1. a. Let \( \nu_{\text{max}} \) be the frequency at which the blackbody radiation function
\[
I(\nu) = \frac{2\pi^2 \nu^2}{c^2} \frac{h\nu}{e^{h\nu/kT} - 1}
\]
is a maximum. Show that \( \nu_{\text{max}} = xkT/h \), where \( x \) is the nonzero solution of \( 3e^{-x^2} + x = 3 \) (i.e., \( x = h\nu_{\text{max}}/kT \)). Since \( x \) is a constant, \( \nu_{\text{max}} \) increases linearly with \( T \).

b. The equation \( 3e^{-x^2} + x = 3 \) cannot explicitly be solved for \( x \). Use a calculator to solve for \( x \) by trial and error. Alternatively, you could use the “Solver” function in Excel.

c. Calculate \( \nu_{\text{max}} \) for a blackbody at 300 K and at 3000 K.

d. The light emitted by the sun conforms closely to the blackbody radiation law and has \( \nu_{\text{max}} = 3.5 \times 10^{14} \text{ s}^{-1} \). Estimate the sun’s surface temperature.

e. The skin temperature of humans is 33°C, and the emission spectrum of human skin at this temperature conforms closely to blackbody radiation. Find \( \nu_{\text{max}} \) for human skin at this temperature.

2. Red light has a wavelength of about 700 nm. Calculate the energy (in joules) of a photon of light at this wavelength.

3. Calculate the de Broglie wavelength of (a) a neutron moving at \( 6.0 \times 10^6 \text{ cm s}^{-1} \) and (b) a 50 g particle moving at 120 cm s\(^{-1}\).

4. True or false? If a statement is false, explain why.
   a. In the equation \( \int \left| \psi \right|^2 \, dt = 1 \), the integral is an indefinite integral.
   b. The wavefunction \( \psi \) takes on only real values.
   c. If \( z \) is a complex number, then \( zz^* = |z|^2 \).
   d. If \( z \) is a complex number, then \( z + z^* \) is always a real number.
   e. If \( z = a + bi \), where \( a \) and \( b \) are real numbers, and we plot \( a \) on the \( x \)-axis and \( b \) on the \( y \)-axis, the distance of the point \( (a, b) \) from the origin in the \( xy \) plane is equal to \( |z| \).
   f. The absolute value \( |z| \) of a complex number must be a real nonnegative number.

5. Consider the following wavefunction
\[
\psi = Ae^{(x^2 + y^2 + z^2)/c^2}, \quad -\infty < x, y, z < +\infty
\]
Determine the constant \( A \).

6. Consider the following functions:
   \( \sin 3x, 6\cos 4x, 5x^5, 1/x, 3e^{-5x}, \ln 2x \)
Which of these are eigenfunctions of the second derivative operator, \( d^2/dx^2 \)? If it is an eigenfunction of this operator, state what the eigenvalue is.