Week 2 – Modeling Headway Distribution

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Microscopic Flow Characteristics
Time Headway - Distribution

Time Headway Definition
Time Headway versus Gap

Headway Characteristics
Some Applications

Uninterrupted Traffic:
- Driver Behavior Studies (minimum headway)
- Saturation Flow Studies
- Freeway Simulation models
- Freeway Merging Characteristics

Headway Characteristics
Some Applications

Interrupted Traffic:
- Gap Acceptance (Unsignalized intersection capacity)
- Saturation Flow Studies
- Traffic Signal Control
Microscopic Flow Characteristics
Time Headway - Distribution

Microscopic Flow Characteristics
Time Headway - Classification

- Random Headway State
  (Negative Exponential-Poisson count Distribution)

- Constant Headway State
  (Normal Distribution)

- Intermediate Headway State
  (Pearson type II, Gamma, Enlarg, Negative Exponential, shifted Negative Exponential)

Microscopic Flow Characteristics
Random Headway State-Poisson Distribution

\[ P(x) = \frac{m^x e^{-m}}{x!} \]

- \( P(x) \) = Probability that exactly \( x \) number of events occur during time interval (t)
- \( m \) = Average number of events during time interval (t)
- \( e \) = Napierian base of logarithms (\( e = 2.71828 \))
- Population mean = population variance
Examples of using Poisson Distribution

On an intersection approach with a left turn volume of 120 vph (Poisson), what is the probability of skipping the green phase for the left turn traffic? The intersection is controlled by an actuated signal with an average cycle length of 90 seconds.

\[ m = \text{Average number of left turn vehicles per Cycle} \]
\[ m = 120 \text{ vph} / 40 = 3 \text{ vehicles/cycle} \]
\[ \text{Probability of } x = 0 \]
\[ P(0) = 0.049787 = 4.9 \% \text{ (1.96 cycles/hours)} \]

A parking study was conducted for a parking lot that has 60 parking spaces. The study used 5 minutes intervals over a 2 hour period for 5 days. The number of empty parking spaces observed during the total time period is 200. Assuming a Poisson distribution, what is the probability that a parking space will be available at any time?

\[ m = \text{Average number of empty spaces per time period} \]
\[ m = 200 \text{ vph} / 120 = 1.67 \text{ space/time period} \]
\[ P(x > 0) = 1 - P(0) \]
\[ P(x > 0) = 1 - 0.18 = 0.82 \text{ } 82\% \text{ Probability of finding an empty parking space} \]

An intersection is controlled by a fixed time signal having a cycle length of 55 seconds. From the northbound, there is a permitted left turn movement of 175 vph. If two vehicles can turn each cycle without causing delay, on what percent of the cycles will delay occur?

\[ m = \text{Average number of left turn vehicles per Cycle} \]
\[ m = 175 \text{ vph} / 65.46 = 2.67 \text{ LT vehicles/cycle} \]
\[ \text{Probability of } x > 2 \]
\[ P(x > 2) = 1 - [P(0) + P(1) + P(2)] \]
\[ = 1 - [0.069 + 0.185 + 0.247] = 49.9\% \text{ [32.67 Cycles/hour]} \]

Using the following data to construct a distribution curve, determine the probability that a vehicle approaching on a side street will have to wait 1, 2, or 3 gaps before entering the traffic stream. (the minimum gap acceptance is 4.0 seconds)

<table>
<thead>
<tr>
<th>Gap size</th>
<th>Observed frequency</th>
<th>P (h=x)</th>
<th>P (h ≥ x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>2 – 3</td>
<td>9</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>3 – 4</td>
<td>14</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>4 – 5</td>
<td>15</td>
<td>0.21</td>
<td>0.56</td>
</tr>
<tr>
<td>5 – 6</td>
<td>13</td>
<td>0.19</td>
<td>0.74</td>
</tr>
<tr>
<td>6 – 7</td>
<td>9</td>
<td>0.13</td>
<td>0.87</td>
</tr>
<tr>
<td>7 – 8</td>
<td>6</td>
<td>0.09</td>
<td>0.96</td>
</tr>
<tr>
<td>&gt; 8</td>
<td>3</td>
<td>0.04</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Total of 70 Observations
Using the following data to construct a distribution curve, determine the probability that a vehicle approaching on a side street will have to wait 1, 2, or 3 gaps before entering the traffic stream. (the minimum gap acceptance is 4.0 seconds)

Examples of using Poisson Distribution

| Probability that the gap in the main street < 4.0 seconds | P (h < 4.0) = 0.34 |
| Probability that the gap in the main street ≥ 4.0 seconds | P (h ≥ 4.0) = 1 - 0.34 = 0.64 |
| Probability that a vehicle on the side street will not wait (turn in the first gap) | = Probability of first gap ≥ 4.0 = 0.64 (64%) |
| Probability that a vehicle on the side street will wait ONE gap | = Probability of first gap < 4.0 AND second gap ≥ 4 = 0.34 x 0.64 = 0.217 |
| Probability that a vehicle on the side street will wait TWO gaps | = Probability of first gap < 4.0 AND second gap < 4 AND third gap ≥ 4 = 0.34 x 0.34 x 0.64 = 0.074 (7.4%) |

Random Headway State-Poisson Distribution

\[ P(x) = \frac{m^x e^{-m}}{x!} \]

P (0) = e^{-m}

If no vehicles arrive in time interval (t), then the time headway must be equal to or greater than (t)

Therefore, P (h ≥ t) = P(0) = e^{-m}

If V = hourly flow rate, then

m = (V/3600) t (veh/time interval)

And, P (h ≥ t) = e^{-m} = e^{-vt/3600}

Example: traffic volume = 720 vph

<table>
<thead>
<tr>
<th>t = 3600/V</th>
<th>P (t ≤ h &lt; t+Δt)</th>
<th>P (h ≥ t)</th>
<th>F (t ≤ h &lt; t+Δt)</th>
<th>F (t ≤ h &lt; t+Δt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.819</td>
<td>0.181</td>
<td>131</td>
</tr>
<tr>
<td>1</td>
<td>0.819</td>
<td>0.710</td>
<td>0.249</td>
<td>87</td>
</tr>
<tr>
<td>2</td>
<td>0.710</td>
<td>0.549</td>
<td>0.451</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>0.549</td>
<td>0.449</td>
<td>0.551</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>0.449</td>
<td>0.368</td>
<td>0.632</td>
<td>99</td>
</tr>
<tr>
<td>5</td>
<td>0.368</td>
<td>0.301</td>
<td>0.673</td>
<td>147</td>
</tr>
<tr>
<td>6</td>
<td>0.301</td>
<td>0.247</td>
<td>0.719</td>
<td>206</td>
</tr>
<tr>
<td>7</td>
<td>0.247</td>
<td>0.202</td>
<td>0.756</td>
<td>258</td>
</tr>
<tr>
<td>8</td>
<td>0.202</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean Time Headway \( t = \frac{3600}{V} \)

And, P (h ≥ t) = e^{-vt/3600}

Therefore, P (h ≥ t) = e^{-vt/3600}

To obtain the frequency of headways:

F (t ≤ h < t+Δt) = N [P (t ≤ h < t+Δt)]

Where N is total number of observed headway
**Constant Headway State-Normal Distribution**

- Mean time headway \( h = \frac{3600}{V} \)
- Normal distribution with mean \( \mu \) and standard deviation \( s \)
- 95% confidence interval in the range \( (\mu \pm 2s) \)
- Minimum headway \( \alpha = \mu - 2s \) or \( t - 2s \)
  
  Then \( s = \frac{h - \alpha}{2} \)

**Normal Distribution:**

To get the probability \( P(A \leq X < B) \)

\[ Z = \frac{B - A}{s} \]

From the table using the value \( Z/s \)

We are interested in the probability:

\[ P(t \leq h < t) \]

**Example:** \( V = 2000 \text{ vph} \)

- Mean time headway = \( 3600/2000 = 1.8 \) seconds
- Assuming a minimum headway of 0.6 seconds

\( s = \frac{1.8 - 0.6}{2} = 0.6 \)

To obtain the probability: \( P(1.0 \leq h < t) \)

\[ Z = 1.8 - 1.0 = 0.8 \] or \( Z/s = 0.8/0.6 = 1.333 \)

\[ P(1.0 \leq h < 1.8) = 0.408 \]

40.8% of the headway between the mean headway and 1.0 second

**Intermediate Headway State**

- Generalized Mathematical Model Approach
- Composite Model Approach
- Other Approaches
Intermediate Headway State
Generalized Mathematical Model Approach

1. Pearson Type III
2. Gamma
3. Erlang
4. Negative Exponential
5. Shifted Negative Exponential

Pearson Type III
\[ f(t) = \frac{\lambda}{\Gamma(K)} [\lambda(t-\alpha)]^{K-1} e^{-\lambda(t-\alpha)} \]

\( f(t) \): Probability density function
\( \alpha \) and \( \lambda \): User select parameters that affect the shape and the shift of the distribution
\( \lambda \): Parameter that is a function of the mean time headway and the two user specified parameters (\( K \) and \( \alpha \))
\( t \): Time headway being investigated
\( \Gamma(K) \): Gamma function equivalent to \((K-1)!\)

\( \alpha = 0 \) and \( K > 0 \)

Gamma Distribution
\[ f(t) = \frac{\lambda}{\Gamma(K)} [\lambda t]^{K-1} e^{-\lambda t} \]

\( \alpha = 0 \) and \( K = 1 \)

Negative Exponential Distribution
\[ f(t) = \lambda e^{-\lambda t} \]
Intermediate Headway State
Generalized Mathematical Model Approach

Pearson Type III
\[ f(t) = \frac{\lambda}{\Gamma(K)} [\lambda (t - \alpha)]^{K-1} e^{-\lambda(t-\alpha)} \]
\( \alpha > 0 \) and \( K = 1 \)

\( \hat{f}(t) = \lambda e^{-\lambda(t-\alpha)} \)

Intermediate Headway State
Generalized Mathematical Model Approach

For any Model
\[ P(h \geq t) = \int_t^\infty f(t)dt \]

Intermediate Headway State
Composite Model Approach

\[ \bar{t} = \bar{t}_P P_P + \bar{t}_{NP} P_{NP} \]
\( P_p = \) Proportion of vehicles in Platoon
(Shifted Negative Exponential Distribution)

\( P_{NP} = \) Proportion of vehicles NOT in Platoon
Normal Distribution

Intermediate Headway State
Generalized Mathematical Model Approach

Pearson Type III
\[ f(t) = \frac{\lambda}{\Gamma(K)} [\lambda (t - \alpha)]^{K-1} e^{-\lambda(t-\alpha)} \]
\( \alpha > 0 \) and \( K = 1 \)

\( \alpha = 0 \) and \( K = 1 \)
\[ f(t) = \lambda e^{-\lambda t} \]
Negative Exponential Distribution

\( \alpha = 0 \) and \( K > 0 \)
\[ f(t) = \frac{\lambda}{(k-1)!} [\lambda t]^{k-1} e^{-\lambda t} \]
Gamma Distribution

\( \alpha = 0 \) and \( K = 1, 2, 3 \)
\[ f(t) = \frac{\lambda}{(k-1)!} [\lambda t]^{k-1} e^{-\lambda t} \]
Erlang Distribution

For any Model
\[ P(h \geq t) = \int_t^\infty f(t)dt \]