm/mm})(100 \text{ mm}) = -0.1690 \text{ mm}$$ **Ans.**

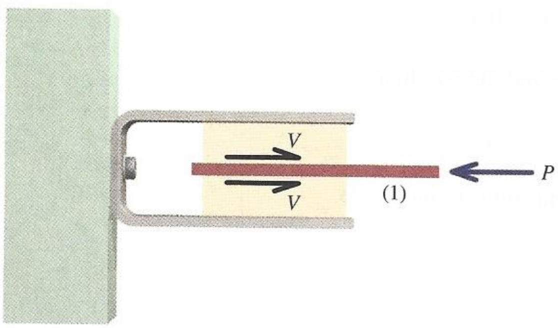
EXAMPLE 3.3



Two blocks of rubber, each 80 mm long by 40 mm wide by 20 mm thick, are bonded to a rigid support mount and to a movable plate (1). When a force $P = 2{,}800$ N is applied to the assembly, plate (1) deflects 8 mm horizontally. Determine the shear modulus G of the rubber used for the blocks.

Plan the Solution

Hooke's law [Equation (3.5)] expresses the relationship between shear stress and shear strain. The shear stress can be determined from the applied load P and the area of the rubber blocks that contact the movable plate (1). Shear strain is an angular measure that can be determined from the horizontal deflection of plate (1) and the thickness of the rubber blocks. The shear modulus G is computed by dividing the shear stress by the shear strain.

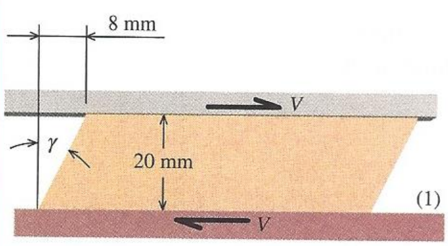


SOLUTION

Consider a free-body diagram of movable plate (1). Each rubber block provides a shear force that opposes the applied load P. From a consideration of equilibrium, the sum of forces in the horizontal direction is

$$\Sigma F_x = 2V - P = 0$$

$$\therefore V = P/2 = (2{,}800 \text{ N})/2 = 1{,}400 \text{ N}$$



Next, consider a free-body diagram of the upper rubber block in its deflected position. The shear force V acts on a surface that is 80 mm long and 40 mm wide. Therefore, the average shear stress in the rubber block is

$$\tau = \frac{1{,}400 \text{ N}}{(80 \text{ mm})(40 \text{ mm})} = 0.4375 \text{ MPa}$$

The 8 mm horizontal deflection causes the block to skew as shown. The angle γ (measured in radians) is the shear strain:

$$\tan\gamma = \frac{8 \text{ mm}}{20 \text{ mm}} \qquad \therefore \gamma = 0.3805 \text{ rad}$$

The shear stress τ, the shear modulus G, and the shear strain γ are related by Hooke's law:

$$\tau = G\gamma$$

Therefore, the shear modulus of the rubber used for the blocks is

$$G = \frac{\tau}{\gamma} = \frac{0.4375 \text{ MPa}}{0.3805 \text{ rad}} = 1.150 \text{ MPa}$$ **Ans.**

MecMovies

EXERCISE

M3.1 Figure M3.1 depicts basic problems requiring the use of Hooke's law.

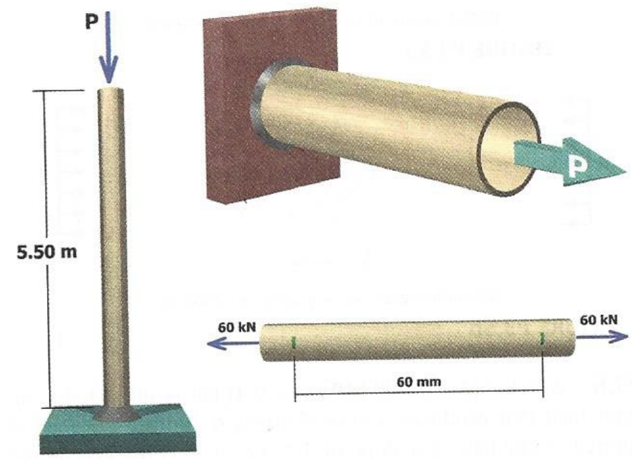


FIGURE M3.1

PROBLEMS

P3.1 At the proportional limit, a 2 in. gage length of a 0.500 in. diameter alloy rod has elongated 0.0035 in. and the diameter has been reduced by 0.0003 in. The total tension force on the rod was 5.45 kips. Determine the following properties of the material:

(a) the proportional limit.
(b) the modulus of elasticity.
(c) Poisson's ratio.

P3.2 A solid circular rod with a diameter $d = 16$ mm is shown in Figure P3.2. The rod is made of an aluminum alloy that has an elastic modulus $E = 72$ GPa and a Poisson's ratio $\nu = 0.33$. When subjected to the axial load P, the diameter of the rod decreases by 0.024 mm. Determine the magnitude of load P.

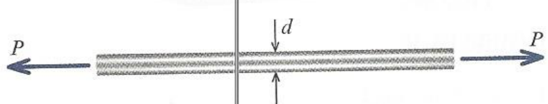


FIGURE P3.2

P3.3 The polymer bar shown in Figure P3.3 has a width $b = 50$ mm, a depth $d = 100$ mm, and a height $h = 270$ mm. At a compressive load $P = 135$ kN, the bar height contracts by $\Delta h = -2.50$ mm and the bar depth elongates by $\Delta d = 0.38$ mm. At this load, the stress in the polymer bar is less than its proportional limit. Determine

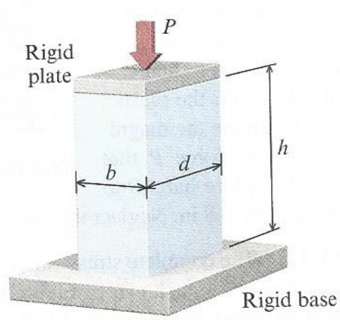


FIGURE P3.3

(a) the modulus of elasticity.
(b) Poisson's ratio.
(c) the change in the bar width b.

P3.4 A 0.625 in. thick rectangular alloy bar is subjected to a tensile load P by pins at A and B as shown in Figure P3.4. The width of the bar is $w = 2.00$ in. Strain gages bonded to the specimen measure the following strains in the longitudinal (x) and transverse (y) directions: $\varepsilon_x = 1{,}140\ \mu\varepsilon$ and $\varepsilon_y = -315\ \mu\varepsilon$.

(a) Determine Poisson's ratio for this specimen.
(b) If the measured strains were produced by an axial load $P =$ 17.4 kips, what is the modulus of elasticity for this specimen?

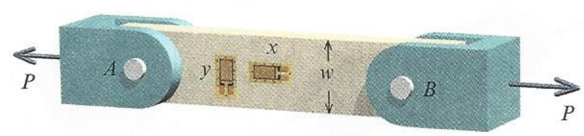
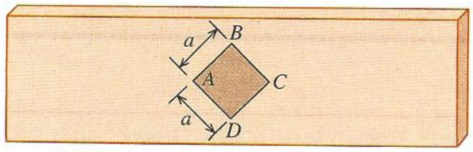


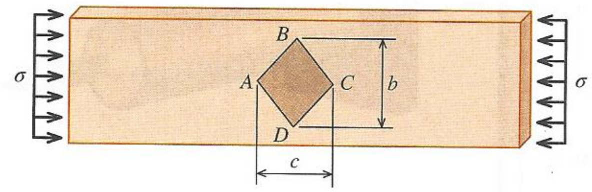
FIGURE P3.4

P3.5 A 40 mm by 40 mm square $ABCD$ (i.e., $a = 40$ mm) is drawn on a rectangular bar prior to loading. (See Figure P3.5a). A uniform normal stress $\sigma = 54$ MPa is then applied to the ends of the rectangular bar, and square $ABCD$ is deformed into the shape of a rhombus, as shown in the Figure P3.5b. The dimensions of the rhombus after loading are $b = 56.88$ mm and $c = 55.61$ mm. Determine the modulus of elasticity for the material. Assume that the material behaves elastically for the applied stress.



Initial square drawn on bar before loading.

FIGURE P3.5a



Rhombus after bar is loaded by stress σ.

FIGURE P3.5b

P3.6 A nylon [$E = 2{,}500$ MPa; $\nu = 0.4$] bar is subjected to an axial load that produces a normal stress σ. Before the load is applied, a line having a slope of 3:2 (i.e., 1.5) is marked on the bar as shown in Figure P3.6. Determine the slope of the line when $\sigma = 105$ MPa.

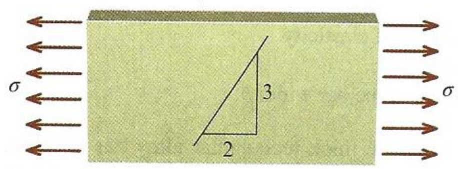


FIGURE P3.6

P3.7 A nylon [$E = 360$ ksi; $\nu = 0.4$] rod (1) having a diameter $d_1 = 2.00$ in. is placed inside a steel [$E = 29{,}000$ ksi; $\nu = 0.29$] tube (2) as shown in Figure P3.7. The inside diameter of the tube is $d_2 = 2.02$ in. An external load P is applied to the rod, compressing it. At what value of P will the space between the rod and the tube be closed?

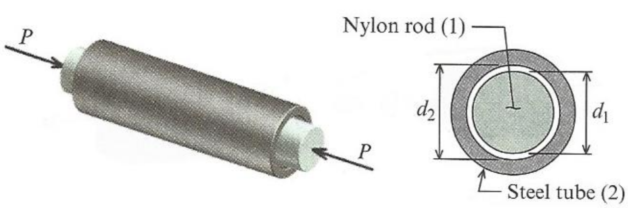


FIGURE P3.7

P3.8 A metal specimen with an original diameter of 0.500 in. and a gage length of 2.000 in. is tested in tension until fracture occurs. At the point of fracture, the diameter of the specimen is 0.260 in. and the fractured gage length is 3.08 in. Calculate the ductility in terms of percent elongation and percent reduction in area.

P3.9 A portion of the stress–strain curve for a stainless steel alloy is shown in Figure P3.9. A 350 mm long bar is loaded in tension until it elongates 2.0 mm, and then the load is removed.

(a) What is the permanent set in the bar?
(b) What is the length of the unloaded bar?
(c) If the bar is reloaded, what will be the proportional limit?

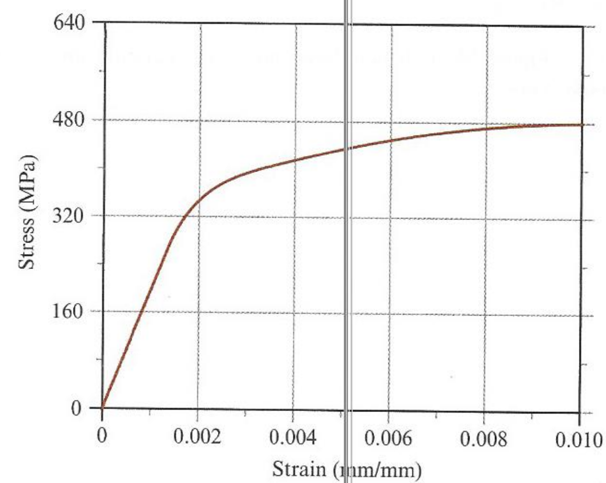


FIGURE P3.9

P3.10 A plastic block is bonded to a fixed base and to a horizontal rigid plate as shown in Figure P3.10. The shear modulus of the plastic is $G = 45{,}000$ psi, and the block dimensions are $a = 4.0$ in., $b = 2.0$ in., and $c = 1.50$ in. A horizontal force $P = 8{,}500$ lb is applied to the plate. Determine the horizontal deflection of the plate.

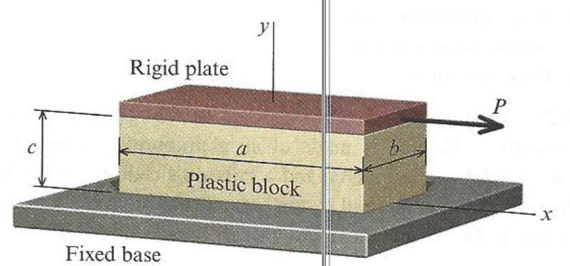


FIGURE P3.10

P3.11 A 0.5 in. thick plastic panel is bonded to the pin-jointed steel frame shown in Figure P3.11. Assume that $a = 4.0$ ft, $b = 6.0$ ft, and $G = 70{,}000$ psi for the plastic, and determine the magnitude of the force P that would displace bar AB to the right by 0.8 in. Neglect the deformation of the steel frame.

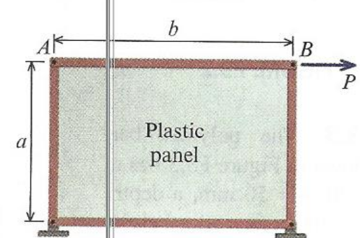


FIGURE P3.11

P3.12 The complete stress–strain diagram for a particular stainless steel alloy is shown in Figure P3.12*a*/13*a*. This diagram has been enlarged in Figure P3.12*b*/13*b* to show in more detail the linear portion of the stress-strain diagram. A rod made from this material is initially 800 mm long at a temperature of 20°C. After a tension force is applied to the rod and the temperature is increased by 200°C, the length of the rod is 804 mm. Determine the stress in the rod, and state whether the elongation in the rod is elastic or inelastic. Assume the coefficient of thermal expansion for this material is 18×10^{-6}/°C.

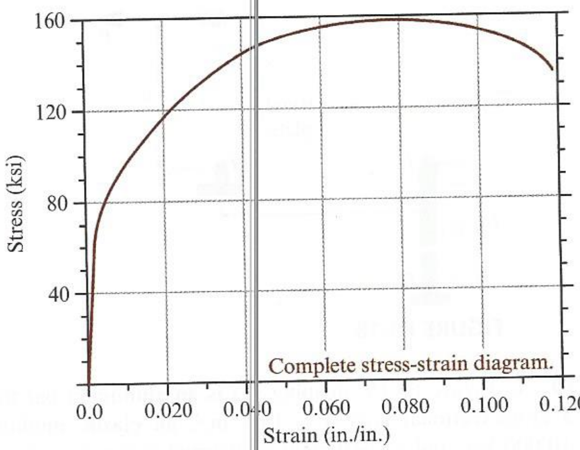


FIGURE P3.12*a*/13*a*

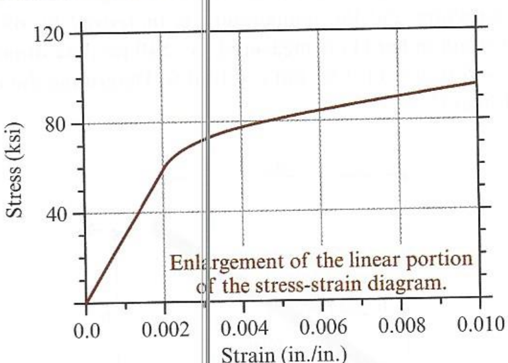


FIGURE P3.12*b*/13*b*

P3.13 A tensile test specimen of stainless steel alloy having a diameter of 12.6 mm and a gage length of 50 mm was tested to fracture. The complete stress–strain diagram for this specimen is shown in Figure P3.12*a*/13*a*. This diagram has been enlarged in Figure P3.12*b*/13*b* to show in more detail the linear portion of the stress-strain diagram. Determine

(a) the modulus of elasticity.
(b) the proportional limit.
(c) the ultimate strength.
(d) the yield strength (0.20% offset).
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 8.89 mm.

P3.14 A 7075-T651 aluminum alloy specimen with a diameter of 0.500 in. and a 2.0 in. gage length was tested to fracture. Load and deformation data obtained during the test are given in the accompanying table. Determine

(a) the modulus of elasticity.
(b) the proportional limit.
(c) the yield strength (0.20% offset).
(d) the ultimate strength.
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 0.387 in.

Load (lb)	Change in Length (in.)	Load (lb)	Change in Length (in.)
0	0	14,690	0.0149
1,221	0.0012	14,744	0.0150
2,479	0.0024	15,119	0.0159
3,667	0.0035	15,490	0.0202
4,903	0.0048	15,710	0.0288
6,138	0.0060	16,032	0.0581
7,356	0.0072	16,295	0.0895
8,596	0.0085	16,456	0.1214
9,783	0.0096	16,585	0.1496
11,050	0.0110	16,601	0.1817
12,247	0.0122	16,601	0.2278
13,434	0.0134	16,489	0.2605
		16,480	fracture

P3.15 A Grade 2 Titanium tension test specimen has a diameter of 12.60 mm and a gage length of 50 mm. In a test to fracture, the stress and strain data shown in the accompanying table were obtained. Determine

(a) the modulus of elasticity.
(b) the proportional limit.
(c) the yield strength (0.20% offset).
(d) the ultimate strength.
(e) the fracture stress.
(f) the true fracture stress if the final diameter of the specimen at the location of the fracture was 9.77 mm.

Load (kN)	Change in Length (mm)	Load (kN)	Change in Length (mm)
0.00	0.000	52.74	0.314
4.49	0.017	56.95	0.480
8.84	0.032	60.76	0.840
13.29	0.050	63.96	1.334
17.57	0.064	66.61	1.908
22.10	0.085	68.26	2.562
26.46	0.103	69.08	3.217
30.84	0.123	69.41	3.938
35.18	0.144	69.39	4.666
39.70	0.171	69.25	5.292
43.95	0.201	68.82	6.023
48.44	0.241	68.35	6.731
		68.17	fracture

P3.16 Compound axial member *ABC* shown in Figure P3.16 has a uniform diameter $d = 1.50$ in. Segment (1) is an aluminum [$E_1 =$ 10,000 ksi] alloy rod with length $L_1 = 90$ in. Segment (2) is a copper [$E_2 =$ 17,000 ksi] alloy rod with length $L_2 = 130$ in. When axial force *P* is applied, a strain gage attached to copper segment (2) measures a normal strain of $\varepsilon_2 = 2{,}100$ μin./in. in the longitudinal direction. What is the total elongation of member *ABC*?

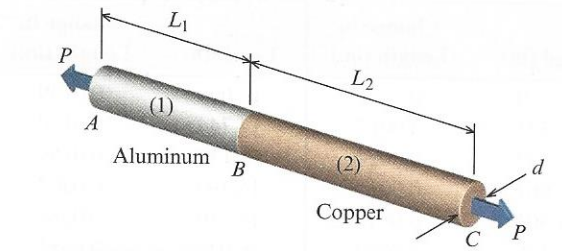


FIGURE P3.16

P3.17 An aluminum alloy [$E =$ 70 GPa; $\nu = 0.33$; $\alpha = 23.0 \times 10^{-6}$/°C] plate is subjected to a tensile load *P* as shown in Figure P3.17. The plate has a depth $d =$ 260 mm, a cross-sectional area $A = 6{,}500$ mm², and a length $L =$ 4.5 m. The initial longitudinal normal strain in the plate is zero. After load *P* is applied and the temperature of the plate has been increased by $\Delta T = 56$°C, the longitudinal normal strain in the plate is found to be 2,950 με. Determine

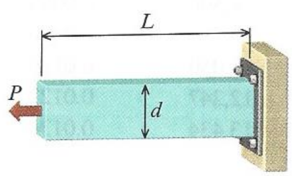


FIGURE P3.17

(a) the magnitude of load *P*.
(b) the change Δd in plate depth.

P3.18 The rigid plate in Figure P3.18 is supported by bar (1) and by a double-shear pin connection at *B*. Bar (1) has a length $L_1 = 60$ in., a cross-sectional area $A_1 = 0.47$ in.², an elastic modulus $E = 10{,}000$ ksi, and a coefficient of thermal expansion of $\alpha = 13 \times 10^{-6}$/°F. The pin at *B* has a diameter of 0.438 in. After load *P* has been applied and the temperature of the entire assembly has been *decreased* by 30°F, the total strain in bar (1) is measured as 570 με (elongation). Assume dimensions of $a = 12$ in. and $b = 20$ in. Determine

(a) the magnitude of load *P*.
(b) the average shear stress in pin *B*.

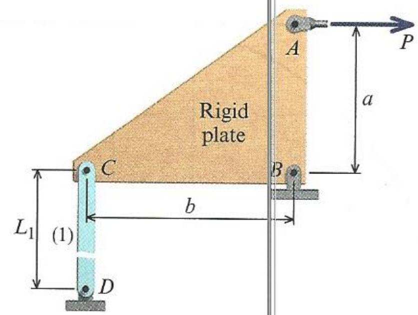


FIGURE P3.18

P3.19 In Figure P3.19, member (1) is an aluminum bar that has a cross-sectional area $A = 1.05$ in.², an elastic modulus $E = 10{,}000$ ksi, and a coefficient of thermal expansion of $\alpha = 12.5 \times 10^{-6}$/°F. After a load *P* of unknown magnitude is applied to the structure and the temperature is *increased* by 65°F, the normal strain in bar (1) is measured as −540 με. Use dimensions of $a = 24.6$ ft, $b = 11.7$ ft, and $c = 14.0$ ft. Determine the magnitude of load *P*.

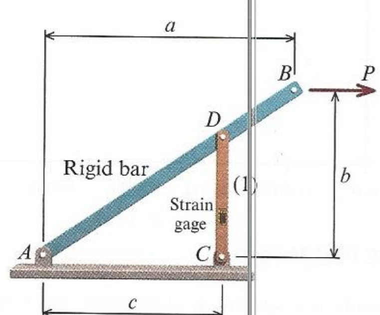


FIGURE P3.19