1. True or false? If a statement is false, explain why.
   a. The speed \( v \) of a molecule is never negative.
   b. The velocity component \( v_x \) of a molecule is never negative.

   **Solution**
   a. T.
   b. F. Velocity is a vector which means that it has both magnitude and direction; so it can be negative.

2. Derive an expression for the ratio of the root mean square speeds of two different types of gas molecules. Use this expression to calculate \( \frac{v_{\text{rms}}(\text{Ne})}{v_{\text{rms}}(\text{He})} \).

   **Solution**
   The expression for the root mean square speed is
   \[
   v_{\text{rms}} = \left( \frac{1}{2} \right) \left( \frac{3kT}{m} \right)^{1/2}.
   \]
   The ratio for two different types of gas molecules is
   \[
   \frac{v_{\text{rms}}(\text{Ne})}{v_{\text{rms}}(\text{He})} = \left( \frac{m_{\text{He}}}{m_{\text{Ne}}} \right)^{1/2} = \left( \frac{6.64 \times 10^{-27} \text{ kg}}{3.35 \times 10^{-26} \text{ kg}} \right)^{1/2} = 0.445.
   \]
   Notice that the ratio is independent of temperature. For Ne relative to He at 298 K we have
   \[
   \frac{v_{\text{rms}}(\text{Ne})}{v_{\text{rms}}(\text{He})} = \left( \frac{m_{\text{He}}}{m_{\text{Ne}}} \right)^{1/2} = \left( \frac{6.64 \times 10^{-27} \text{ kg}}{3.35 \times 10^{-26} \text{ kg}} \right)^{1/2} = 0.445.
   \]

3. For \( \text{CH}_4(\text{g}) \) at 300 K and 1 bar, calculate the probability that a molecule picked at random has its speed in the range 400.000 to 400.001 m s\(^{-1}\). This interval is small enough to be considered infinitesimal.

   **Solution**
   The probability that a molecule has a particular speed can be found using the speed distribution function:
   \[
   Pr(v) = f(v) dv = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-m v^2 / 2kT} dv.
   \]
   Since the interval is infinitesimal, we can evaluate this directly without having to integrate:
   \[
   Pr(v) = 4\pi (400 \text{ ms}^{-1})^2 \left( \frac{2.66 \times 10^{-26} \text{ kg}}{2\pi (1.38 \times 10^{-23} \text{ JK}^{-1})(300\text{ K})} \right)^{3/2} e^{-2.66 \times 10^{-26} \text{ kg} / (400 \text{ ms}^{-1})^2 / 2(1.38 \times 10^{-23} \text{ JK}^{-1})(300\text{ K})(0.001 \text{ ms}^{-1})} = 1.24 \times 10^{-6}.
   \]

4. In the mythical world of Flatland, everything is two-dimensional. Find the expression for the probability that a molecule in a two-dimensional ideal gas has its speed in the range \( v \) to \( v + dv \).

   **Solution**
   Speed is the magnitude of the velocity (which is a vector). In two-dimensions this is
   \[
   v = \sqrt{v_x^2 + v_y^2}.
   \]
   Where \( v_x \) and \( v_y \) are the \( x \) and \( y \) components of the velocity, respectively. The probability that a molecule will have velocity components \( v_x \) to \( v_x + dv_x \) and \( v_y \) to \( v_y + dv_y \) is found using the two-dimensional velocity distribution function: (continued on next page)
This question is asking us to find an expression describing the probability that a molecule will have a speed within the shaded regions (i.e., from \(v\) to \(v + dv\)). We can determine this by converting from Cartesian coordinates to plane polar coordinates and integrating over all values of \(\phi\) (where \(\phi\) is the angle that \(v\) makes in the \(v_x,v_y\) plane). In plane polar coordinates
\[
dv_x dv_y = v dv d\phi
\]
The range of \(\phi\) is 0 to \(2\pi\). So we have:
\[
Pr(v) = \frac{m}{2\pi k T} \int_0^{2\pi} e^{-\frac{mv^2}{2kT}} dv d\phi
\]
\[
= \frac{mv}{kT} e^{-\frac{mv^2}{2kT}} dv
\]
Note than an alternative (albeit less rigorous) way to approach this is to calculate the shaded area directly. The area of the inner circle is
\[
A_{in} = \pi v^2
\]
The area of the outer circle is:
\[
A_{out} = \pi (v + dv)^2
\]
\[
= \pi v^2 + 2\pi vd v + \pi (dv)^2
\]
The shaded area is the difference:
\[
A_{out} - A_{in} = \pi v^2 + 2\pi vd v + \pi (dv)^2 - \pi v^2
\]
\[
= 2\pi vd v + \pi (dv)^2
\]
Since \(dv\) is infinitesimal, the term \((dv)^2\) will be small enough to neglect so that the shaded area is just:
\[
A_{out} - A_{in} = 2\pi vd v
\]
Using this for \(dv_x dv_y\) in the expression at the top of the page gives the same result.

5. A container holding solid scandium in equilibrium with its vapor at 1690 K shows a weight loss of 10.5 mg in 49.5 minutes through a circular hole of diameter 0.1763 cm. Find the vapor pressure of Sc at this temperature in torr.

**Solution**

This is a way to experimentally measure the collisional flux. We derived the expression for this in class:
\[
J_N = \frac{\Delta M}{mA \Delta t}
\]
\(\downarrow\) the collisional flux is related to the vapor pressure
\[
\rho = \frac{1}{(2\pi mkT)^{1/2}}
\]
Solving for the pressure gives
(continued on next page)
\[ P = \frac{(2\pi mkT)^{1/2}}{m \Delta t} \Delta M \]
\[ = \frac{2\pi (7.47 \times 10^{-26} \text{ kg})(1.38 \times 10^{-23} \text{ JK}^{-1})(1690 \text{ K})^{1/2} (1.05 \times 10^{-5} \text{ kg})}{(7.47 \times 10^{-26} \text{ kg})(2.44 \times 10^{-6} \text{ m}^2)(2970 \text{ s})} \]
\[ = 2.03 \text{ kg m}^{-1} \text{ s}^{-2} \]

\[ \deq \]
\[ 1 \text{ Pa is } 1 \text{ kg m}^{-1} \text{ s}^{-2} \]
\[ = 2.03 \text{ Pa} \]
\[ \deq \times 0.0075 \text{ torr Pa}^{-1} \]
\[ = 0.015 \text{ torr} \]

6. A certain sample of a pure gas has a mean speed of 450 m s\(^{-1}\) and the average time between two successive collisions of a given molecule with other molecules is 4.0 \(\times 10^{-10}\) s. Find the mean free path in this gas.

**Solution**

This is pretty straightforward. If you know the average speed (we do) and you know the average time between collisions (we do), then the average distance traveled between collisions (i.e., the mean free path) is just the product of these:

\[ \lambda = \langle v \rangle \Delta t \]
\[ = 1.8 \times 10^{-7} \text{ m} \]

7. For N\(_2\) (hard sphere diameter = 3.7 \(\times 10^{-10}\) m) at 298 K and 1 atm, calculate the following
   a. the collisional frequency, \(z\);
   b. the collisional density, \(z_{\text{tot}}\);
   c. Repeat the calculations from (a) and (b) using a pressure of 10\(^{-6}\) torr (i.e., under vacuum).

**Solution**

a. The expression for the collisional frequency is

\[ z = \sqrt{2\pi \rho d^2 \langle v \rangle} \]
\[ = \sqrt{2\pi \rho d^2 \left( \frac{8kT}{\pi m} \right)^{1/2}} \]

If we treat the gas as ideal, then we can use

\[ P = \frac{P}{kT} \]
\[ \deq \]

\[ z = \sqrt{2\pi P d^2 \left( \frac{8kT}{\pi m} \right)^{1/2}} \]

To calculate this, we need to convert \(P = 1 \text{ atm}\) to pascal (which has units of kg m\(^{-1}\) s\(^{-2}\)):

\[ 1 \text{ atm} \times \frac{101325 \text{ Pa}}{1 \text{ atm}} = 101325 \text{ kg m}^{-1} \text{ s}^{-2} \]

The collisional frequency is

\[ z = \frac{\sqrt{2\pi (101325 \text{ kg m}^{-1} \text{ s}^{-2}) (3.7 \times 10^{-10} \text{ m})^2 \left[ 8(1.38 \times 10^{-23} \text{ kgm}^2 \text{ K}^{-1}) (298 \text{ K}) \right]^{1/2}}}{(1.38 \times 10^{-23} \text{ JK}^{-1})(298 \text{ K}) \left[ \pi (4.65 \times 10^{-26} \text{ kg}) \right]} \]
\[ = 7.11 \times 10^8 \text{ s}^{-1} \]

b. The collisional density is just the collisional frequency from (a) multiplied by one half of the particle density of the gas:

\[ z_{\text{tot}} = \frac{1}{2} \rho z \]
\[ = \frac{zP}{2kT} \]

(continued on next page)
\[ z_{\text{tot}} = \frac{(7.11 \times 10^9 \text{ s}^{-1})(101325 \text{ kgm}^{-1} \text{s}^{-2})}{2(1.38 \times 10^{-22} \text{ JK}^{-1})(298 \text{K})} \]
\[ = 8.76 \times 10^{14} \text{ s}^{-1} \text{m}^{-3} \]

c. In Pa, this pressure is
\[ 10^4 \text{ torr} \times \frac{133 \text{ Pa}}{1 \text{ torr}} = 1.33 \times 10^{-4} \text{ kgm}^{-1} \text{s}^{-2} \]
The collisional frequency at this pressure is
\[ z = \sqrt{2\pi \left(1.33 \times 10^{-4} \text{ kgm}^{-1} \text{s}^{-2}\right)(3.7 \times 10^{-10} \text{ m})^2} \frac{8(1.38 \times 10^{-23} \text{ JK}^{-1})(298 \text{K})}{\pi (4.65 \times 10^{-26} \text{ kg})} \]
\[ = 9.34 \text{ s}^{-1} \]
and the collisional density at this pressure is
\[ z_{\text{tot}} = \frac{(9.34 \text{ s}^{-1})(1.33 \times 10^{-4} \text{ kgm}^{-1} \text{s}^{-2})}{2(1.38 \times 10^{-23} \text{ JK}^{-1})(298 \text{K})} \]
\[ = 1.51 \times 10^{17} \text{ s}^{-1} \text{m}^{-3} \]