Principles of Braking

- Vehicle braking is key for roadway design and traffic analysis.
  - stopping-sight distance
    - yellow interval for traffic signals
    - speed limits
    - Liability cases (crash clinical investigation)

Use a free-body diagram to determine $F_{br}$ and $F_{tr}$.

Notice that this figure is identical to Fig. 2.3, except that the braking forces have replaced the tractive forces, and are in the opposite direction because the braking forces are counteracting the forward motion. Also, $ma$ points in opposite direction (because this force is counteracting braking force).
Principles of Braking

- Taking moments about axles
- Assuming \( \cos \theta_g = 1 \)
- Front and rear normal loads, respectively:
  \[
  W_f = \frac{1}{2} \left[ W_l + h \left( ma - R_u \pm W \sin \theta_g \right) \right] \quad \text{Eq. 2.23}
  \]
  \[
  W_r = \frac{1}{2} \left[ W_l - h \left( ma - R_u \pm W \sin \theta_g \right) \right] \quad \text{Eq. 2.24}
  \]
- Grade resistance \((W \sin \theta_g)\) is negative for uphill grades and positive for downhill grades
- These equations are identical to their tractive effort equation counterparts (eq. 2.10 for \( W \)), except that ‘\( ma \)’ is of the opposite sign

Principles of Braking

- Summing forces along the longitudinal axis gives:
  \[
  F_h + f_r W = ma - R_u \pm W \sin \theta_g \quad \text{Eq. 2.25}
  \]
  
  where: \( F_h = F_{hfr} + F_{hr} \)

  - Substituting equation (2.25) into previous two equations (2.23, 2.24):
    \[
    W_f = \frac{1}{2} \left[ W_l + h \left( F_h + f_r W \right) \right] \quad \text{Eq. 2.26}
    \]
    \[
    W_r = \frac{1}{2} \left[ W_l - h \left( F_h + f_r W \right) \right] \quad \text{Eq. 2.27}
    \]

Principles of Braking

- The maximum vehicle braking force \((F_{br, \text{max}})\) is equal to the coefficient of road adhesion times the weights normal to the roadway surface.
  \[
  F_{br, \text{max}} = \mu W_f \quad \text{(front braking force)}
  \]
  \[
  F_{br, \text{max}} = \mu W_r \quad \text{(rear braking force)}
  \]

  \( \mu \) = coefficient of road adhesion (coefficient of friction)

Principles of Braking

- Substituting Eqs. 2.26 and 2.27 gives:
  \[
  F_{hfr, \text{max}} = \frac{\mu W}{L} \left[ F_h + h \left( \mu + f_r \right) \right] \quad \text{Eq. 2.28}
  \]
  \[
  F_{hr, \text{max}} = \frac{\mu W}{L} \left[ F_h - h \left( \mu + f_r \right) \right] \quad \text{Eq. 2.29}
  \]

  - Maximum braking forces occurs at the point of impending slide.
  - With sliding, significant reduction in road adhesion results. (See Table 2.4).
Braking Force Ratio and Efficiency

- Max deceleration: is $\mu g$ (similar to the derivation of max acceleration).

- Distributing braking forces: vehicle braking systems must correctly distribute braking forces between the vehicle’s front and rear brakes to achieve max deceleration.

- What happens with a different distribution?

Max deceleration: $\mu g$

Distributing braking forces: vehicle braking systems must correctly distribute braking forces between the vehicle’s front and rear brakes to achieve max deceleration.

What happens with a different distribution?

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Braking Force Ratio and Efficiency

Optimal proportioning of braking forces (deceleration rate equal to $\mu g$) when in equivalent to the ratio of the maximum braking forces on the front and rear axles:

$$BFR_{\text{f/rmax}} = \frac{F_{\text{fmax}}}{F_{\text{rmax}}}$$

Maximum braking forces occur when the brake force ratio (front force over the rear force) is:

$$BFR_{\text{f/rmax}} = \frac{F_{\text{fmax}}}{F_{\text{rmax}}} = \frac{1 + (\mu f)}{1 - (\mu + f)}$$

Eq. 2.30

BFR—brake force ratio that results in maximum (optimal) braking forces

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Braking Force Ratio and Efficiency

Percent braking force allocation for maximum braking is:

$$PBF_f = 100 \cdot \frac{100}{1 + BFR_{f/rmax}}$$  \hspace{1cm} \text{Eq. 2.31}$$

$$PBF_r = 100 \cdot \frac{100}{1 + BFR_{f/rmax}}$$  \hspace{1cm} \text{Eq. 2.32}

- If $W_f$ is 1000 lbs and $W_r$ is 2000 lbs then what should the percent of brake force allocation be ($PBF_f$ and $PBF_r$)?

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Braking Force Ratio and Efficiency

Percent braking force allocation for maximum braking is:

$$PBF_f = 100 \cdot \frac{100}{1 + BFR_{f/rmax}}$$  \hspace{1cm} \text{Eq. 2.31}$$

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- If $W_f$ is 1000 lbs and $W_r$ is 2000 lbs then what should the percent of brake force allocation be ($PBF_f$ and $PBF_r$)?

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Eq. 2.30

BFR—brake force ratio that results in maximum (optimal) braking forces
Practical Stopping Distance

\[ V_i^2 = V_f^2 + 2ad \]

AASHTO Standard

\[ d = \frac{V_i^2 - V_f^2}{2a} \]

\[ d = \frac{V_i^2 - V_f^2}{2g\left(\frac{a}{g} \pm G\right)} \]

Theoretical

\[ S = \frac{\eta_i (V_i^2 - V_f^2)}{2g(\eta_i \mu + f'_W \pm \sin \theta_y)} \]

Assuming: \( \eta_i \) and \( V_f \) are small and offset each other

\[ S = \frac{(V_i^2 - V_f^2)}{2g(\eta_i \mu + G)} \]


Braking Force Ratio and Efficiency

- Reality of less-than-optimal braking: Because true optimal brake-force proportioning is seldom achieved in standard non-antilock braking systems, we use a term that reflects the degree to which the braking system is operating below optimal.

- Braking efficiency: defined as the ratio of the maximum rate of deceleration, expressed in \( g \)'s (denote \( g_{\text{max}} \)), achievable prior to any wheel lockup, to the coefficient of road adhesion.

\[ \eta_b = \frac{g_{\text{max}}}{\mu} \]

Eq. 2.33


Antilock Braking Systems (ABS)

- What is the braking efficiency of an ABS?

- What impact does roadway adhesion have on braking efficiency?

- For an ABS to work, what should it do after you load a vehicle?


Theoretical Stopping Distance

- Review Section 2.9.4 for derivation details

- Equation for theoretical stopping distance is given by (assuming \( V_f = 0 \))

\[ S = \frac{f_D W}{2gK_a} \times \ln \left( \frac{1 + \frac{K_i V_i^3}{\eta_i \mu W + f'_W + W \sin \theta_y}}{\eta_i \mu W + f'_W + W \sin \theta_y} \right) \]

Eq. 2.42

grade resistance term is + for uphill, - for downhill

where: \( \gamma_k \) is the mass factor accounting for moments of inertia during braking, given value of 1.04 for automobiles, and

\[ K_a = \frac{\rho}{2} C_o A_f \]
Theoretical Stopping Distance

- Assume that the effect of speed on the coeff. of rolling resistance is constant, and can be approximated as $V = (V_1 + V_2)/2$ (use this $V$ in Eq. 2.5 for $f_r$)
- Ignoring aerodynamic resistance (small factor in deceleration):

\[
S = \frac{\gamma_s (V_1^2 - V_2^2)}{2g(\eta, \mu + f_a \pm \sin \theta_e)}
\]

Eq. 2.43