Chapter 1

Introduction

1.1 Examples of turbulent flows

The vast majority of flows of technological interest are turbulent, distinguished from laminar flows by their highly disorderly, random appearance. The randomness of the velocity and pressure fields makes it necessary to use statistical methods. Although statistical differential equations applicable to turbulent flows were formulated more than a century ago, it has also long been realized that their “exact” solution is impossible, because the number of unknown parameters exceeds the number of available equations (closure problem). Of particular interest are the properties of the fine structure of turbulence, as this controls the rate of mixing and, thus, greatly influences the rates of combustion and chemical reactions. Another issue of interest is the macroscopic evolution of turbulent flows, which, in most cases, is affected by the formation of identifiable, distinct, large-scale flow patterns, referred to as coherent structures.

Examples of highly disorderly, or irregular, or “turbulent” fluid flow can be found in abundance in the natural as well as in the man-made environment. When visualized by natural or technological means, turbulent motion can be fascinating and provoking in its complexity. The very perceptive drawings made by Leonardo da Vinci around (e.g., Fig. 1.1) clearly capture the evident features of most turbulent flows:

- the presence of large-scale, vortical, coherent structures, superimposed on a background of small-scale motions;
• an apparent directional randomness ("isotropy") of the fine structure; and

• a sharp "interface" between turbulent and non-turbulent regions of the flow.

Figure 1.1: Sketch of water spout by Leonardo da Vinci (ca. 1510).

da Vinci himself recognized some of these features in his description of water motion on a free surface: "the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion."

Such features have long occupied prominent positions as topics of modern turbulence research. The appearance of typical turbulent flows is illustrated in Figs. 1.2-1.6.
1.1. EXAMPLES OF TURBULENT FLOWS

Figure 1.2: Turbulent flow generated by a perforated plate and visualized by the smoke wire method (Van Dyke (1982)).
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Figure 1.3: Turbulent water jet visualized using a fluorescent dye (Van Dyke (1982)).

Figure 1.4: Turbulent mixing layer between two gases (Van Dyke (1982)).
1.1. EXAMPLES OF TURBULENT FLOWS

Figure 1.5: Laminar and turbulent boundary layer separation from convex surfaces and corners (Van Dyke (1982)).
Figure 1.6: Turbulent flows, including the “Giant Red Spot” in the atmosphere of the planet Jupiter.
1.2 Characteristics of turbulence

Turbulence is a feature of fluid flow and not a property of any particular fluid. All fluids will become turbulent under certain conditions (e.g., at sufficiently large Reynolds number). Every turbulent flow is unique in its details and exhibits strong variations in space and time. Even so, most turbulent flows have common characteristics, which can be used to distinguish them from other phenomena. Among the most important characteristics are the following ones.

- Turbulent flows are highly disorderly, or random. Even so, when examined statistically, turbulent features display some regularity. Randomness is a necessary but not sufficient condition for turbulence, as there exist many types of fluid motion, which can be very complex and even contain random elements, but cannot be characterized as turbulent. For example, a low Reynolds number flow induced by the interactions of a large number of laminar vortices may have a random velocity field but it is not turbulent.

- Turbulent flows are highly diffusive, causing rapid mixing and increased rates of momentum, mass and heat transfer (Fig. 1.7). Therefore, a random-looking fluid which does not quickly diffuse its properties is not turbulent, even if it had been turbulent at some previous time. For example, consider the “skyline” left far behind a jet airplane flying at high altitude; this is a nearly frozen pattern of condensation induced by nucleation in the jet.

- Turbulent flows are rotational and three dimensional, characterized by high levels of fluctuating vorticity and the mechanism of vortex stretching. An exceptional case of approximately two-dimensional turbulence occurs in a stably stratified atmosphere, in which vertical motions are inhibited by gravitational forces.

- Turbulent flows are strongly dissipative, converting kinetic energy of the velocity fluctuations to internal energy through the action of viscous shear stresses. Therefore, turbulence would eventually decay, unless provided with a source of kinetic energy; such sources include non-uniformities of the mean flow velocity (shearing), buoyancy due to density differences, and centrifugal actions due to curvature of the stream.
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Figure 1.7: Laminar vs. turbulent mixing: the sequence on the left shows the mixing of creamer in coffee, while the sequence on the right shows the mixing of white paint in molasses. Due to the high viscosity of molasses, the Reynolds number is relatively low and the flow induced by rotation of the spoon is laminar (Corrsin (1961)).
1.3. **Definition of Turbulence**

or rotation of the fluid. The phenomenon of kinetic energy dissipation distinguishes turbulence from random wave propagation, which is non-dissipative or only slightly dissipative.

- Turbulent flows contain motions with *wide ranges of amplitudes, length scales and time scales*. The smallest scales of dynamic importance in turbulence, called the *Kolmogorov scales*, are generally much larger than the scales of molecular motions, so that turbulence may be studied as a *continuum* phenomenon, governed by the basic principles of Newtonian mechanics and thermodynamics (i.e., conservation of mass, momentum and energy). Although turbulent motions involve movement of the elementary particles of matter, there is no apparent need to consider individual molecular motions for the study of most properties of turbulence. Theories of turbulence do not cover low-density gases for which distances between molecules are relatively large.

- Turbulent flows have *spatial intermittency*; this means that the small-scale motions that contribute to energy dissipation are not evenly distributed in space but highly “spotty”.

- All fluid flows will become turbulent when the Reynolds number (or other dynamic parameters such as the Richardson number or the Taylor number) exceeds a certain value. The process of instability of a laminar flow that leads to a turbulent one is called *transition to turbulence*. The reverse phenomenon, called *relaminarization*, may also occur under certain conditions.

### 1.3 Definition of turbulence

The word “turbulent” is derived from the latin word “turbulentus”, which literally means restless (“turba” = turmoil + “lentus” = full of). Da Vinci used the term “turbulenzia”, but O. Reynolds used the term “sinuous” to describe turbulent pipe flow (Fig. 1.8). The term “turbulence” was established in scientific use in the early decades of the twentieth century. The common, everyday meaning of the word, as defined in the dictionaries, is “disturbed”, “agitated”, “in a state of violent disorder or commotion”. The scientific meaning of turbulence is nearly impossible to define by a single sentence and is best demonstrated through examples and through description.
of common characteristic features (see earlier section). Among the various attempts to define turbulence one could mention the following:

_Taylor and von Kármán (1937):_ Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighbouring streams of the same fluid flow past or over one another.

_Hinze (1975):_ Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned.
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Bradshaw (1971): Turbulence is a three dimensional time dependent motion in which vortex stretching causes velocity fluctuations to spread to all wavelengths between a minimum determined by viscous and a maximum determined by the boundary conditions of the flow. It is the usual state of fluid motion except at low Reynolds number.

A.J. Reynolds (1974): Turbulence is a fluid motion involving random macroscopic mixing, with a large range of length and time scales.

1.4 Turbulence in Engineering

The study of turbulence as a fascinating physical phenomenon has its own scientific merit. On the other hand, its prediction and control is a challenging engineering task, considering its extensive occurrence and its important effects, desirable or undesirable, in biological, technological and environmental systems. A few examples of such effects are given below.

- Compared to laminar flows, turbulent flows are generally characterized by increased frictional forces exerted by the fluid on solid surfaces. This applies to both internal and external flows. An example of great importance in engineering is flow through pipes. For any given pipe-fluid configuration, a flow may switch from laminar to turbulent or vice versa if one adjusts the flow velocity through the pipe. Transition occurs as soon as the Reynolds number exceeds a certain critical value. The appearance of the flow changes dramatically during transition, as may be demonstrated by injecting a dye filament along the centreline of the pipe. The pressure loss along the pipe also changes significantly following transition.

- The drag on an aircraft, ship or automobile, the pressure drop in a pipe, the vibrations of a bridge or a telecommunications tower and the efficiency of a turbomachine all depend greatly on the state of the flow. In most cases, turbulence is accompanied by undesirable effects, such as decrease of efficiency and the need for higher power and thrust. Large turbulent stresses are known to cause hemolysis, i.e. destruction of blood cells, in artificial hearts. On the other hand, turbulence is sometimes deliberately introduced in order to alleviate other problems. A example very familiar to aerodynamicists is the
“tripping” of a laminar boundary layer on a wing to prevent separation at relatively large angles of attack (Fig. 1.5).

- Pressure fluctuations in turbulent flow often result in increased noise level. Jet noise control is still a serious engineering problem. They are also associated with scattering of electromagnetic waves that affect telecommunications.

- A very important and useful aspect of turbulence is that it increases the rates of mixing and of momentum, heat and mass transport (Fig. 1.7). Heat exchangers, nuclear reactors, chemical reactors and combustion chambers would be very inefficient without turbulence. Rapid dispersion of pollutants in the atmosphere and the oceans are caused by atmospheric and oceanic turbulence. Examples of technological applications of turbulent mass transport include sediment and slurry transport, fluidized beds and sprays.

1.5 Generation and maintenance of turbulence

1.5.1 Instability and transition

The normal state of any flow at relatively small Reynolds number is laminar. As the Reynolds number is gradually increased, the flow may become unstable to even extremely small disturbances and would not continue at its current state, but either change to another, more stable, laminar state or become turbulent. When the Reynolds number exceeds a critical value, characteristic for each type of flow, a laminar flow will be transformed to a turbulent one, following the process called transition to turbulence. Besides the Reynolds number, transition is affected by several other factors, including the level and type of disturbances, mean pressure gradients and wall shape and roughness. The details of transition remain a topic of intense research, as no generally acceptable theory has yet been formulated.

Instability and transition usually occur as a result of excessive growth of disturbances, which always exist as a background in a physical system. As a simple analogy, consider a sphere, in stable (mono- or multi-stable), unstable or neutrally stable equilibrium (Fig. 1.9). Even the slightest displacement or impulse will cause the sphere to move. After some time, under the action of friction, the sphere will reach a new equilibrium state, which, depending on
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Mathematical analyses are currently capable of predicting instability of a few relatively simple flow configurations. The mathematical study of the evolution (amplification or decay) of small disturbances in laminar flows and the prediction of critical Reynolds numbers for instability comprise a branch of fluid mechanics, called hydrodynamic stability. Its results are often presented in the form of a stability diagram. In many configurations, there exists a “critical” Reynolds number Re_{cr} below which all disturbances, however strong they may be, will decay and the flow is unconditionally stable. For Re > Re_{cr}, depending on their amplitude and wavelength, some disturbances may decay, while others may not, leading to instability and to a change in flow pattern, although not necessarily to turbulence.

For several flow configurations, including boundary layers and circular Couette flows, theoretical predictions of instability are in good agreement with experimental results. However, classical hydrodynamic stability analysis is unable to predict instability and transition for pipe flows, for which it finds that Re_{cr} \to \infty. Under usual laboratory conditions transition in pipe flow
occurs at $Re \approx 2300$. However, in certain carefully designed experiments with extremely smooth pipes and “quiet” surroundings, laminar flow was maintained at much higher values of $Re$, reputedly up to 100,000.

Examples of unstable laminar flows that undergo a change to another laminar state or transition to turbulence are shown in Figs. 1.10-1.15.

1.5.2 Turbulence production and dissipation

Turbulence is always dissipative, converting its kinetic energy into internal fluid energy. Therefore, unless provided with kinetic energy from another source, it cannot maintain itself and would eventually decay. A common source of energy is mean shear, namely a non-uniformity of the mean flow velocity, which converts kinetic energy of the mean flow into turbulent kinetic energy. Shear is produced by viscous actions in the proximity of solid walls and in mixing regions between fluid streams of different velocities. An inter-
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Figure 1.11: Turbulent (or Emmons) spot in a laminar boundary layer over a smooth flat plate (Van Dyke (1982)).

Figure 1.12: Flow instability and successive modes of motion in Taylor-Couette flow (Van Dyke (1982)).
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Figure 1.13: Flow instability and transition to turbulence of an axisymmetric jet (Van Dyke (1982)).

Figure 1.14: Flow instability of a thermal plume (Van Dyke (1982)).
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Figure 1.15: Breakdown of delta wing vortices (Van Dyke (1982)).
nal source of turbulence production is buoyancy due to density differences within the fluid, which could be caused by mixing of different fluids or by heating. Another possible mechanism that produces turbulence is centrifugal actions due to curvature of the stream or rotation of the fluid.

If no continuous energy source exists, turbulence decays and, eventually, the flow will relaminarize. The process of transition may also be obstructed by mechanisms that consume energy so that a flow may not become turbulent, even at relatively large Reynolds numbers. Examples of energy consuming mechanisms are electro-magnetic fields acting on electrically conductive fluids, stable density stratification in atmospheric and oceanic flows, flow acceleration due to a favourable pressure gradient, streamline curvature and phase change.

1.6 Historical perspective

The general equations of motion that apply to laminar as well as turbulent flows (Navier-Stokes equations) were developed by the first half of the 19th century, by the great scientists and mathematicians J.R. d’Alembert, L. Euler, P.S. Laplace, J.L. Lagrange, L.M.H. Navier, A.L. Cauchy, S.D. Poisson, J.-C. B. Saint-Venant, G. Green and G.G. Stokes. The first systematic analytical and experimental studies of turbulence may also be traced into the 19th century. Experiments showing transition to turbulence in pipe flow and its dependence on diameter, velocity and temperature were performed by G.H.L. Hagen (1839, 1854, 1869), while Saint-Venant (1843) and J. Boussinesq (1877) demonstrated the analogy between transport of momentum by turbulent eddies and transport by molecular motions. The true classical landmarks of modern turbulence research may, however, be identified as follows.

- The experiments by Osborne Reynolds (1883), which clearly established the transition criterion for pipe flows.
- The decomposition of the turbulent fluid motion into a mean (average) and a fluctuation and the derivation of statistical equations for the averages, also due to O. Reynolds (1894).
- The formulation of the first turbulence models by Geoffrey I. Taylor (1915, 1932), Ludwig Prandtl (1925) and Theodore von Kármán (1930),
as attempts to circumvent the “closure problem” of the statistical equations.

- The development of the hot-wire anemometer as a means to measure turbulence fluctuations (Louis Vessot King, 1914, 1925 and 1916; Dryden and Kuethe, 1929).

- The concept of homogeneous and isotropic turbulence (Taylor, 1935; von Kármán and L. Howarth, 1938).

- The theory of local isotropy and the concept of the inertial spectral range, due to A. N. Kolmogorov (1941).

- The development of similarity laws for different free and bounded turbulent flows (starting in the 1940s).

### 1.7 Methods of study

Like many other branches of applied science and engineering, turbulence has been studied with the use of all available tools, which include mathematical, experimental and, more recently, numerical techniques. A review of turbulence literature will clearly show that there is a close interaction and feedback among theory, experiment and computation. Therefore, one may conclude that comprehension of turbulent phenomena and application of this knowledge would be impossible without some familiarity with the basic principles and the results of all three methods.

#### 1.7.1 Analytical methods

The mathematical formulation of the turbulence problem, based on the fundamental laws of mechanics, was achieved by Reynolds more than a century ago. It has also long been realized that an exact solution is impossible with the use of currently available mathematical procedures. The random character of turbulence requires a statistical approach, which, however, introduces an unsurpassable difficulty: it results in an “open hierarchy” of statistical differential equations, namely a system in which the number of unknowns is larger than the number of equations. This is due to the non-linearity of the Navier-Stokes equations and is known as the closure problem of turbulence theory. This impass can be resolved only approximately, with the use
of assumed additional relations between various turbulent quantities, thus rendering the number of equations equal to the number of unknowns. Such relations, known as turbulence models are necessarily arbitrary, because they are not consequences of fundamental principles of fluid mechanics. They can be entirely empirical, i.e. best fits to observations or experimental results or semi-empirical, combining empiricism with some physical intuition into the mechanisms involved.

A powerful semi-empirical approach is dimensional analysis. Some turbulent quantity is assumed to depend only on certain imposed parameters, which in turn are arranged to form groups having the proper dimension. In most cases, a numerical coefficient, to be determined from experimental results, is included in the relation. For example, self-preservation or local invariance is the assumption that the structure of a turbulent flow is continuously evolving according to a fixed pattern and remains similar to itself, if properly non-dimensionalized with the local length, velocity and/or time scales. This permits the theoretical prediction of useful features of entire classes of turbulent flows, called self-preserving flows. Another useful approach is to assume that, in the limiting case of Reynolds number approaching infinity, certain turbulence properties become universal. This leads to the so-called universal equilibrium theories.

An indirect approach is to study mathematical problems which, although much simpler, exhibit certain analogies to turbulence. For example, Burgers’ (sic) is the study of a non-linear equation, which is the one-dimensional analog of the Navier-Stokes equations. The analogy between turbulence and low-order dynamical systems has also been explored by several researchers. Needless to say that generalizations based on such analogies are, at best, risky.

1.7.2 Numerical methods

Current numerical simulations of turbulent flows may be classified into three types.

Direct Numerical Simulations (DNS). These methods solve numerically the “exact” time dependent Navier-Stokes equations under stochastic boundary and/or initial conditions. The statistical information on turbulence parameters is then obtained by averaging a large number of individual solutions. Initially developed for some very simple turbulent flows (e.g. isotropic turbulence and uniformly sheared turbulence) at relatively small Reynolds
numbers, DNS have been successively applied to more complex problems, promising that more problems may become solvable as the computer’s size and speed increase. Even so, there is little hope that DNS will be capable of solving large-scale, large Reynolds number practical problems in the near future, because of the enormous memory and computational time requirements.

Large-Eddy Simulations (LES). These methods “filter” away the fine structure of turbulence and solve “exact” equations for the relatively large-scale motions, while modelling the fine structure effects. Despite earlier high expectations, LES have not yet delivered the promise of accurate computation of practical problems, but future improvements may enhance their applicability.

Solutions of the Reynolds-Averaged Navier-Stokes equations (RANS). These are solutions of the statistical equations, rendered “closed” with the use of some turbulence model. Despite the fragility of its physical and mathematical foundations, semi-empirical closure of turbulence equations has led to useful predictions of highly complex flows. For predictions of industrial flows, RANS is the most widely used approach, because it requires far less memory and computational time than DNS and LES. The complexity and computational requirements of different turbulence models vary widely. Although it has been recognized that more complex models may be made to provide more accurate predictions, the current trend is to fine tune the simpler models, such as the eddy viscosity model and the $k - \epsilon$ model, that may be introduced in complex numerical codes with minimal additional effort.

Solutions of the Unsteady Reynolds-Averaged Navier-Stokes equations (URANS). This approach is suitable to turbulent flows that have distinct large-scale coherent motions, which are separated significantly in length and time scales from the non-coherent motions. Like the RANS method, URANS utilize turbulence models for the non-coherent turbulence, but, unlike RANS, they solve the unsteady equations of motion for the coherent one, which may be missed by RANS analyses. The total turbulence activity can be then found by superimposing the coherent and non-coherent contributions. URANS require significantly less computational resources that DNS and LES.

1.7.3 Experimental methods

The most reliable information for most practical turbulent flows is still based on experiment. Even so, one must bear in mind that the design and re-
alization of meaningful experiments, especially the ones corresponding to complicated technological flows, is not an easy task. For this reason, much of turbulence research has been conducted in simplified flow set-ups, which are relatively easily controlled in the laboratory. The available instrumentation is quite sophisticated and powerful, including hot-wire anemometry (HWA), laser-Doppler velocimetry (LDV or LDA) and particle image velocimetry (PIV). In addition to the statistics of flow velocity, it has been possible to measure the statistics of pressure, temperature and concentration fluctuations. A well equipped laboratory can provide reliable "local" measurements of the statistical moments, correlations, spectra, probability density functions and scales of a flow parameter and its derivatives, as well as "global" measurements with the use of multi-probe and optical techniques. The acquisition of turbulence measurements and the interpretation of the results require an in-depth understanding of the techniques involved and of the physical properties of the flow under study.

1.8 Canonical turbulent flows

Each turbulent flow is unique in its macroscopic details. Furthermore, most flows of technological interest, as, for example, flow in a gas turbine or in the human lung, are subjected to serious geometrical and physical complications. The main objective of engineering is, of course, to solve such problems, but this cannot be achieved without first developing a thorough understanding of the basic concepts and mechanisms of the underlying phenomena, which can only be achieved by studying simpler, exemplary systems. Turbulence research has historically evolved through the study of simplified cases, to be denoted as "canonical turbulent flows". These include isotropic turbulence, homogeneous shear flows, free shear layers, boundary layers, pipe and channel flows and two-dimensional and axisymmetric jets, wakes and plumes. The schematics and mean velocity profiles of these flows are shown in Fig. 1.16. Such cases rarely occur in their ideal form, however, they often constitute elements of more complicated flows. For example, a wall jet can be perceived as a combination of a jet and a boundary layer. These elementary turbulent flows have been studied extensively analytically, experimentally and numerically and will be used as vehicles for introducing the definitions, concepts and solution approaches to the problem of turbulence.
Figure 1.16: Schematic representation of “canonical” turbulent flows.