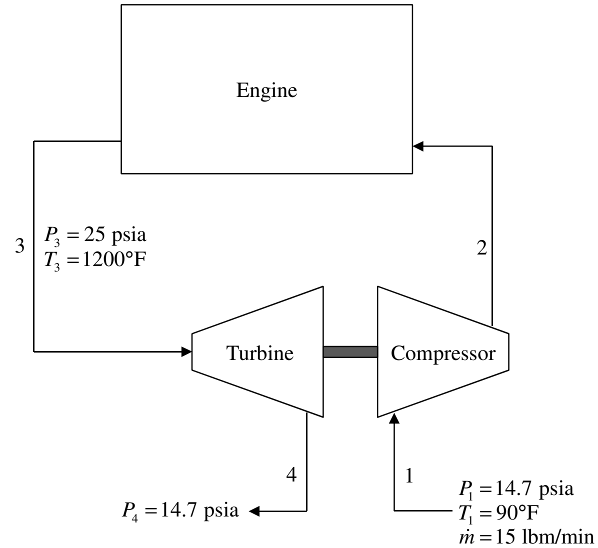
TURBOCHARGER ANALYSIS

GIVEN: A turbocharger used to boost the pressure of the air entering the combustion process of an engine as shown below. As a first estimate, the turbine and compressor can be modeled as isentropic.

FIND: (a) The turbine exit temperature and power delivery
(b) The compressor exit pressure and temperature
(c) Calculate (a) and (b) for the case where the turbine has an isentropic efficiency of 85% and the compressor has an isentropic efficiency of 80%.



Consider air as an ideal gas with constant heat capacity (c_p = 0.24 Btu/lbm-R – Table C.13a). Then,

$$\eta_t = \frac{\dot{W}_t}{W_{ts}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Analyzing the Isentropic Turbine

State 4s … process is isentropic.

$$\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k} \quad \rightarrow \quad T_{4s} = (1200 + 459.67)\,\text{R}\left(\frac{14.7 \text{ psia}}{25 \text{ psia}}\right)^{(1.4-1)/1.4} = 1426 \text{ R} = \underline{\underline{966.4°\text{F}}}$$

$$\dot{W}_{ts} = \dot{m}\left(h_3 - h_{4s}\right) = \dot{m}c_p\left(T_3 - T_{4s}\right)$$

$$\dot{W}_{ts} = \left(15\frac{\text{lbm}}{\text{min}}\right)\left(0.24\frac{\text{Btu}}{\text{lbm-R}}\right)(1200 - 966.4)\,\text{R}\left|\frac{60 \text{ min}}{\text{hr}}\,\frac{\text{hp-hr}}{2545 \text{ Btu}}\right. = \underline{\underline{20.56 \text{ hp}}}$$

Analyzing the Real Turbine

For the non-isentropic turbine,

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad \rightarrow \quad T_4 = T_3 - \eta_t\left(T_3 - T_{4s}\right) = 1200°\text{F} - (0.85)(1200 - 966.4)°\text{F} = \underline{\underline{1001.4°\text{F}}}$$

$$\dot{W}_t = \dot{m}c_p(T_3 - T_4) = \left(15\frac{\text{lbm}}{\text{min}}\right)\left(0.24\frac{\text{Btu}}{\text{lbm-R}}\right)(1200 - 1001.4)\,\text{R}\left|\frac{60 \text{ min}}{\text{hr}}\,\frac{\text{hp-hr}}{2545 \text{ Btu}}\right. = \underline{\underline{16.85 \text{ hp}}}$$

Analyzing the Isentropic Compressor

For the isentropic (ideal) compressor,

$$\dot{W}_{cs} = \dot{m}c_p\left(T_1 - T_{2s}\right)$$

$$T_{2s} = T_1 - \frac{\dot{W}_c}{\dot{m}c_p} = 90°\text{F} - \frac{-20.56\ \text{hp}\left|\frac{2545\ \text{Btu}}{\text{hp-hr}}\right.}{\left(15\frac{\text{lbm}}{\text{min}}\right)\left(0.24\frac{\text{Btu}}{\text{lbm-R}}\right)\left|\frac{60\ \text{min}}{\text{hr}}\right.} = \underline{\underline{332.2°\text{F}}}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \rightarrow \frac{P_2}{P_1} = \left(\frac{T_{2s}}{T_1}\right)^{k/(k-1)} \rightarrow P_2 = \left(14.7\ \text{psia}\right)\left(\frac{332.2+459.67}{90+459.67}\right)^{1.4/(1.4-1)} = \underline{\underline{52.8\ \text{psia}}}$$

Analyzing the Real Compressor

For the real compressor,

$$\dot{W}_c = \dot{m}c_p\left(T_1 - T_2\right)$$

$$T_2 = T_1 - \frac{\dot{W}_c}{\dot{m}c_p} = 90°\text{F} - \frac{-16.85\ \text{hp}\left|\frac{2545\ \text{Btu}}{\text{hp-hr}}\right.}{\left(15\frac{\text{lbm}}{\text{min}}\right)\left(0.24\frac{\text{Btu}}{\text{lbm-R}}\right)\left|\frac{60\ \text{min}}{\text{hr}}\right.} = \underline{\underline{288.5°\text{F}}}$$

The temperature at the end of the isentropic process is determined from the isentropic efficiency,

$$\eta_c = \frac{\dot{W}_{cs}}{W_c} = \frac{h_1 - h_{2s}}{h_1 - h_2} = \frac{T_1 - T_{2s}}{T_1 - T_2}$$

$$T_{2s} = T_1 - \eta_c\left(T_1 - T_2\right) = 90°\text{F} - \left(0.80\right)\left(90 - 288.5\right)°\text{F} = \underline{\underline{248.8°\text{F}}}$$

This allows us to find the exhaust pressure (for both cases since pressure is the same for both!).

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \rightarrow \frac{P_2}{P_1} = \left(\frac{T_{2s}}{T_1}\right)^{k/(k-1)} \rightarrow P_2 = \left(14.7\ \text{psia}\right)\left(\frac{248.8+459.67}{90+459.67}\right)^{1.4/(1.4-1)} = \underline{\underline{35.7\ \text{psia}}}$$

Comparison to values calculated with EES using 'air_ha'.

Case	$\dot{W}_t$ (hp)	T_4 (°F)	P_2 (psia)	T_2 (°F)
$\eta_t = 1.0$ $\eta_c = 1.0$	20.56 (IDG) 19.99 (RF)	966.4 (IDG) 987.5 (RF)	52.8 (IDG) 51.2 (RF)	332.2 (IDG) 324.2 (RF)
$\eta_t = 0.85$ $\eta_c = 0.80$	16.85 (IDG) 16.99 (RF)	1001.4 (IDG) 1020.0 (RF)	35.7 (IDG) 35.9 (RF)	288.5 (IDG) 289.2 (RF)

Solution strategy - EES

The exit state of the air leaving the turbine can be found since the isentropic efficiency of the turbine is known,

$$\eta_t = \frac{\dot{W}_t}{W_{ts}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

The properties at state 3 are known from pressure and temperature. At state 4s, the pressure and entropy are known. This allows the calculation of h_4 which allows us to find the exit temperature of the turbine. The power delivered to the compressor is determined from a First Law balance on the turbine,

$$\dot{W}_t = \dot{m}\left(h_3 - h_4\right)$$

The power delivered by the turbine becomes the input to the compressor. Therefore,

$$\dot{W}_c = -\dot{W}_t$$

The First Law on the compressor allows us to find the enthalpy at the exit of the compressor.

$$\dot{W}_c = \dot{m}\left(h_1 - h_2\right)$$

However … this is not enough! We also know the isentropic efficiency of the compressor,

$$\eta_c = \frac{\dot{W}_{cs}}{W_c} = \frac{h_1 - h_{2s}}{h_1 - h_2}$$

The discharge pressure can be found because we now know the enthalpy and entropy at the end of the *isentropic* process! The temperature can also be found! EES time!