8 Modeling Traffic Flow at Signalized Intersections

PURPOSE

The purpose of this activity is to introduce you to queuing systems as they apply to the operation of signalized intersections.

LEARNING OBJECTIVE

- Describe the basic components of a queuing model representing traffic flow at a signalized intersection

DELIVERABLES

- Define the terms and variables in the Glossary
- Prepare a document that includes answers to the Critical Thinking Questions

GLOSSARY

Provide a definition for each of the following terms and variables. Paraphrasing a formal definition (as provided by your text, instructor, or another resource) demonstrates that you understand the meaning of the term or phrase.

average delay	
cumulative vehicle diagram	
platoon	
queue accumulation polygon	
queuing system	
saturation flow rate	
time space diagram	

v	
t	
r	
g	
g_q	
s	
d_i	
t_d	
t_a	
d_a	
c (lowercase)	
C (uppercase)	
X	

CRITICAL THINKING QUESTIONS

When you have completed the reading, prepare answers to the following questions.

1. What is a queuing system?

2. Which elements of a traffic control system are included in the queuing system?

3. Which elements of traffic flow can you represent in a time space diagram?

4. What is a *queue accumulation polygon* and what information does it show about intersection operation and performance?

5. How realistic is the uniform delay equation or model?

6. What performance measures can the cumulative vehicle diagram show?

7. What are the elements common to a flow profile diagram, a cumulative vehicle diagram, and a queue accumulation polygon?

INFORMATION

In the previous chapter, we described the process of vehicles arriving at a signalized intersection and the response of the drivers of these vehicles to the various vehicle displays. We considered driver responses during the red interval, the beginning of the green interval when the queue is clearing, during the green interval after the queue clears and entering the intersection without delay, and during the yellow and red intervals. In some ways, this is the most basic kind of intersection operation, the ebb and flow of vehicles arriving at and traveling through the intersection, as time progresses from the red interval to the green interval to the yellow interval, and back again to red.

In this chapter, we will consider an idealized representation, or model, of vehicular traffic arriving at and departing from the intersection. It is useful to abstract or model this narrative description into more mathematical terms that we can use and extend to more complex conditions. As we proceed to these idealized representations, which are important as learning and visualization tools, we shouldn't forget that the real world isn't quite this clean and sharp, something that we shall see in later in this chapter!

Representing traffic flow using a time-space diagram

Let's first consider vehicles arriving at the intersection. Figure 27 shows three vehicles representing the ideal condition of uniform or constant headways, as they approach an intersection. The "space" of the intersection approach is shown on the y-axis, while time is shown on the x-axis. The vehicle display is shown at the bottom of the figure, with each of the three horizontal spaces available to show the time histories of the red, yellow, and green displays. This representation is known as a time-space diagram.

Figure 27. Vehicles evenly spaced approaching intersection

Figure 28 shows the trajectories for the three vehicles. The time space diagram shows that the vehicles had been traveling at constant (and equal) speeds as the slopes of the trajectories are parallel and linear. The display status is red.
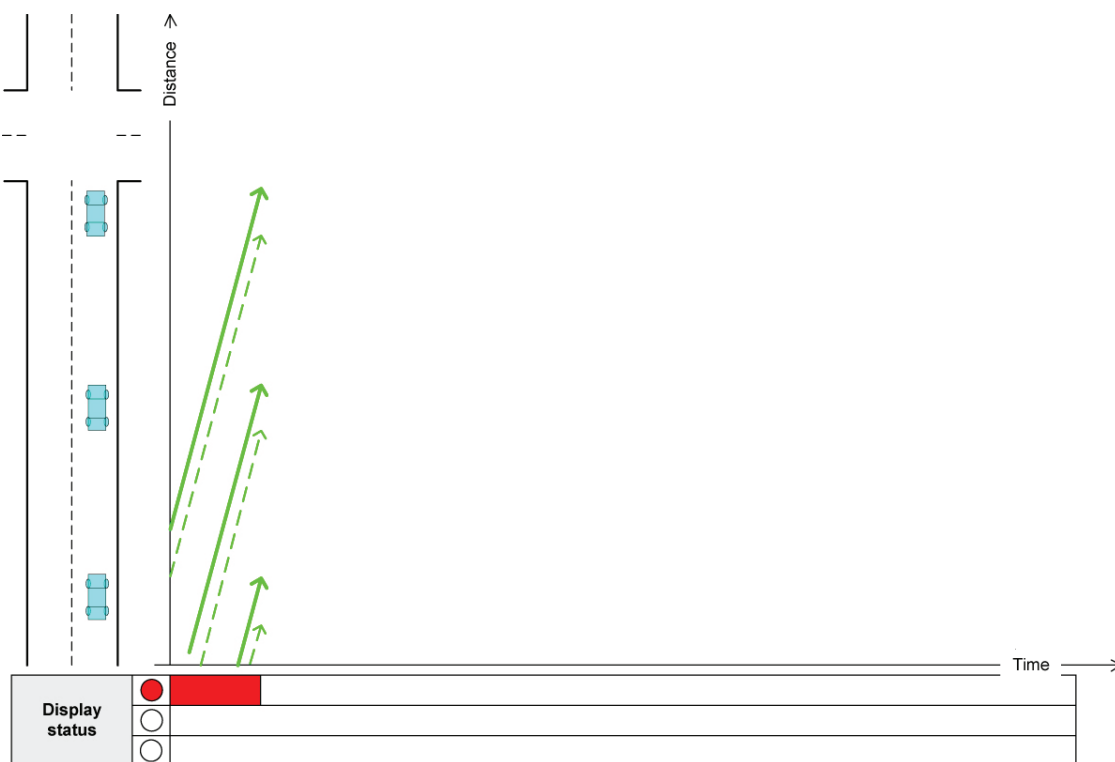
Figure 28. Vehicle trajectories, for evenly spaced vehicles approaching intersection during red

The response of the vehicles to the red indication is shown in Figure 29. The trajectories of the vehicles show this response as vehicles decelerate and their speeds go to zero. There is a queue of five vehicles at the end of the red interval.
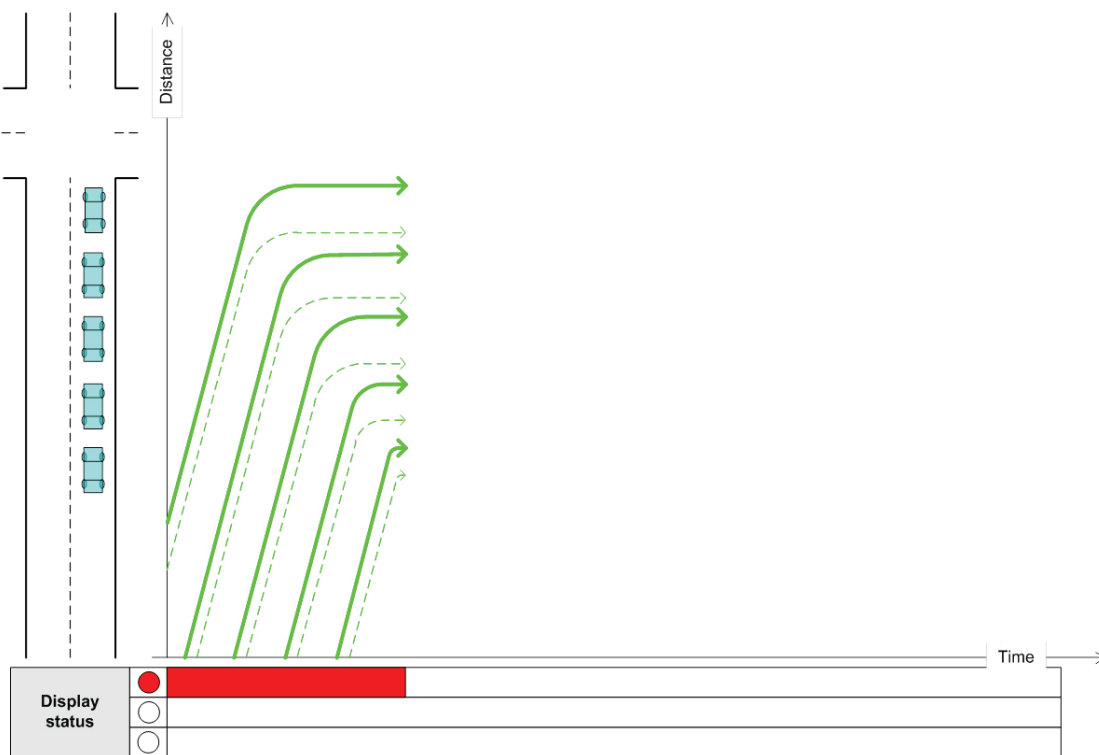
Figure 29. Vehicles respond to red display

Figure 30 shows the response of the queued vehicles to the change in display to green, as they begin to accelerate and move into the intersection.
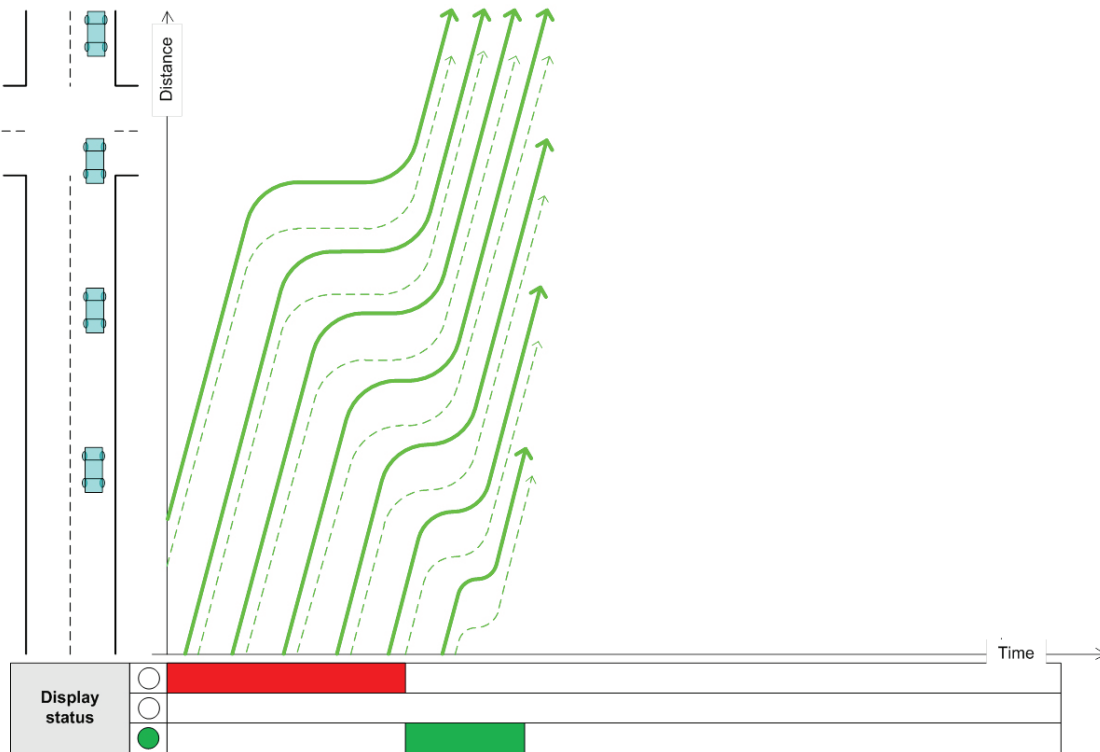

Figure 30. Vehicles respond to green display as queue begins to move and clear.

Figure 31 shows the vehicles that arrive after the queue has cleared. They arrive and leave with no delay, as shown by the constant slope of the vehicle trajectories.
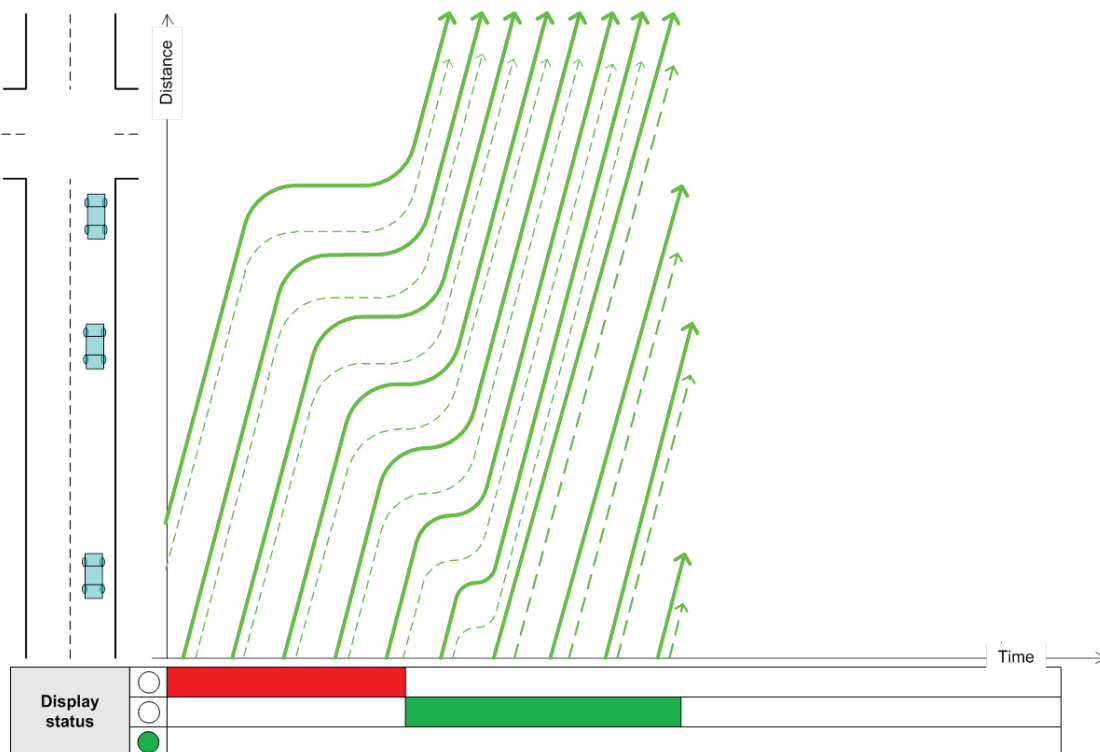

Figure 31. Vehicles responding to the green display after the clearance of the queue

Figure 32 shows vehicles approaching the intersection at the onset of the yellow interval. The last two vehicles shown will not enter the intersection but will begin to stop.
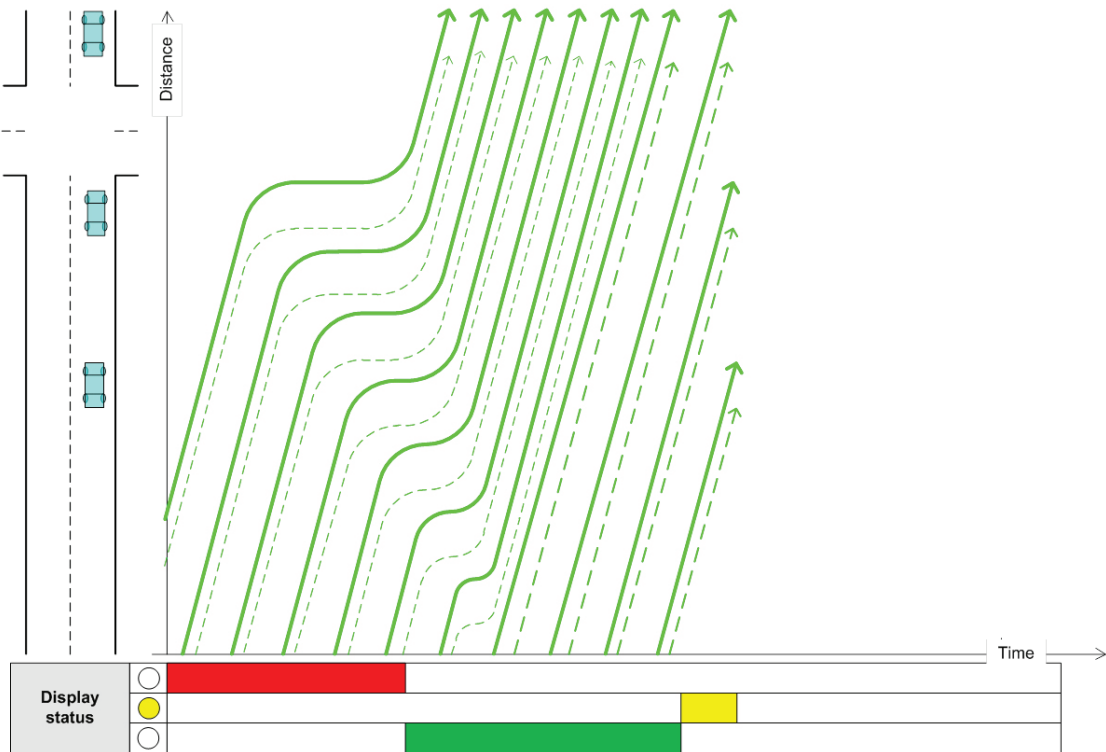

Figure 32. Vehicles responding to yellow display

Figure 33 shows the last two vehicles responding to the red display, stopping at the intersection and forming a new queue.
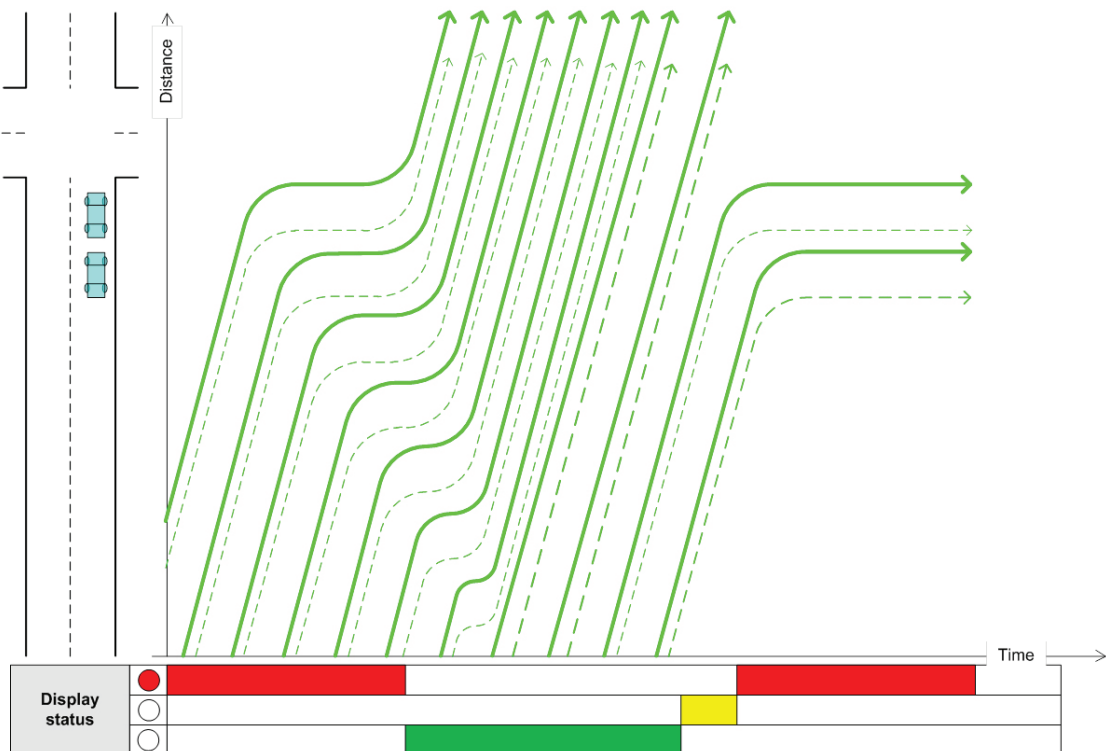

Figure 33. Vehicles responding to red display

While the time-space diagram is an approximation or model of the vehicles traveling through the intersection, particularly with our assumption of uniform headways, we can learn a lot from this diagram. The following six bullets are illustrated in Figure 34 (as adapted from May, 1989) for the vehicles that we've just observed.

1. Shock wave of the queue forming during red.

2. Shock wave of the queue clearing during green.

3. Delay for each vehicle, the horizontal line between the time that the vehicle arrives and the time that it leaves.

4. The saturation headway, the headway between two vehicles departing as part of the clearing queue at the beginning of green, measured at the stop line.

5. The slope of the vehicle trajectory, the vehicle speed.

6. The time between the passage of the end of the leaving vehicle and the front of the following vehicle by a given point, the time gap.
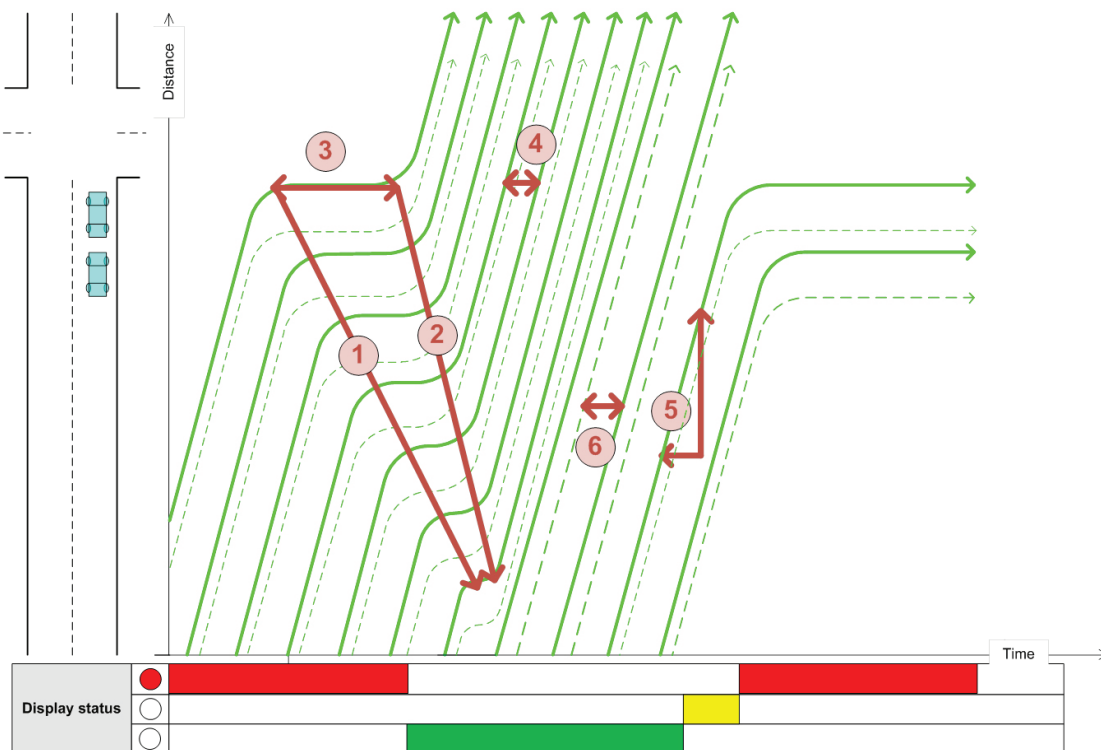

Figure 34. Examples of information available from time-space diagram

We can summarize the categories of driver response in Figure 35, showing:

1. Drivers responding to the red indication by stopping.

2. Drivers responding to the green indication by beginning to move through the intersection.

3. Drivers responding to the green indication (after the queue is cleared) by traveling through the intersection without stopping (without delay).

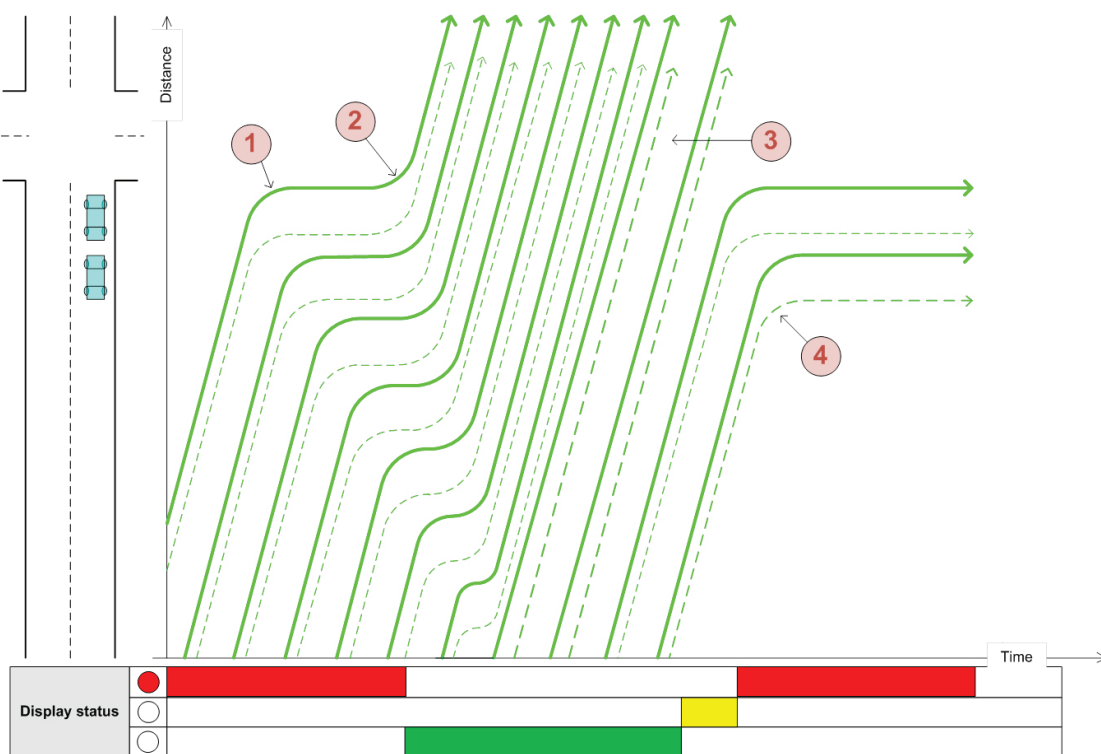4. Drivers responding to the yellow indication by stopping.

Figure 35. Four categories of driver response

Queuing System Representation

Another way of modeling the traffic flow at one approach of a signalized intersection is to use queuing theory. Queuing theory, originally developed to model and help design the nation's telephone communications system, is based on a system that includes users who desire to be served in some way, a server, and a process for serving these users. For example, consider shoppers who are being checked out at a supermarket in Figure 36. They arrive at the check stand with the shopping carts, they wait in line, and they are checked out or served by the checker at the check stand. Formal queuing theory includes specifying the arrival pattern, the service pattern, and the number of service channels, among other factors. The model that we will consider here is called the D/D/1 model, for deterministic (D) arrivals, a deterministic (D) service pattern, and one service channel.

Figure 36. The supermarket checkout line as a queuing system

Figure 37 shows the elements of a queuing system for vehicles traveling on the through lane of a signalized intersection. A vehicle at the stop line, waiting to travel through the intersection, is said to occupy the server as it is waiting to be served. The act of service for a vehicle involves having a green indication and responding to it. Any vehicles waiting behind the server position are said to be waiting in queue. It should be noted that the term "queue" as used by traffic engineers includes vehicles in both the server and the queue. In queuing theory terminology, the queue does not include the first-in-line position (the server). In addition, in queuing theory the queue is assumed to be stacked vertically at the intersection stop bar.
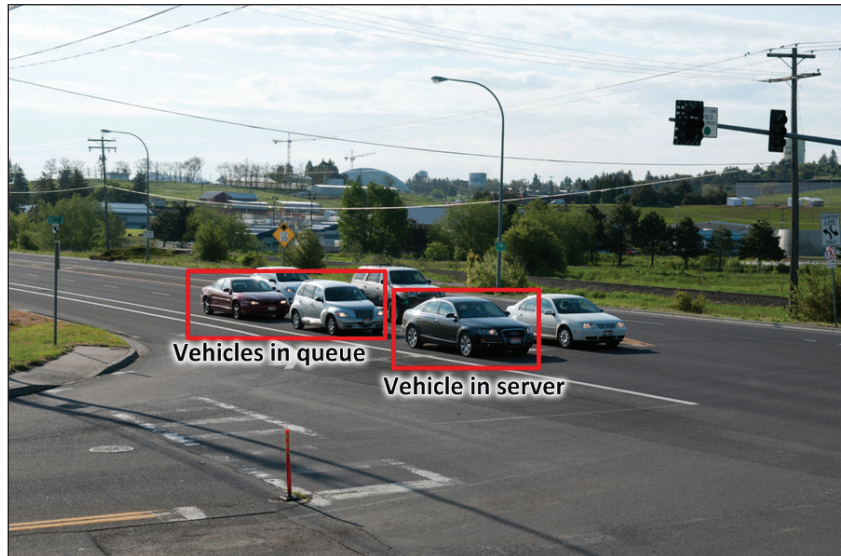

Figure 37. Elements of queuing system

Since one vehicle can be served at a time, the number of service channels is said to be one. And, since vehicles are served in the order that they arrive at the intersection, the queue discipline is "first in, first out."

The flow rate as measured upstream of the approach stop line is called the demand or arrival rate. The flow rate measured at the stop bar, as vehicles are being served, is called the service rate. As we will see in the next section, the service rate varies by time during the signal cycle and can be divided into three segments:

- zero, during the red interval

- the saturation flow rate, during the initial period of green when the queue is clearing

- the arrival rate, during green after the queue has cleared

While the time space diagram shows the response of vehicles to the signal display, we can use other graphical representations of the process, each related to the arrival and departure patterns of vehicles at the intersection. We can represent the flow patterns for this queuing system in three different ways, using a flow profile diagram, a cumulative vehicle diagram, and a queue accumulation polygon. Each diagram will be described in the following sections of this chapter.

Flow Profile Diagram

The flow profile diagram represents the flow rates of vehicles arriving at and departing from the intersection over time. Figure 38 shows a graph of the rates that vehicles arrive at the intersection and then depart from it during one cycle. During both the red and green intervals, vehicles arrive at a constant or uniform rate that we will call the arrival rate or approach demand. This rate is shown in Figure 38 as a solid dark line. The departure, or service, rate (shown as the lighter dashed line) depends on the status of the signal. During the red interval, the service rate is zero. During the initial part of the green interval (which we will call the queue service time, g_q), the service rate is equal to the saturation flow rate (shown below as s). After the queue has dissipated, the service rate equals the arrival rate for the remainder of the green interval, which we will call g_u, or the unsaturated portion of the green interval. Note that the total green interval (g) is what is sometimes called the "effective green", or the total time available to and usable by vehicles traveling through the intersection.
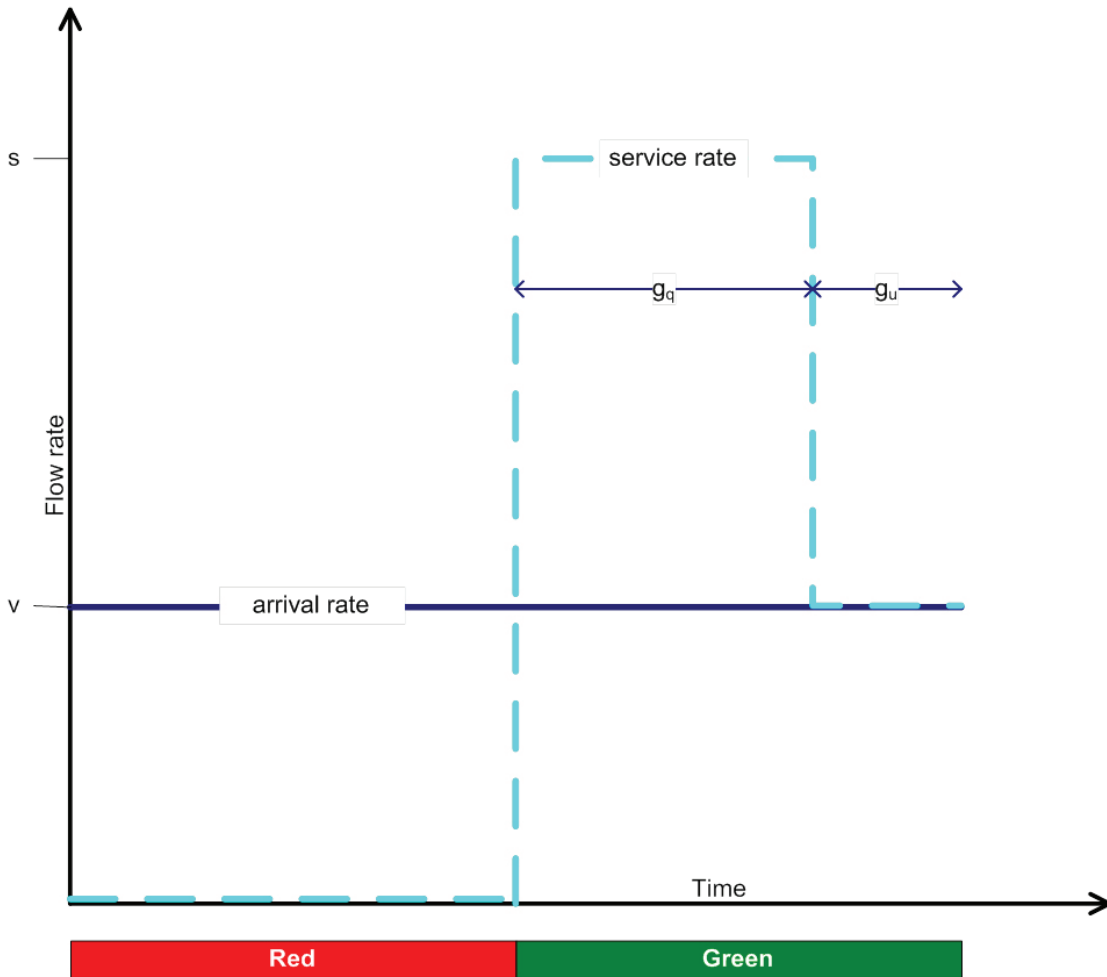

Figure 38. Flow profile diagram

Cumulative Vehicle Diagram

Another representation of this same process is the cumulative vehicle diagram, or the number of vehicles that have arrived at and departed from the intersection at any point in time during the cycle. We call these functions the cumulative arrivals and cumulative departures and we can graph them as shown in Figure 39. The slope of the arrival curve is equal to the arrival rate, while the slope of the second segment of the departure curve (its steepest portion) is equal to the saturation flow rate.
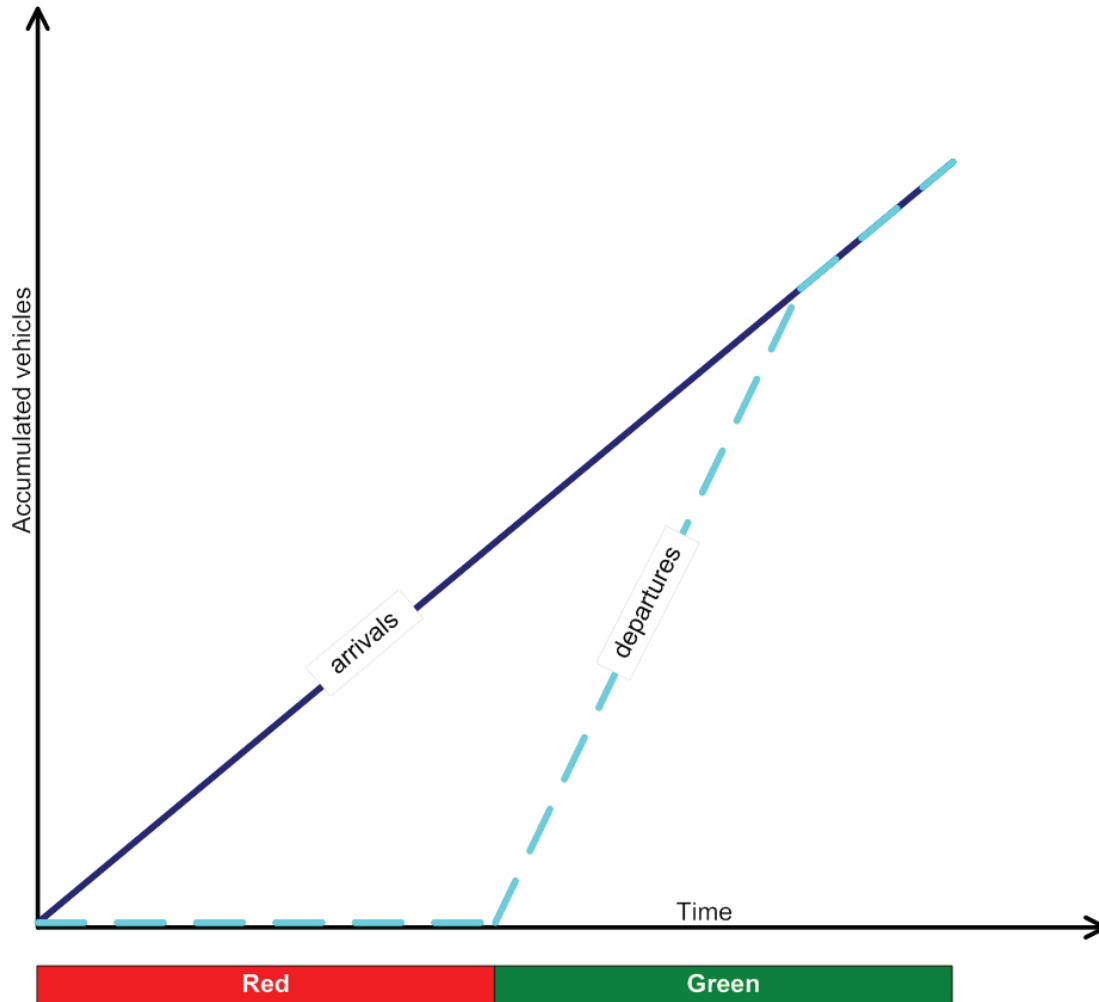
Figure 39. Cumulative vehicle diagram

We will now look at several important mathematical representations that can be derived from the graphical representation of the cumulative arrivals and departures shown in Figure 39. First consider the duration of time from the beginning of the red interval to the time during the green interval that the queue has just cleared. This duration is equal to $r + g_q$, and is shown in Figure 40. We are particularly interested in the time duration g_q, or the time that it takes for the queue to clear after the beginning of green.
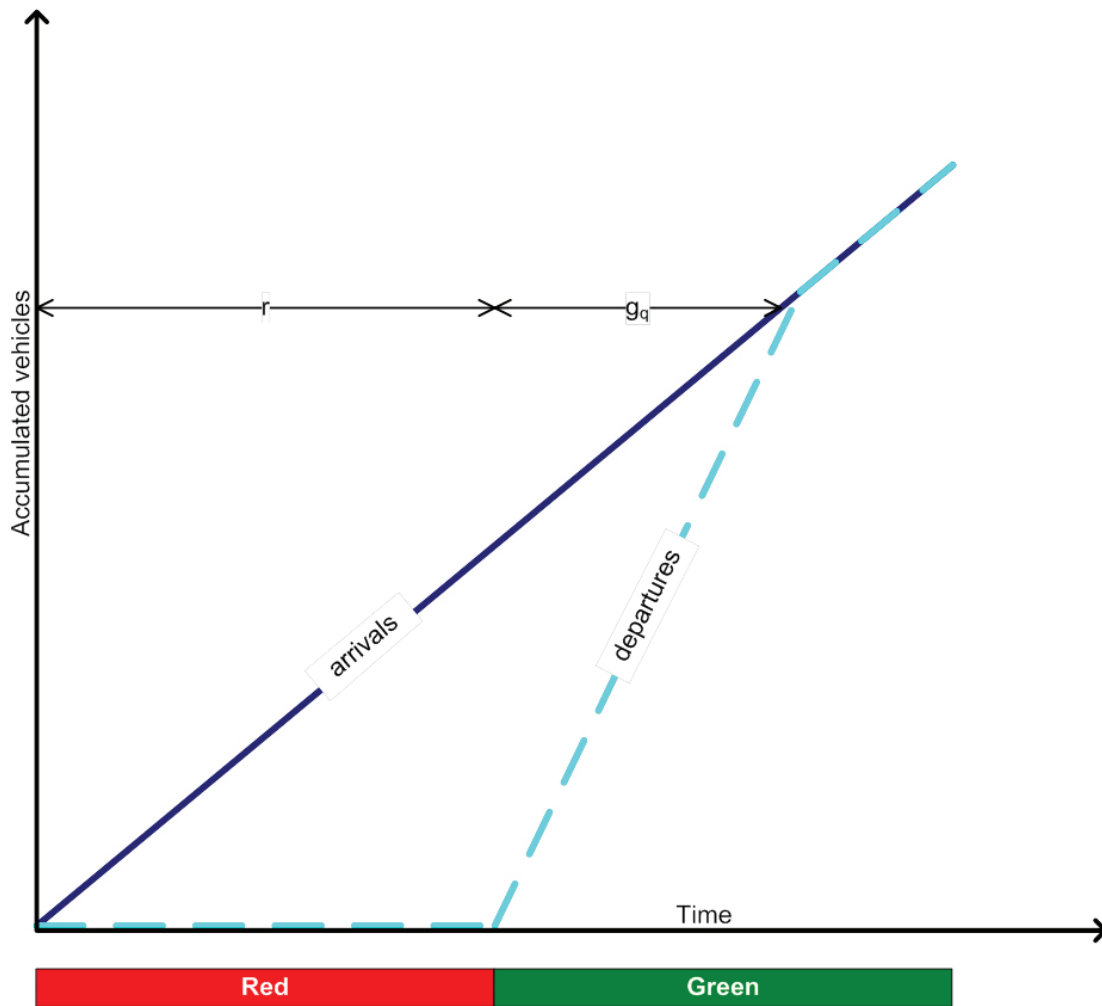
Figure 40. Queue clearance time, g_q

A derivation of a method to compute g_q follows. Assuming a uniform arrival rate, the number of vehicles that have arrived at the intersection at any time t after the start of the red interval is the product of the arrival flow rate v and the time t after the start of red, or vt. The number of vehicles that have arrived at the intersection at the point that the queue clears can be written as:

$$vt = v(r + g_q)$$

The rate at which vehicles depart from the intersection depends on the time after the start of red. During the red interval, the rate is zero. During the time that the queue is clearing, the departure rate is s, or the saturation flow rate. The number of vehicles that depart from the intersection during this same time interval $(r + g_q)$ can be written as the product of the saturation flow rate, s, and the duration of the queue clearance time, g_q, since the departure flow rate during red is zero.

Since the number of vehicles that arrive during this time interval must equal the number of vehicles that depart during this time:

$$v(r + g_q) = sg_q \qquad \text{Solving for } g_q, \qquad g_q = \frac{vr}{s - v}$$

In words, the duration of time for the queue to dissipate after the beginning of the green interval, g_q, is the number of vehicles that are in the queue at the end of the red interval (or vr) divided by the net rate that the queue dissipates, or $s - v$.

Clearly, the longer the length of the red interval, the longer it will take for the queue to dissipate after the beginning of the green interval. This conclusion has a very important implication for signal timing design, a point to which we will return periodically in this book: how long should we remain in the green interval? (And, the longer the green, the longer the cycle, and the longer the resulting delay.) While this question could generate complex responses, the simple answer is just enough time to serve the standing queue of vehicles that is present at the beginning of the green interval. And, we've seen how to compute this time, which we've called the queue service time or g_q. Once the standing queue has been served, and the flow rate drops below the saturation flow rate, the green interval should end so that the traffic on the other approaches can be served. Again, we want to keep the cycle length as short as possible to keep the delay as low as possible. However, it must also be noted that cycle lengths that are too short could lead to oversaturation.

The cumulative vehicle diagram also tells us some important information about the performance of the intersection, as shown in Figure 41. We can note the arrival of the ith vehicle at time t_a and its departure at t_d. The horizontal line connecting these two time points represents the delay d_i experienced by this vehicle.

$$d_i = t_d - t_a \qquad \text{where} \qquad d_i \text{ is the delay experienced by the } i^{th} \text{ vehicle}$$

t_d is the time that the ith vehicle departs the intersection

t_a is the time the that i^{th} vehicle arrives at the intersection

Here, the delay experienced by the vehicle is simply its time in the system, from its arrival at t_a to its departure at t_d.
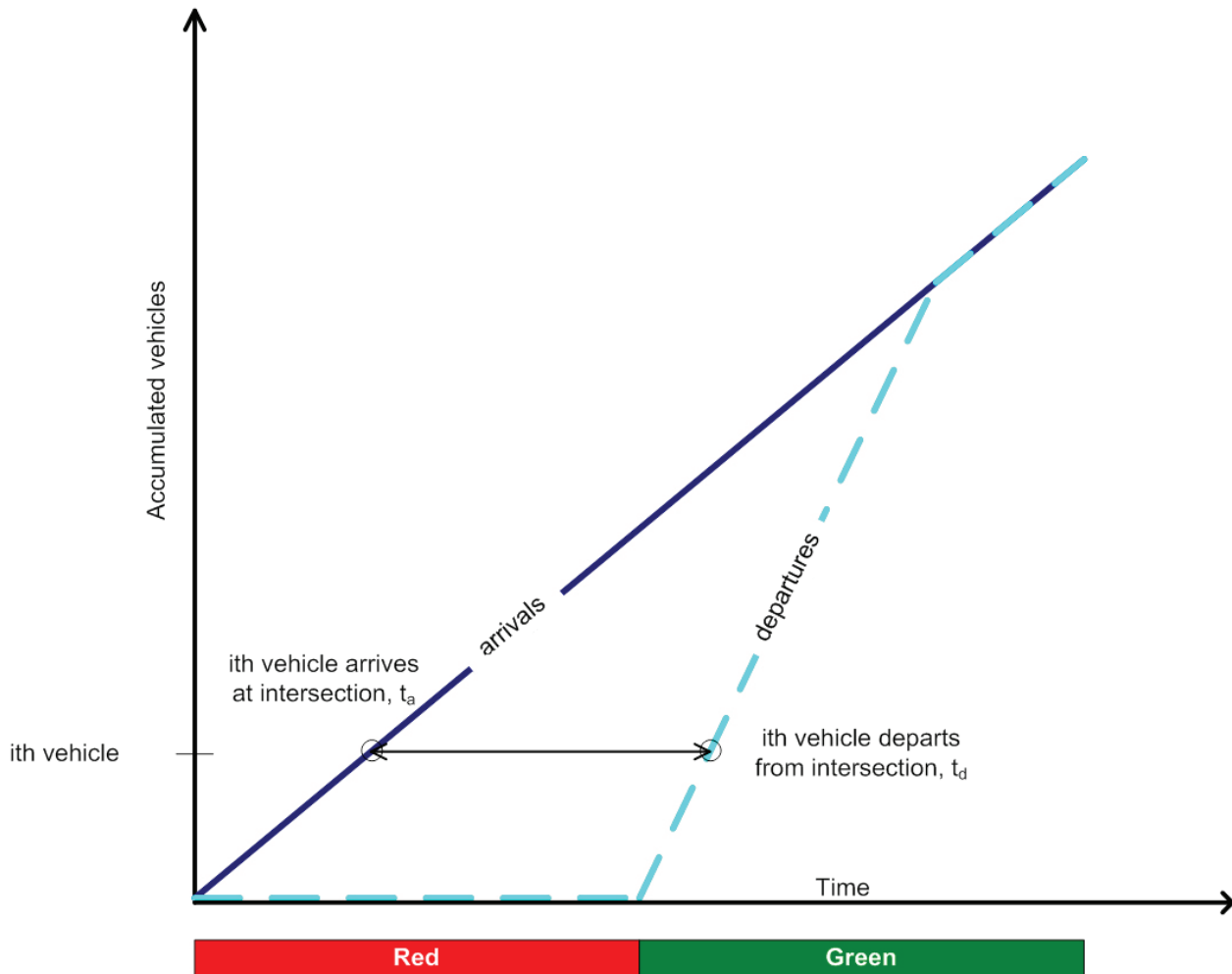

Figure 41. Cumulative vehicle diagram showing delay for i^{th} vehicle

If we consider each vehicle that arrives and departs from the intersection, we can represent the delay that each vehicle experiences as the horizontal line that connects the arrival curve with the departure curve. Examples of four such vehicles, and their individual delays, are shown in Figure 42. Further, if we integrate the area between the arrival and departure curves or add these individual delays, the total area (shown as shaded in the figure) is the total delay experienced by all vehicles that arrive at the intersection during the red and green intervals.
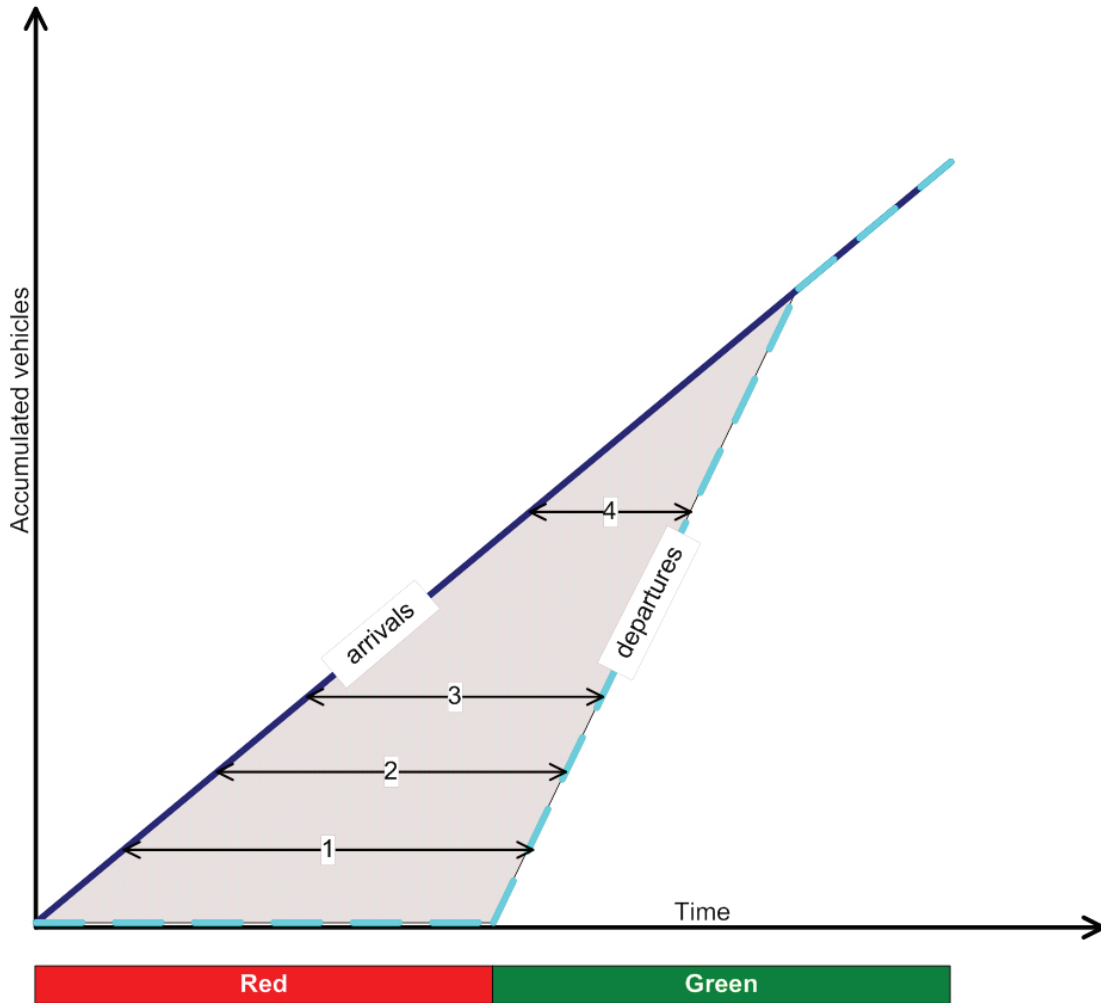

Figure 42. Cumulative vehicle diagram with total delay

Mathematically, we can write an expression for this total delay, as the area of the triangle: one half the product of the base and height of the triangle. The base is the length of the red interval and the height is the number of vehicles that have arrived between the start of red and the time that the queue clears:

$$d_t = (0.5)(base)(height)$$
$$d_t = (0.5)(r)(sg_q)$$
$$d_t = (0.5)(r)(s)\left(\frac{vr}{s-v}\right)$$

Rearranging terms, we can write the total delay experienced by all vehicles arriving during the red and green intervals as:

$$d_t = \frac{0.5r^2v}{1-v/s}$$

We can also compute the average delay experienced by all vehicles by dividing this expression by the total number of vehicles that arrive during the cycle, vC, where C is the cycle length:

$$d_a = \frac{0.5r^2v}{(vC)(1-v/s)} = \frac{0.5r^2}{(C)(1-v/s)} = \frac{0.5r(1-g/C)}{1-v/s} = \frac{0.5C(1-g/C)^2}{1-X(g/C)}$$

This delay is sometimes called the uniform delay since this model assumes that vehicles arrive at the intersection at a uniform rate. X is the degree of saturation or volume/capacity (v/c) ratio.

Note also that a vertical line drawn between the arrival curve and the departure curve at any point in time shows the number of vehicles in the queue on the intersection approach at that point in time. An example of this is shown in Figure 43. If we draw this line continuously from left to right, it would start at zero, grow to its maximum value at the end of the red interval, and then decrease until the arrival and departure curves join together at g_q, after the beginning of the green interval.
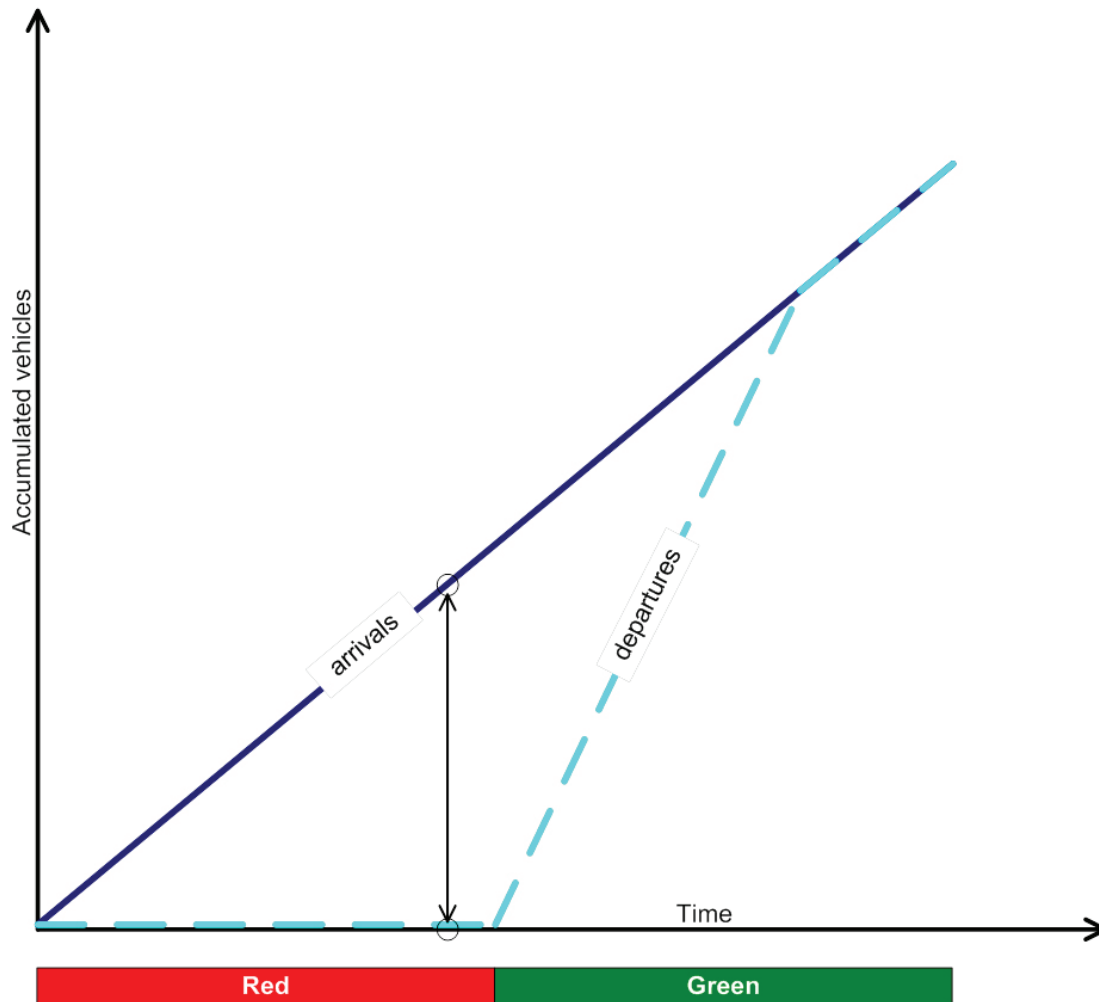

Figure 43. Cumulative vehicle model showing instantaneous queue

Queue Accumulation Polygon

A third representation is the queue accumulation polygon (QAP), the queue length at any point in time or the vertical distance between the arrival and departure curves shown in Figure 43. The QAP is shown in Figure 44. Here the queue grows from zero at the beginning of the red interval reaching a maximum at the end of the red interval. The queue begins to dissipate at the beginning of the green interval and finally clears at a point g_q after the beginning of the green interval. The queue is zero after this time and continues to be zero until the end of the green interval.

The area of the QAP is the total delay experienced by all vehicles arriving at the intersection during both the red and green intervals. This area is equal to the area between the arrival and departure curves presented in the cumulative vehicle diagram in Figure 42.
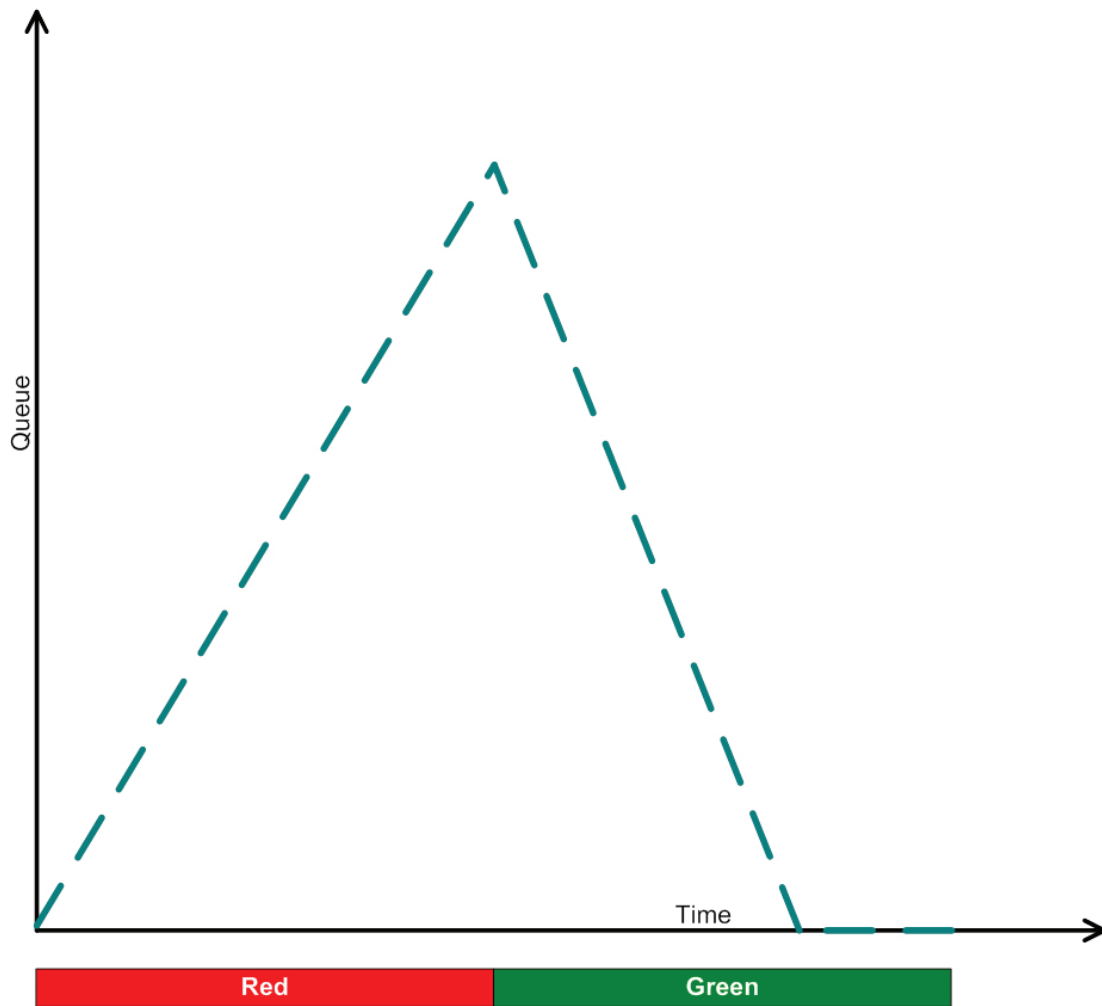

Figure 44. Queue accumulation polygon

Example Calculation of Delay

Let's consider an example using the queuing diagrams to illustrate a point that we discussed earlier: longer cycle lengths result in longer delay. Here we will assume the following input data for two cycle lengths, $C = 60$ seconds and $C = 120$ seconds, for one approach of a signalized intersection in which the arrival flow is uniform throughout the cycle:

- green ratio, g/C, $= 0.5$

- volume, v, $= 800$ vehicles per hour

- saturation flow rate, s, $= 1900$ vehicles per hour of green

The capacity (c) is calculated as the product of the saturation flow rate and the green ratio:

$$c = (g/C)s = (0.5)(1900) = 950 \ veh/hr$$

The average delay for the two cases is calculated below:

$$d_{a1} = \frac{0.5C(1-g/C)^2}{1-X(g/C)} \qquad\qquad d_{a2} = \frac{0.5C(1-g/C)^2}{1-X(g/C)}$$

$$d_{a1} = \frac{0.5(60)(1-0.5)^2}{1-(800/950)(0.5)} \qquad\qquad d_{a2} = \frac{0.5(120)(1-0.5)^2}{1-(800/950)(0.5)}$$

$$d_{a1} = 13.0 \ sec/veh \qquad\qquad d_{a2} = 25.9 \ sec/veh$$

We can also observe the increase in average delay per vehicle for a range of cycle lengths, from 40 seconds to 150 seconds, as shown in Figure 45, using these same conditions. This finding will be important later as you determine the maximum green time parameter, as this parameter directly affects the cycle length. Keeping the maximum green time (and thus the cycle length) low, will keep delay lower.

We should also note that when the cycle length is decreased to much smaller values, the delay begins to increase. This concept will be discussed in Chapter 7.
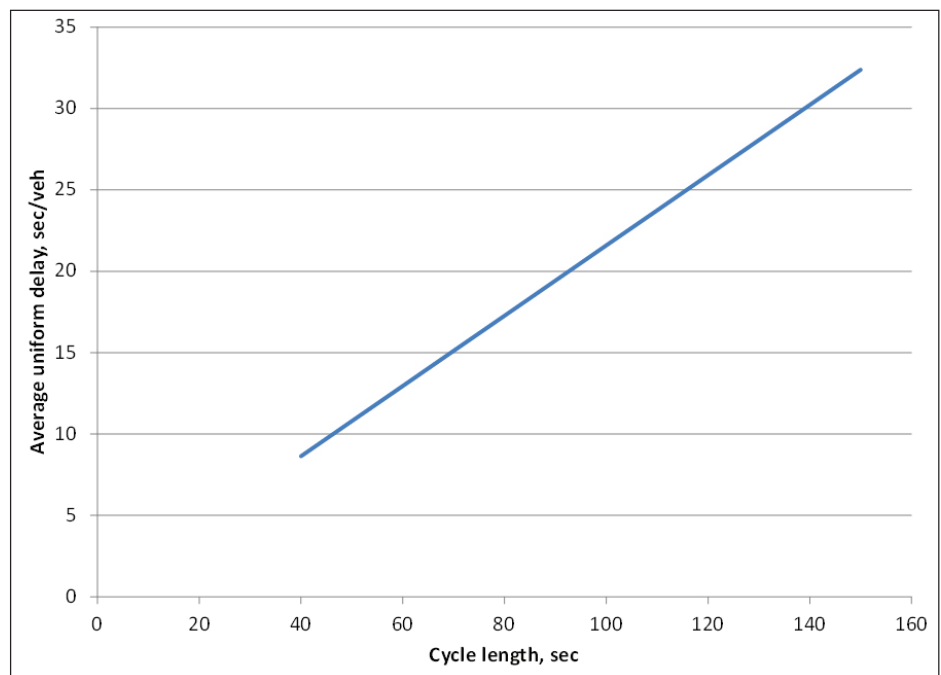

Figure 45. Uniform delay as a function of cycle length