9. The total number of cells can be calculated as

$$X_T = \sum_{i=1}^{4} (X_{vi} + X_{di})$$

C. *Case Studies*

1. The system of equations can be solved for any distribution of reactor volumes, influent flow, and recycle flow.

2. There are four cases of particular interest

   a) **Case 1: All feed and recycle going to reactor 1**

   b) **Case 2: The feed and recycle are distributed evenly among the four tanks.**

   c) **Case 3: The feed is distributed evenly among the four tanks and the recycle is added to tank 1.**

   d) **Case 4: All recycle is returned to tank 1 and all feed enters tank 3.**

3. The relative impact of each of the cases on X, S, and RO will be illustrated using the following values: u_m=0.5 hr^{-1}; K_s = 100 mg/L; Y_{max}=0.5 mg cells/mg BOD; b=0.004 hr^{-1}; γ=0.004 hr^{-1}; β=1.2 mg O_2/mg cells; F=1.0 L/hr; S_o=500 mg/L BOD.

4. Case 1:

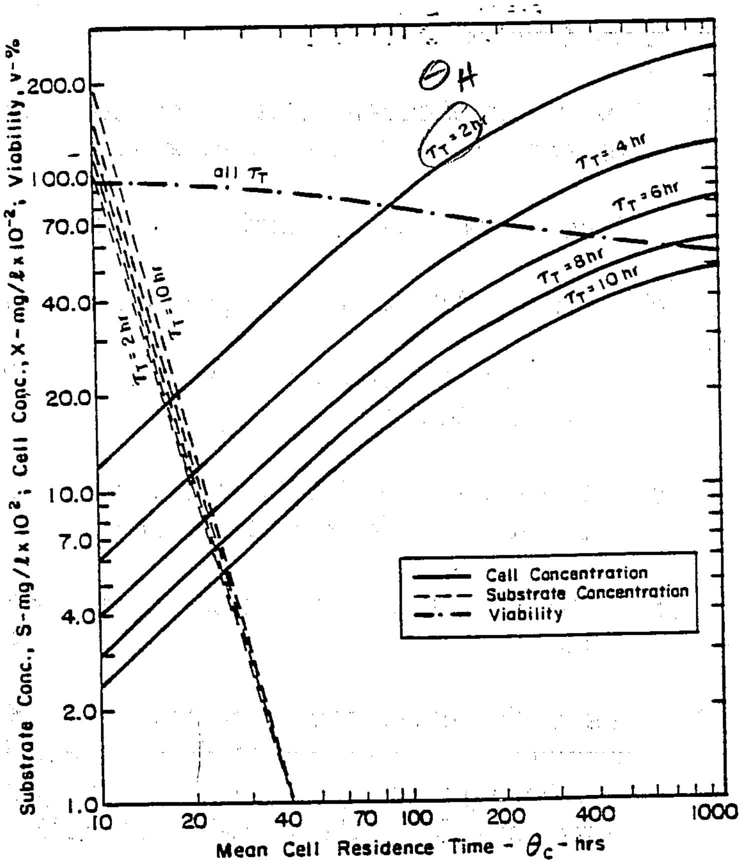
   a) **Under the specified conditions, ρ1 and φ1=1.0; ρ2, ρ3, ρ4=0 and φ2, φ3, φ4 = 0.**

   b) **The performance can be related to MCRT, but MCRT is no longer directly related to the specific growth rate as it was in a single CFSTR with recycle. The specific growth rate is different in each reactor in the chain. MCRT reflects average overall growth rate.**

$$\mu = \frac{\mu_m S}{K_s + S}$$

6/22

c) Figure illustrating substrate and cell concentration in the effluent of the fourth tank (α=0.25):



1.7   41

θc ; d

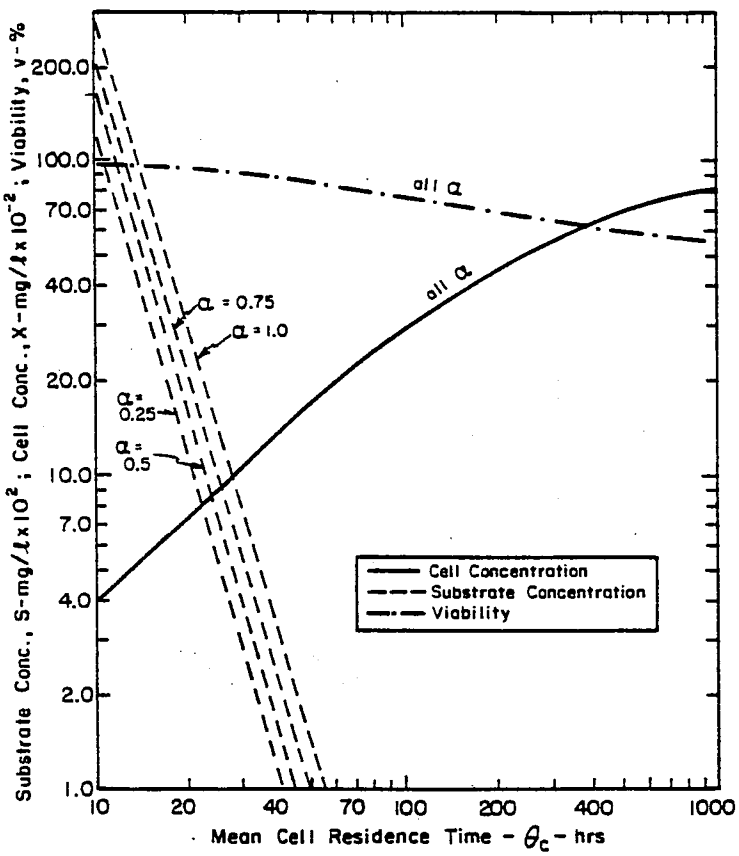
(1) Points:

(a) *The substrate concentration leaving the chain is much less than in a single CFSTR with recycle.*

(b) *The total hydraulic residence time influences the effluent substrate concentration. For a fixed MCRT, the effluent substrate concentration increases with increasing values of θ_h. This is in contrast to a single CFSTR w/recycle.* $\theta_H \uparrow$

(c) *The cell concentration in a series of CFSTR's will be similar to a single CFSTR w/recycle for a given MCRT as long as the MCRT is greater than θ_h. Because the cells are recycled around the chain many times within a cell residence time the cells approach a completely mixed condition even though the fluid passes through a cascade.*

d) Impact of the recycle ratio on cell and substrate concentration (θh = 6.0 hrs; all concentrations are for the fourth tank).

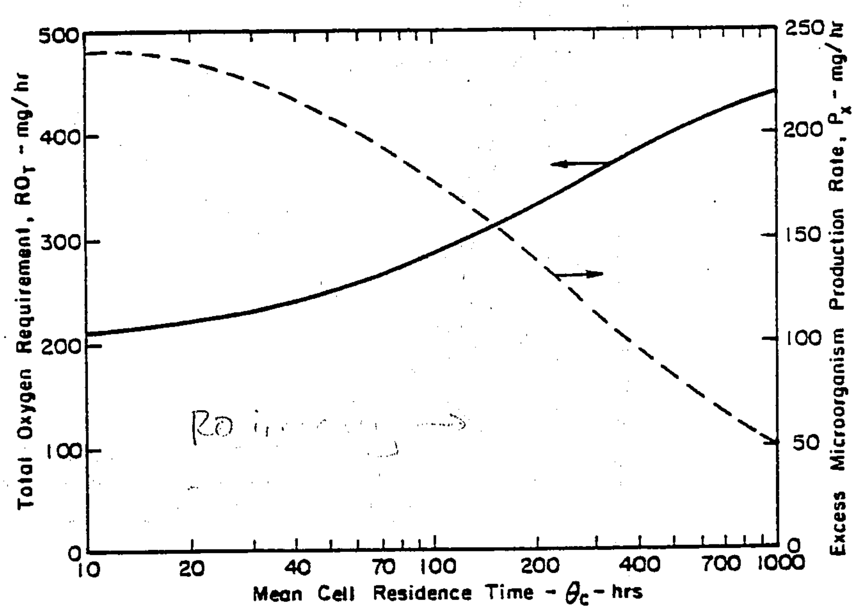


(1) Points:

(a) *The recycle ratio has no impact on the cell concentration in a series of CFSTRs, similar to a single CFSTR w/recycle.*

(b) *The recycle ratio does have an influence on the effluent substrate concentration (which was not the case in a single CFSTR). The substrate concentration increases with increasing values of the recycle ratio. However, for typical MCRT's of activated sludge processes (discussed later), the relative influence of* α *on* S_e *is negligible.*

e) **Effect of MCRT on the total oxygen requirement ($0.25 \leq \alpha \leq 1.0$; 2.0 liters $\leq V_T \leq 10.0$ liters).**



(1) Points:

(a) *The recycle ratio and θ_h has no effect on the oxygen requirement.*

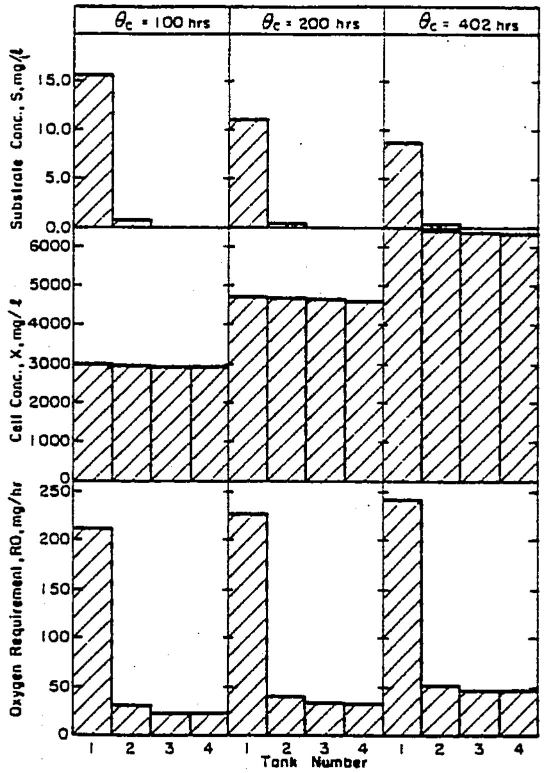
(b) *The total oxygen requirement for a series of CFSTR's is almost identical to a single CFSTR.*

(i) At MCRTs greater than 60 hours almost all soluble substrate has been removed from both systems. The total oxygen requirement depends mostly on decay. The extent of decay independent of configuration.

(ii) When MCRT small, the RO_T greater in chain than in single CFSTR due to greater degree of substrate removal, not differences in decay.

X S RO

f) Cell concentration, Substrate Concentration, and Oxygen Requirement in each of the four CFSTR's in series (V_T = 6 L; α = 0.5).



(1) Points:

(a) *The cell concentration is approximately the same in all reactors. A significant decline would occur only if θ_h and θ_c were both large and of approximate magnitude such that decay would begin to play a role.*

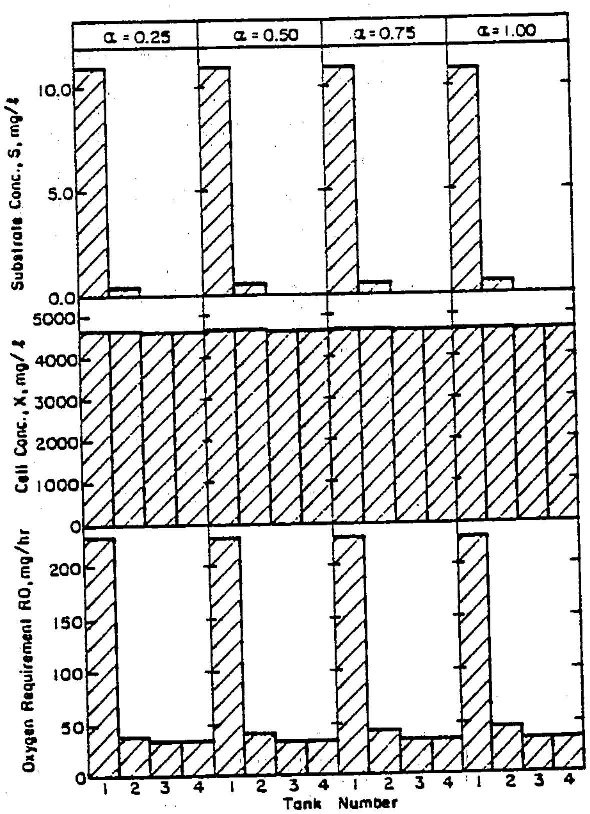
(b) *Rate of substrate removal declines as flow moves down the series of tanks. If a plot were made of the specific growth rate, it would follow the substrate plot (illustrating why MCRT is not representative of actual growth rates).*

(c) *The oxygen requirement in each tank reflects the metabolic activity of the cells. The oxygen requirement is greatest in the first tank where most of the substrate removal occurs. Most of the metabolic*

activity in the tanks 3 and 4 is due to endogenous metabolism or decay.

(d) *Increasing the MCRT increases the concentration of cells, which in turn increases the rate of substrate removal and oxygen requirement. Additionally, an increase in MCRT allows more time for decay resulting in an increase in the oxygen requirement in all tanks.*

g) Impact of recycle on the substrate concentration, cell concentration, and the oxygen requirement (V_T = 6 L; θ_c = 200 hours)



(1) Points:

(a) *The recycle ratio has no influence on S, X, or RO.*

(b) *Recycle cannot be used as a tool to alter substrate removal or oxygen uptake.*

(c) *Recycle does not influence the volume of cells that must be wasted from the 4th tank to maintain a desired MCRT.*

h) **The general trends above apply to other kinetic parameters typical of activated sludge processes. The specific values will change with other kinetic parameters. The biggest system variable influencing the specific values is temperature. The relative impact of temperature, particularly annual fluctuations, should be investigated in the design process to allow system modifications over the course of the year (e.g., changes in the rate of oxygen transfer to match changes in oxygen uptake).**

5. Case 2: Chain with Recycle and Feed Distributed Evenly Among Tanks

a) **One way to reduce the oxygen requirement in the first reactor and increase it in the others is to evenly distribute the feed to each reactor.**

b) **Under this situation, the chain of CFSTRs behaves identically to a single CFSTR with recycle. Performance curves have already been presented for a single CFSTR w/recycle.**

c) **While flow and recycle distribution evens out oxygen requirement, reduces effluent quality to single CFSTR.**

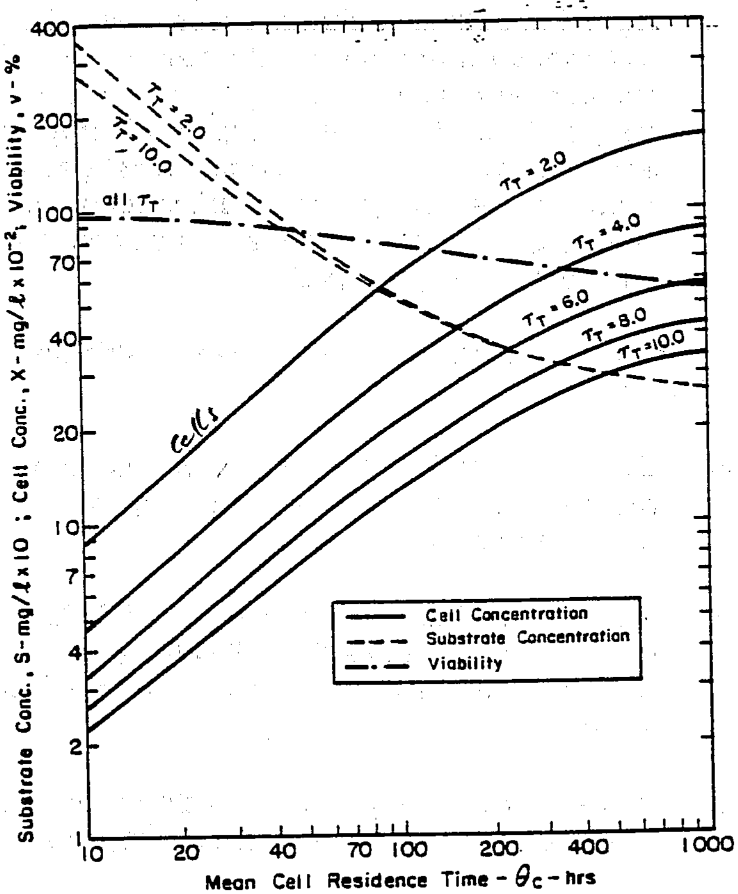
d) **Operationally, this situation provides flexibility because it allows the system to operate like a single CFSTR or a series.**

6. Case 3: Chain with Recycle to Tank 1 and Feed Distribution Evenly Among Tanks

a) **Under the specified conditions, $\phi1=\phi2=\phi3=\phi4=0.25$; $\rho1=1.0$; $\rho2$, $\rho3$, $\rho4=0$.**

b) **The performance can be related to MCRT, but MCRT is no longer directly related to the specific growth rate as it was in a single CFSTR with recycle. The specific growth rate is different in each reactor in the chain. MCRT reflects average overall growth rate.**

c) Effect of θ_h and θ_c on X and S_e in Tank 4 (α=0.5)



(1) Points:

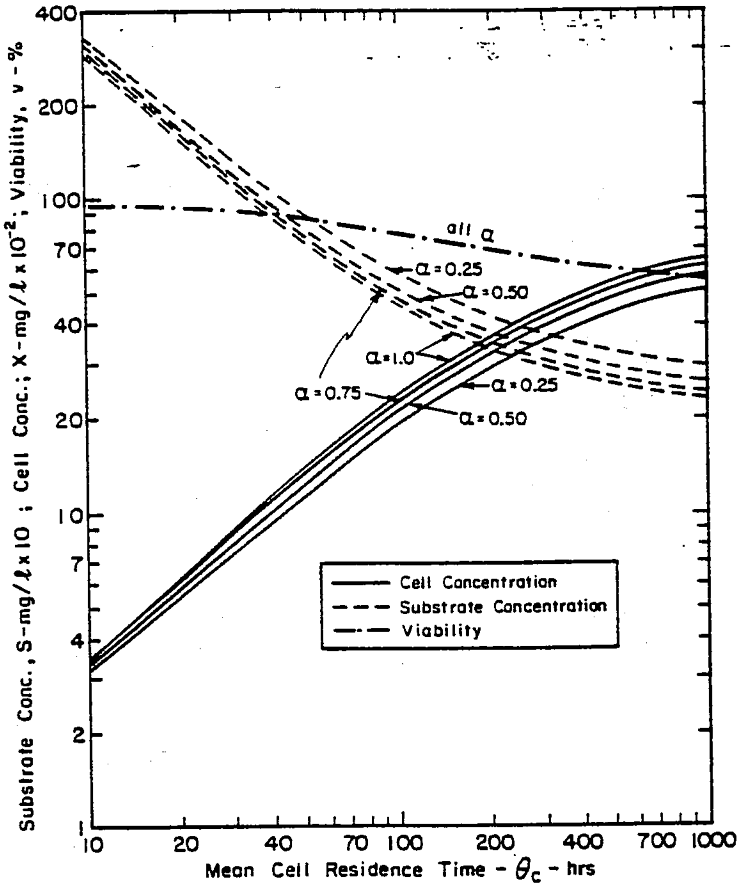
(a) *The substrate concentration is higher than in Case 1, a reflection of the addition of feed to the final tank.*

(b) *The substrate concentration is slightly higher than in a single CFSTR. The difference is not large, but may be large with the use of other system variables.*

(c) *The hydraulic residence time has a slight influence on substrate removal that is in the opposite direction as in Case 1. The practical significance is small.*

(d) *The concentration of cells is less than Case 1 for all values of θ_h (reason will be illustrated in profiles in each tank).*

d) Impact of recycle ratio on S and X in Tank 4 (V_T = 6 L)



(1) Points:

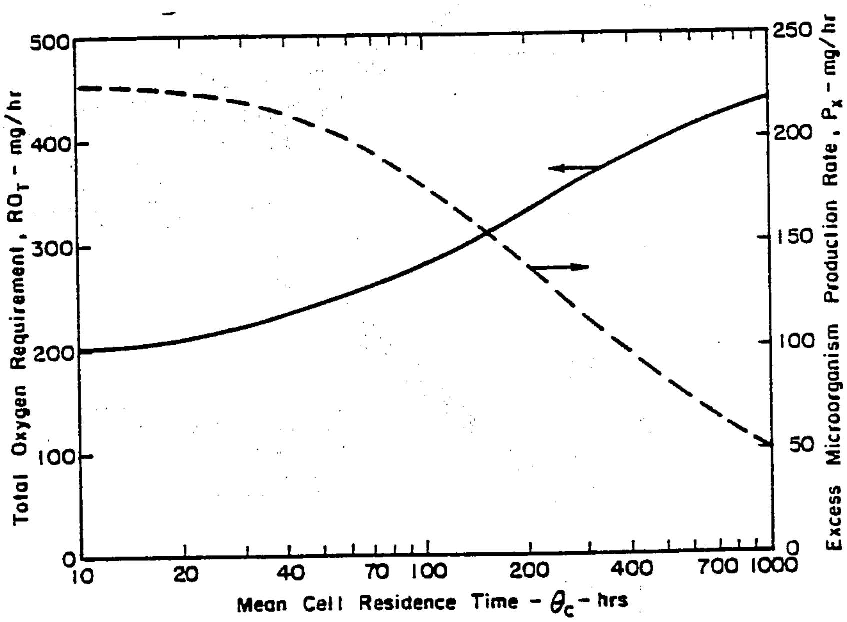
(a) *The recycle ratio influences both X and S to a greater degree than in Case 1.*

(b) *Increase in cell concentration in fourth tank results in a decrease in the substrate concentration.*

(c) *The relative influence of α on X and S is slight, but may be much bigger with different system parameters.*

(d) *This is the first case where α influences performance. Operational scheme in this Case adds a level of complexity because α as well as θ_c can be modified to influence performance.*

e) **Impact of MCRT on the total oxygen requirement (0.25≤ α ≤1.0; 2.0 liters ≤ V_T ≤10.0 liters)**



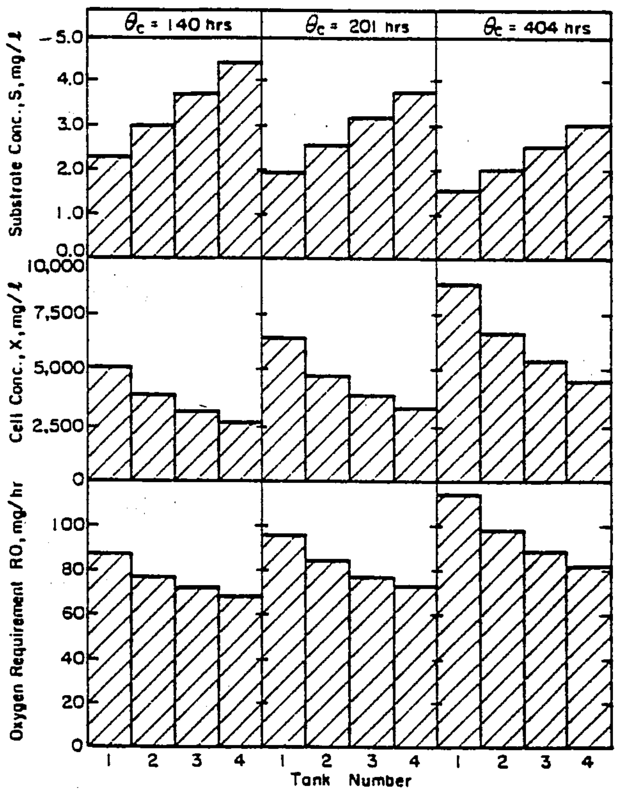
(1) Points:

(a) *Neither α or θ_h influence RO.*

(b) *The curve is almost identical to a single CFSTR and a series of CFSTRs in Case 1.*

(c) *For MCRTs less than 60 hours, the oxygen requirement is slightly less than in Case 1 because less substrate is being removed.*

f) Cell concentration, Substrate Concentration, and Oxygen Requirement in each of the four CFSTR's in series (V_T = 6 L; α = 0.5).



(1) Points:

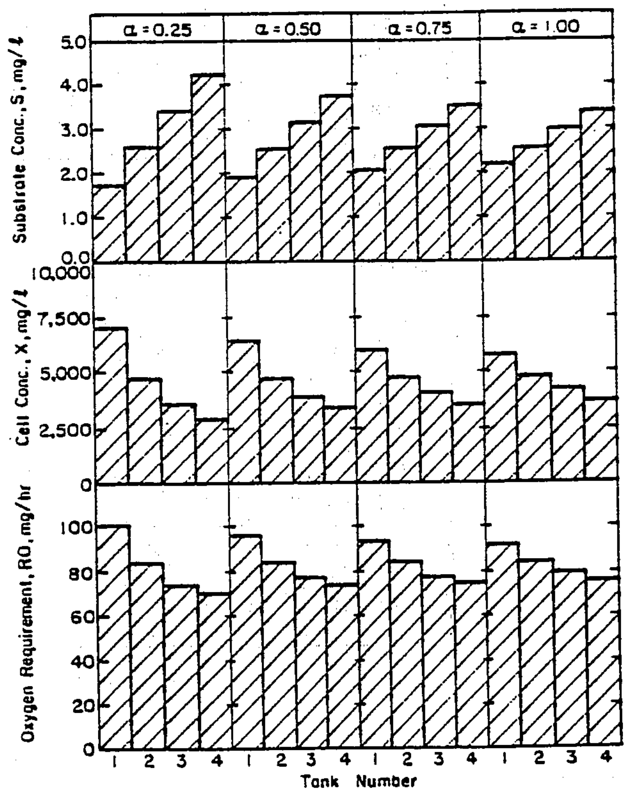
(a) *The concentration of cells declines down the series of tanks. The recycle flow entering tank 1 is diluted from the step feed into each of the four tanks.*

(b) *Substrate concentration has an opposite profile than the cell concentration, a result of the reduction in the cell concentration and the step feed (increase is not large).*

(c) *The oxygen requirement has been smoothed across the four tanks relative to Case 1; it also follows a similar trend to cell concentration.*

(d) *An increase in MCRT increases the cell concentration which reduces the substrate concentration and increases the oxygen requirement. Additionally, RO increase due primarily to decay, as with Case 1.*

g) Impact of recycle on the substrate concentration, cell concentration, and the oxygen requirement (V_T = 6 L; θ_c = 200 hours)



(1) Points:

(a) *The recycle ratio has an impact on S, X and RO, unlike Case 1 and 2. Since the MCRT is fixed in this example, the total concentration of cells in the four tanks is constant. Increasing the recycle ratio, reduces the relative impact of dilution in each of the tanks, resulting in a leveling effect in the concentration of cells in each individual tanks. A leveling in the cell concentration results in a similar effect in S and RO.*

(b) *Recycle can be used as a tool to alter substrate removal or oxygen uptake.*

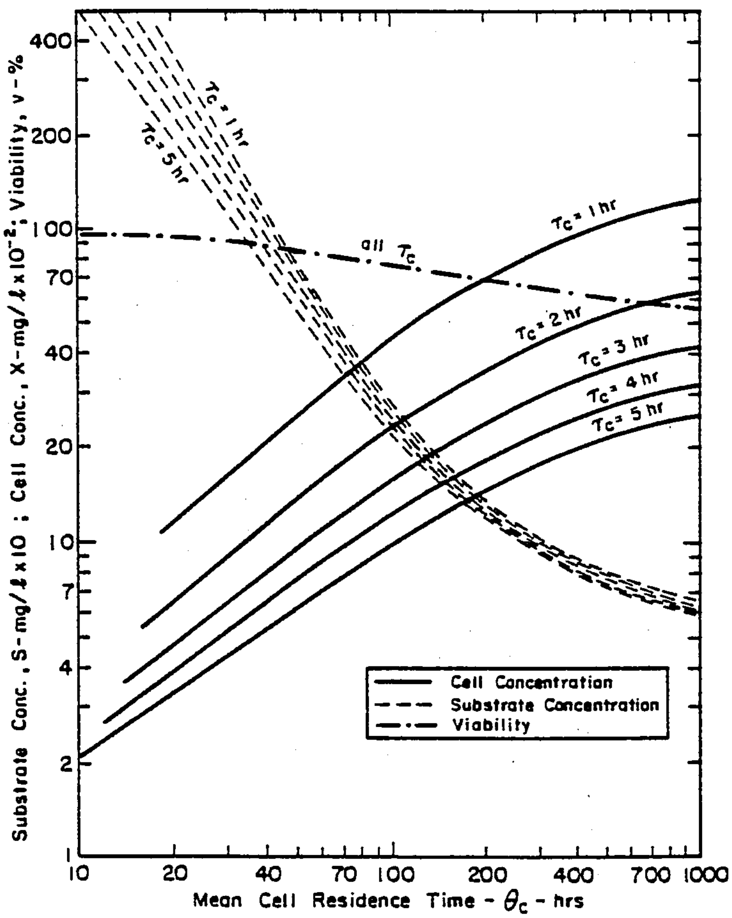
(c) *Recycle does influence the volume of cells that must be wasted from the 4th tank to maintain a desired MCRT.*

7. Case 4: Recycle to Tank 1 and Feed to Tank 3.

a) Under the specified conditions, ρ1 and ϕ3=1.0; ϕ1=ϕ2=ϕ4=0; ρ2, ρ3, ρ4=0.

b) Because some tanks receive substrate and others do not, the MCRT is not a direct reflection of the specific growth rate in each tank, but is a measure of the overall growth rate in the series of tanks.

c) Effect of θ_h and θ_c on X and S_e in Tank 4 (α=0.5)



(1) Points:

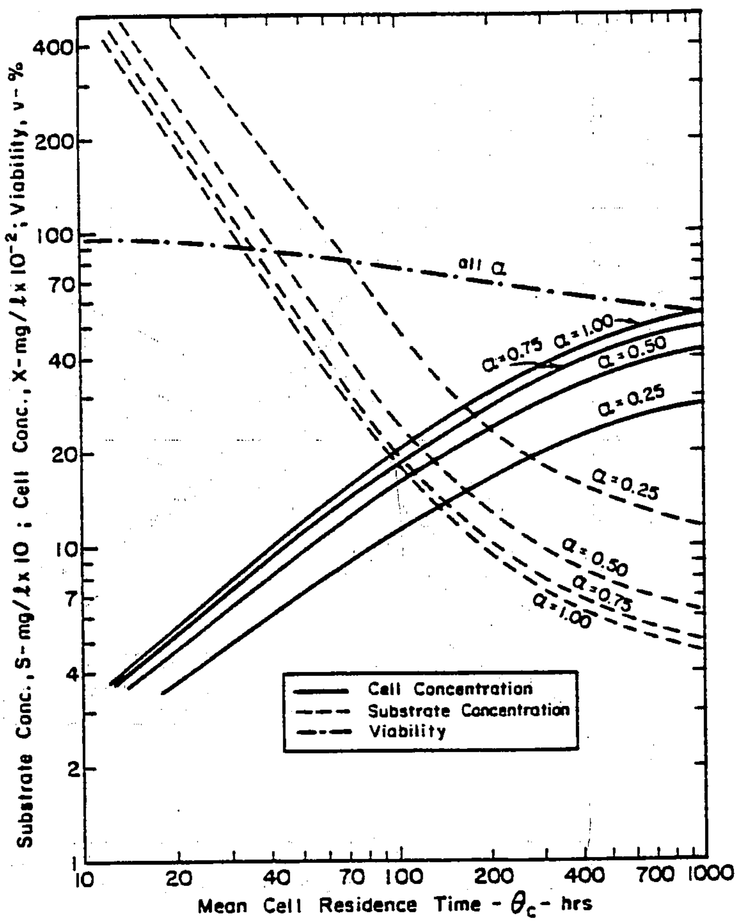
(a) *This system is capable of removing more substrate than a single CFSTR or a series as in Case 3, but not a series as in Case 1.*

(b) *The amount of substrate removed is more sensitive to θ_h than in any of the other systems considered (τ_c is the hydraulic residence time based on the last two tank volumes).*

(c) *A decrease in τ_c results in an increase in S.*

(d) *The cell concentration in the last tank is less than any of the other systems. The reason is similar to the one given in Case 3.*

d) Impact of recycle ratio on S and X in Tank 4 (V_T = 6 L)

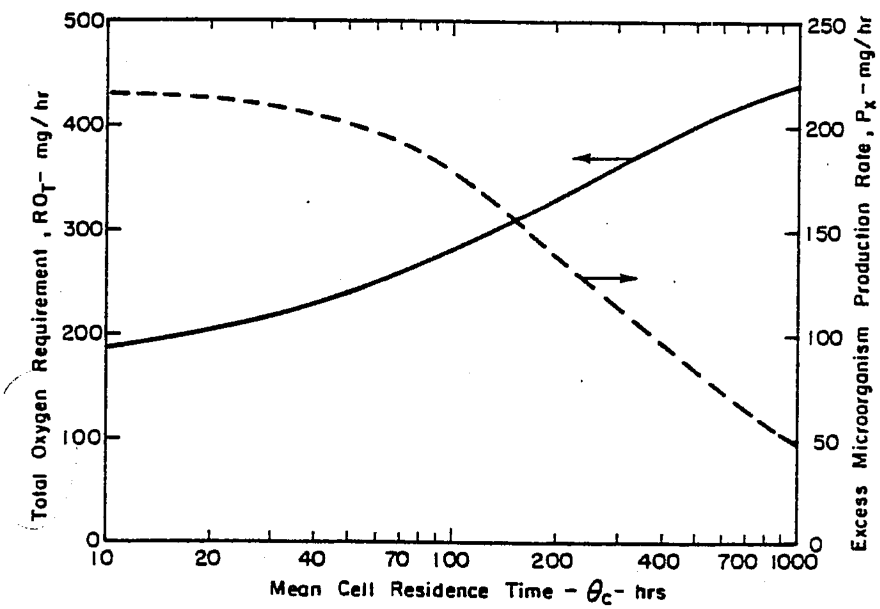


(1) Points:

(a) *The recycle ratio has a much larger impact on the performance of this system than any of the others considered.*

(b) *As α is increased the cell concentration increases and the substrate concentration decreases.*

e) Impact of MCRT on the total oxygen requirement ($0.25 \leq \alpha \leq 1.0$; 2.0 liters $\leq V_T \leq 10.0$ liters)



(1) Points:

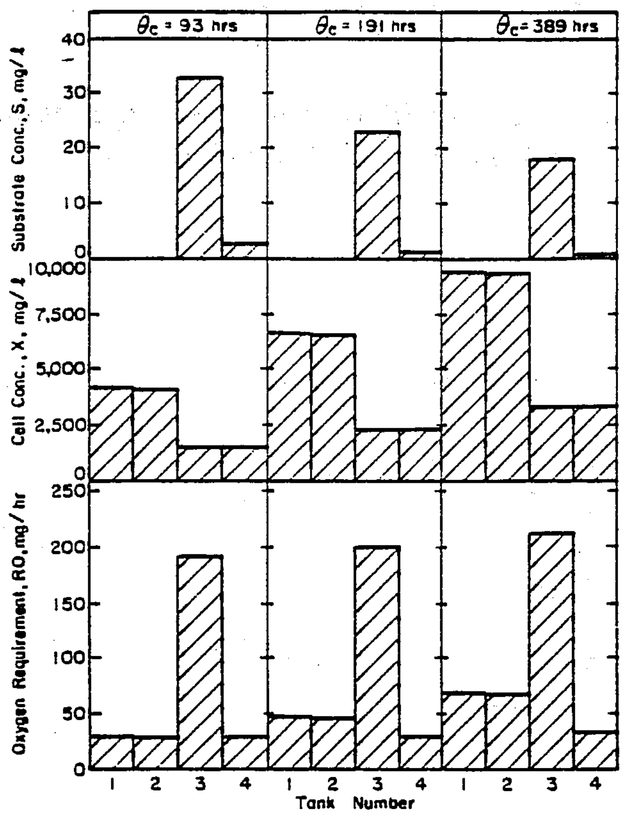
(a) *The recycle ratio does not influence RO.*

(b) *For MCRTs greater than 60, the RO is roughly the same for all systems considered.*

(c) *For MCRTs less than 60, the RO is similar to Case 1.*

20/22

f) **Cell concentration, Substrate Concentration, and Oxygen Requirement in each of the four CFSTR's in series (V_T = 6 L; α = 0.5).**



(1) Points:

(a) *The cell concentration declines in the third and fourth reactors due the dilution resulting from the addition of the feed into the third reactor.*

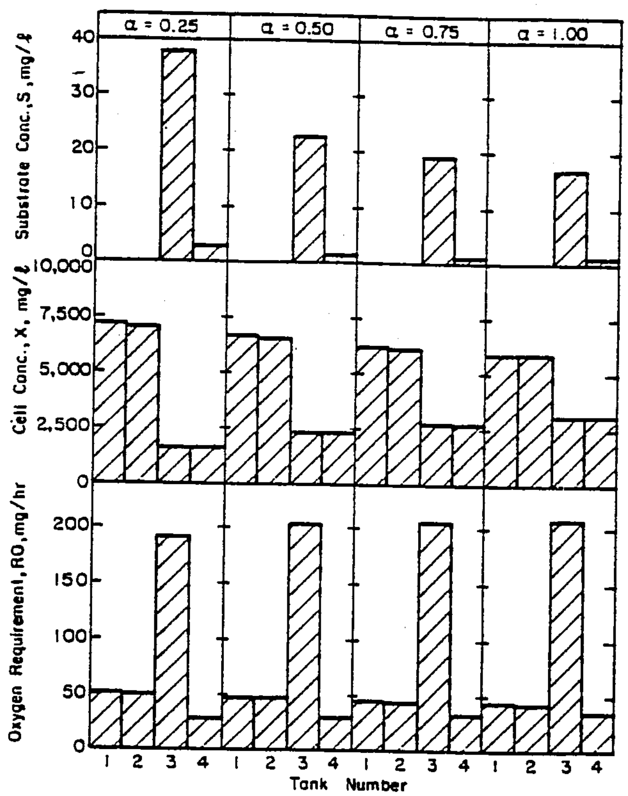
(b) *A substantial amount of substrate is removed by the fourth reactor.*

(c) *The oxygen requirement in the first and second tanks is due to endogenous respiration (e.g., decay).*

(d) *The increase in RO in the third reactor is due to the addition of substrate.*

(e) *Increasing the MCRT increases the cell concentration, resulting in a reduction in the substrate concentration and an increase in the RO.*

g) **Impact of recycle on the substrate concentration, cell concentration, and the oxygen requirement (V_T = 6 L; θ_c = 200 hours)**



(1) Points:

(a) *The recycle ratio has an impact on S, X and RO, similar to Case 3. Since the MCRT is fixed in this example, the total concentration of cells in the four tanks is constant. An increase in the recycle ratio decreases the concentration in the recycle flow and subsequently the concentration in the first tank. A decrease in the first two tanks must be accompanied by an increase in the last two tanks. An increase in the cell concentration in the last two tanks reduces the substrate concentration and increases the RO.*

(b) *Recycle can be used as a tool to alter substrate removal or oxygen uptake.*

(c) *Recycle does influence the volume of cells that must be wasted from the 4th tank to maintain a desired MCRT.*

8. We have considered only five cases out of an infinite number. These five cases represent the most common designs of activated sludge processes (to be discussed next).

9. Selection of the appropriate system configuration (e.g., the type of activated sludge process) is dependent on the overall objective.

   a) **One objective might be to minimize the overall volume for a given effluent quality (e.g., space constraint at a particular site).**

   b) **Another objective might be to minimize the overall cost.**

   c) **The engineer must select the proper configuration (e.g., the appropriate activated sludge process) based on the client's end objectives.**