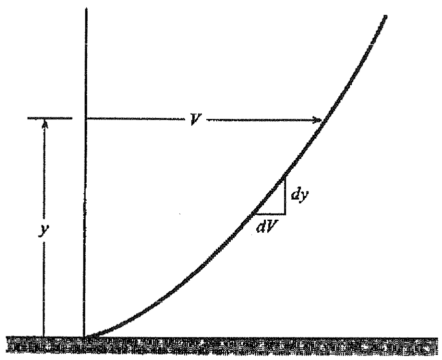
FIGURE 2.1

Velocity distribution next to a boundary.



Consider the flow shown in Fig. 2.1. This velocity distribution is typical of that for laminar (nonturbulent) flow next to a solid boundary. Several observations relating to this figure will help you appreciate the interaction between viscosity and velocity distribution. First, the velocity gradient at the boundary is finite. The curve of velocity variation cannot be tangent to the boundary because this would imply an infinite velocity gradient and, in turn, an infinite shear stress, which is impossible. Second, a velocity gradient that becomes less steep (dV/dy becomes smaller) with distance from the boundary has a maximum shear stress at the boundary, and the shear stress decreases with distance from the boundary. Also note that the velocity of the fluid is zero at the stationary boundary. This is characteristic of all flows dealt with in this basic text. That is, at the boundary surface the fluid has the velocity of the boundary—no slip occurs.

From Eq. (2.6) it can be seen that the units of μ are $N \cdot s/m^2$.

$$\mu = \frac{\tau}{dV/dy} = \frac{N/m^2}{(m/s)/m} = N \cdot s/m^2$$

A common unit of viscosity is the *poise,* which is 1 dyne-s/cm^2 or 0.1 $N \cdot s/m^2$. The viscosity of water at 20°C is one centipoise (10^{-2} poise) or 10^{-3} $N \cdot s/m^2$. The unit of viscosity in the traditional system is $lbf \cdot s/ft^2$.

Many of the equations of fluid mechanics include the combination μ/ρ. Because it occurs so frequently, this combination has been given the special name *kinematic viscosity* (so called because the force dimension cancels out in the combination μ/ρ). The symbol used to identify kinematic viscosity is ν (nu). The units of kinematic viscosity ν are m^2/s.

$$\nu = \frac{\mu}{\rho} = \frac{N \cdot s/m^2}{kg/m^3} = m^2/s$$

The units for kinematic viscosity in the traditional system are ft^2/s.

Whenever shear stress is applied to a fluid, motion occurs. This is the basic difference between fluids and solids. Solids can resist shear stress in a static condition, but fluids deform continuously under the action of a shear stress. Another important characteristic of fluids is that the viscous resistance is independent of the normal force (pressure) acting within the fluid. In contrast, for two solids sliding relative to each other, the shearing resistance is totally dependent on the normal force between the two.

or

$$p = \frac{2\sigma}{r}$$

Case (b) is a bubble of radius r that has internal and external surfaces and the surface-tension force acts on both surfaces, so

$$p = \frac{4r}{\sigma}$$

Case (c) is a cylinder supported by surface-tension forces. The liquid does not wet the cylinder surface. The maximum weight the surface tension can support is

$$Wt = 2F_s = 2\sigma L$$

where L is the length of the cylinder.

Case (d) is a ring being pulled out of a liquid. This is a technique to measure surface tension. The force due to surface tension on the ring is

$$\begin{aligned} F_\sigma &= F_{\sigma,i} + F_{\sigma,o} \\ &= \pi\sigma(D_i + D_o) \end{aligned}$$

In addition to the preceding cases, surface tension is an important force in the shattering of liquid droplets, the shape and motion of bubbles, and the structure of foams.

2.8 Vapor Pressure

The pressure at which a liquid will boil is called its *vapor pressure*. This pressure is a function of temperature (vapor pressure increases with temperature). In this context we usually think about the temperature at which boiling occurs. For example, water boils at 212°F at sea-level atmospheric pressure (14.7 psia). However, in terms of vapor pressure, we can say that by increasing the temperature of water at sea level to 212°F, we increase the vapor pressure to the point at which it is equal to the atmospheric pressure (14.7 psia), so that boiling occurs. When we think of incipient boiling in terms of vapor pressure, it is easy to visualize that boiling can also occur in water at temperatures much below 212°F if the pressure in the water is reduced to its vapor pressure. For example, the vapor pressure of water at 50°F (10°C) is 0.178 psia (approximately 1% of standard atmospheric pressure). Therefore, if the pressure within water at that temperature is reduced to that value, the water boils.* Such boiling often occurs in flowing liquids, such as on the suction side of a pump. When such boiling does occur in flowing liquids, vapor bubbles start growing in local regions of very low pressure

* Actually, boiling can occur at this vapor pressure only if there is a gas–liquid surface present to allow the process to start. Boiling at the bottom of a pot of water is usually initiated in crevices in the material of the pot, in which minute bubbles of air are entrapped even when the pot is filled with water.

4·4

Euler's Equation

In Chapter 3, we learned that the forces acting on a static fluid particle are the pres and gravitational force (weight). With no acceleration, the sum of these forces is z In this section, we extend the analysis to an accelerating fluid particle.

From dynamics we know that the motion of a body is governed by Newton's ond law, $F = ma$. The forces acting on a fluid mass are due to pressure and gra (weight). For the present we are neglecting the forces due to viscous effects. Consider cylindrical fluid element* situated between two streamlines shown in Fig. 4.12. We regard this element as a "free body" in which the presence of the surrounding fluid is placed by pressure forces acting on the element. Here the element is being accelerate the ℓ direction, and the direction of ℓ is arbitrary. Note that the coordinate axis z is v cally upward and that the pressure varies along the length of the element. Applying N ton's second law in the ℓ direction, we have

$$\sum F_{\ell} = ma_{\ell}$$ (4

$$F_{\text{pressure}} + F_{\text{gravity}} = ma_{\ell}$$

The mass of the fluid element is

$$m = \rho \Delta A \Delta \ell$$

Substituting the forces due to pressure and gravity (weight) into Eq. (4.14), we have

$$p\Delta A - (p + \Delta p)\Delta A - \Delta W \sin\alpha = \rho \Delta A \Delta \ell a_{\ell}$$ (4

Notice that the pressure force acting on the sides of the cylindrical element do not c tribute to the force in the ℓ direction. However, $\Delta W = \gamma \Delta \ell \Delta A$, so Eq. (4.15) reduces

$$-\frac{\Delta p}{\Delta \ell} - \gamma \sin\alpha = \rho a_{\ell}$$ (4

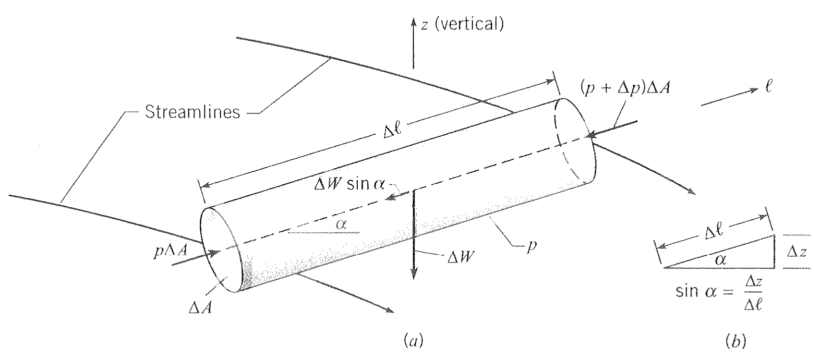


FIGURE 4.12

"Free body" diagram for fluid element accelerating in the ℓ direction. (a) Fluid element. (b) Trigometric relation.

* In this case, the words "fluid element" and "fluid particle" are synonymous. The word "elemer used here because it is more descriptive of the shape.

Pressure is a function of both position and time. Taking the limit of $\Delta p/\Delta\ell$ at a given time as $\Delta\ell$ approaches zero yields the partial derivative

$$\lim_{\Delta\ell \to 0} \frac{\Delta p}{\Delta\ell}\bigg|_t = \frac{\partial p}{\partial\ell}$$

Figure 4.12*b* also shows that $\sin\alpha$ is equal to $\Delta z/\Delta\ell$. Taking the limit as $\Delta\ell$ approaches zero at a given time yields

$$\sin\alpha = \lim_{\Delta\ell \to 0} \frac{\Delta z}{\Delta\ell} = \frac{\partial z}{\partial\ell}$$

Thus the limiting form Eq. (4.16) when $\Delta\ell$ approaches zero is

$$-\frac{\partial p}{\partial\ell} - \gamma\frac{\partial z}{\partial\ell} = \rho a_\ell$$

or, taking γ as a constant,

$$-\frac{\partial}{\partial\ell}(p + \gamma z) = \rho a_\ell \qquad (4.17)$$

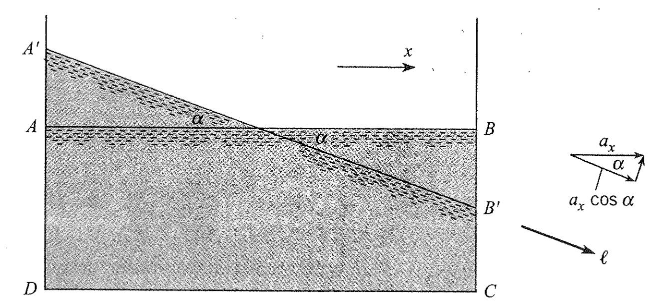
Equation (4.17) is *Euler's equation* for motion of a fluid. It is interesting to note that when the acceleration is zero, Eq. (4.17) reduces to $\partial/\partial\ell(p + \gamma z) = 0$, which corresponds to the familiar hydrostatic equation $p + \gamma z = \text{const.}$ In other words, along a direction in which there is no acceleration the pressure distribution is hydrostatic. For example, in a flow with straight, parallel streamlines, the pressure in the direction normal to the streamlines is hydrostatic because there is no acceleration in this direction. Again, this assumes that the gravity and pressure forces are the only forces acting. When the flow is static, there is no motion (or acceleration), so the viscous stresses are zero and Euler's equation reduces to the hydrostatic equation.

An example application of Euler's equation is to the uniform acceleration of liquid in a tank.

Assume that the open tank of liquid shown in Fig. 4.13 is accelerated to the right, the positive x direction, at a rate of a_x. For this to occur, a net force must act on the liquid in the x direction; this is accomplished when the liquid redistributes itself in the tank as shown by $A'\,B'\,CD$. Under this condition the hydrostatic force at the left end is greater than the hydrostatic force at the right, which is consistent with the requirement of $F = Ma$.

FIGURE 4.13

Uniform acceleration of a tank of liquid



Further quantitative analysis of the acceleration of the tank of liquid is made Eq. (4.17). First consider application of the equation along the liquid surface $A'B'$. the pressure is constant, $p = p_{\text{atm}}$. Consequently, $\partial p/\partial, = 0$. The acceleration $A'B'$ is given by $a_{,} = a_x \cos\alpha$. Hence, Euler's equation reduces to

$$\frac{d}{d,}(\gamma z) = -\rho a_x \cos\alpha \qquad ($$

where the total derivative is used because the variables do not change with time. The cific weight in Eq. 4.18 is constant. Therefore, Eq. (4.18) becomes

$$\frac{dz}{d,} = -\frac{a_x \cos\alpha}{g}$$

But $dz/d, = -\sin\alpha$. Thus we obtain

$$\sin\alpha = \frac{a_x \cos\alpha}{g}$$

or

$$\tan\alpha = \frac{a_x}{g} \qquad (4$$

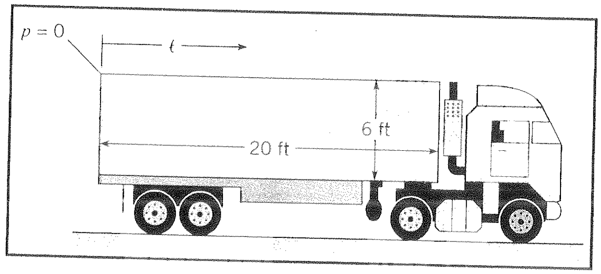
Still further analysis can be made if Euler's equation is applied along a horizo plane in the liquid, such as at the bottom of the tank. Now z is constant and Eu equation reduces to $\partial p/\partial, = -\rho a_x$, which shows that the pressure must decreas the direction of acceleration. The change in pressure is consistent with the chang depth of the liquid because hydrostatic pressure variation prevails in the vertical di tion, since there is no component of acceleration in that direction. Thus as the d decreases in the direction of acceleration, the pressure along the bottom of the tank r also decrease. Another case of uniform acceleration is given in the following exampl

example 4.3

The tank on a tank truck is filled completely with gasoline, which has a specific weig of 42 lbf/ft^3 (6.60 kN/m^3).

a. If the tank on the trailer is 20 ft (6.1 m) long and if the pressure at the top rear e of the tank is atmospheric, what is the pressure at the top front when the truck dec erates at a rate of 10 ft/s^2 (3.05 m/s^2)?

b. If the tank is 6 ft (1.83 m) high, what is the maximum pressure in the tank?



Solution Apply Euler's equation along the top of the tank. Here z is constant and the pressure does not vary with time during this phase of deceleration. Therefore, one may write

$$\frac{dp}{d\ell} = -\rho a_\ell$$

Integrating, one obtains

$$p = -\rho a_\ell \ell + C$$

When $\ell = 0$, $p = 0$; hence, $C = 0$ and $p = -\rho a_\ell \ell$.

Now substituting $-10\ \mathrm{ft/s^2}$ ($-3.05\ \mathrm{m/s^2}$) for a_ℓ, 20 ft (6.1 m) for ℓ, and 1.30 slugs/ft^3 (672 kg/m^3) for ρ, which is equal to γ/g, one obtains

$$p = -1.30\ \mathrm{slugs/ft^3} \times (-10\ \mathrm{ft/s^2}) \times 20\ \mathrm{ft} = 260\ \mathrm{psfg} \qquad \triangleleft$$

SI units $p = -673\ \mathrm{kg/m^3} \times (-3.05\ \mathrm{m/s^2}) \times 6.1\ \mathrm{m}$

$$= 12{,}500\ \mathrm{N/m^2} = 12{,}500\ \mathrm{Pa\ gage} \qquad \triangleleft$$

The maximum pressure in the tank will occur at the front end of the tank bottom. Since the pressure variation is hydrostatic in the vertical direction, one obtains $p + \gamma z = \text{constant}$, or

$$p_{\text{bottom}} + \gamma z_{\text{bottom}} = p_{\text{top}} + \gamma z_{\text{top}}$$

Solving yields

$$p_{\text{bottom}} = 260 + (42)(6)$$

$$p_{\max} = p_{\text{bottom}} = 512\ \mathrm{psfg} \qquad \triangleleft$$

SI units $p_{\max} = p_{\text{bottom}} = 12{,}500\ \mathrm{N/m^2} + 6.6\ \mathrm{kN/m^3} \times 1.83\ \mathrm{m}$

$$= 24.6\ \mathrm{kPa\ gage} \qquad \triangleleft$$

4.5 The Bernoulli Equation

The Bernoulli Equation along a Streamline

From the dynamics of particles in solid-body mechanics, we know that integrating Newton's second law for particle motion along a pathline provides a relationship between the change in kinetic energy and the work done on the particle. Integrating Euler's equation along a pathline in the steady flow of an incompressible fluid yields an equivalent relationship called the Bernoulli equation.

The pressure coefficient at point 3 is

$$C_{p_3} = 1 - \left(\frac{V_3}{V_1}\right)^2$$

$$= 1 - \left(\frac{270}{300}\right)^2$$

$$= 0.19 \qquad \triangleleft$$

The negative C_{p_2} indicates that the local static pressure at point 2 is less than the free-stream value and the positive C_{p_3} shows a larger pressure at point 3 than the free-stream value.

4.6

Rotation and Vorticity

Concept of Rotation

Consider a tank of liquid that is being rotated about a vertical axis. A plan view of such a tank is given in Fig. 4.17. If we focus on a given element, it can be seen that this element will rotate but not deform as time passes. In this process, all lines drawn through the element, such as *a-a* and *b-b* in Fig. 4.17, will rotate at the same rate. This is unquestionably a case of fluid rotation. Now consider fluid flow between two horizontal plates, Fig. 4.18, where the bottom plate is stationary and the top is moving to the right with a velocity *V*. The velocity distribution is linear; therefore, an element of fluid will deform as shown. Here we see that the element face that initially vertical rotates clockwise, whereas the horizontal face does not. It is not clear whether this is a case of rotational motion or not.

Rotation is defined as the average rotation of two initially mutually perpendicular faces of a fluid element. The test is to look at the rotation of the line that bisects both faces (*a-a* and *b-b* in Fig. 4.18*a*). The angle between this line and the horizontal axis is θ. If this line rotates, the flow is rotational. Obviously, in this case, there is rotation because the bisector does rotate. If the bisector does not rotate, the flow is irrotational. The rotation can be monitored by inserting a cruciform (cross) shape in the flow, as shown in Fig. 418*b*, and checking if it rotates. The cross will rotate with the bisector. If there is no rotation, the flow is irrotational.

We will now derive an expression that will give the rate of rotation of the bisector in terms of the velocity gradients in the flow. Consider the element shown in Fig. 4.19. The sides of the element are initially perpendicular. Then the element moves with time and deforms as shown. After time Δt the horizontal side has rotated counterclockwise by $\Delta\theta_A$ and the vertical side clockwise by $\Delta\theta_B$. By definition, counterclockwise rotation is positive. The rotational rate of the bisector is half the sum of the rotational rate of each side, so

$$\dot{\theta} = \frac{1}{2}(\dot{\theta}_A - \dot{\theta}_B)$$

position dot directly over θ

The rotational rate of the element sides is related to the velocity gradients.

Referring to Fig. 4.19, the angle $\Delta\theta_A$ is given by

$$\Delta\theta_A = \frac{\Delta y_B - \Delta y_A}{\Delta x}$$

$$= \frac{\left(v + \frac{\partial v}{\partial x}\right)\Delta t - v\Delta t}{\Delta x}$$

$$= \frac{\partial v}{\partial x}\Delta t$$

or, in the limit as $\Delta t \longrightarrow 0$,

$$\dot{\theta}_A = \frac{\partial v}{\partial x}$$

Similarly, we can show

$$\dot{\theta}_B = \frac{\partial u}{\partial y}$$

so the rotation rate of the element about the z-axis (normal to the page) is

$$\dot{\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

This component of rotational velocity is defined as Ω_z, so

$$\Omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \tag{4.30a}$$

Likewise, the rotation rates about the other axes are

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \tag{4.30b}$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \tag{4.30c}$$

The rate-of-rotation vector is

$$\mathbf{V} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k} \tag{4.31}$$

An irrotational flow ($\mathbf{V} = 0$) requires that

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \tag{4.32a}$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \tag{4.32b}$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \tag{4.32c}$$

The most extensive application of these equations is in ideal flow theory. ideal flow is the flow of an irrotational, incompressible fluid. Flow fields in which cous effects are small can often be regarded as irrotational. In fact, if a flow of an compressible, inviscid fluid is initially irrotational, it will remain irrotational.

Vorticity

Another property used frequently in fluid mechanics is vorticity. The vorticity is tw the rate-of-rotation vector, so the *vorticity equation* is

$$\omega = 2\mathbf{V}$$

change to cap omega bold

$$= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)\mathbf{k} \quad (4.$$

$$= \nabla \times \vec{V}$$

change to cap V bold

An irrotational flow signifies that the vorticity vector is everywhere zero.

example 4.7

cap V bold

The vector $V = 10x\mathbf{i} - 10y\mathbf{j}$ represents a two-dimensional velocity field. Is the flo irrotational?

Solution In a two-dimensional flow in the xy-plane, the flow is irrotational if

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

The velocity components and derivatives are

$$u = 10x \qquad \frac{\partial u}{\partial y} = 0$$

$$v = -10y \qquad \frac{\partial v}{\partial x} = 0$$

So the irrotationality condition is satisfied and the flow is irrotational.

example 4.8

A fluid exists between stationary and moving parallel flat plates, and the velocity is linear shown. The distance between the plates is 1 cm and the upper plate moves at 2 cm/s. Fi the amount of rotation that fluid elements located at 0.25 cm, 0.5 cm, and 0.75 cm will under after they have traveled a distance of 1 cm.

Rotation in Flows with Concentric Streamlines

It is interesting to realize that a flow field rotating with circular streamlines can b tional; that is, the fluid elements do not rotate. Consider the two-dimensional flc shown in Fig. 4.20. The circumferential velocity on the circular streamline is V an dius is r. The x-axis is perpendicular to the page. As before, the rotation of the ele quantified by the rotation of the bisector, which is

(zoo) z/

Move dots directly over the θ's

$$\dot{\theta} = \frac{1}{2}(\dot{\theta}_A + \dot{\theta}_B)$$

From geometry, the angle $\Delta\theta_B$ is equal to the angle $\Delta\phi$. The rotational rate of an V/r, so

$$\dot{\theta}_B = \frac{V}{r}$$

The rate of change of the angle θ_A is

$$\dot{\theta}_A = \frac{\partial V}{\partial r}$$

Since V is a function of r only, the partial derivative can be replaced by the total tive. Therefore the rotational rate about the z-axis is

$$\Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right)$$

For a flow rotating as a solid body, the velocity distribution is $V = \omega r$, so t of rotation is

$$\Omega_z = \frac{1}{2}\left[\frac{d}{dr}(\omega r) + \omega\right]$$
$$= \omega$$

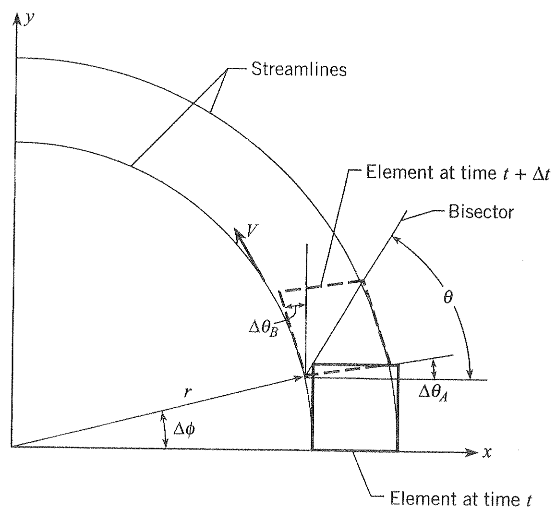


FIGURE 4.20

Deformation of element in flow with concentric, circular streamlines.

mechanics. The prediction of velocities in turbulent flows generated by separation is a continuing challenge for engineers involved with computational fluid mechanics. For additional information on vortices, see Hussaini and Salas (6) and Lugt (7).

Besides separation, there are many other natural processes that generate vortices. For example, the Coriolis effect associated with low-pressure storm centers in the atmosphere develop vortices (cyclonic storms) that extend hundred of miles. Large-scale vortices develop from river discharges into a bay or ocean or when a smoke stack discharges into the atmosphere. See references (8), (9), and (10) for discussion of eddies and basic information on turbulent flow.

4.10 Summary

There are two approaches to describe the velocity of a flowing fluid. In the Lagrangian approach, the position of a specific fluid particle traveling along a pathline is recorded with time. In the Eulerian approach, the properties of fluid particles passing a given point in space are recorded with time. The Eulerian approach is generally used to analyze fluid motion.

The streamline is a curve everywhere tangent to the local velocity vector. The configuration of streamlines in a flow field is called the flow pattern. The pathline is the line traced out by a particle. A streakline is the line produced by a dye introduced at a point in the field. Pathlines, streaklines, and streamlines are coincident in steady flow but differ in unsteady flows.

In a uniform flow, the velocity does not change along a streamline. In a steady flow, the velocity does not change with time at any location.

The tangential acceleration of a fluid element along a pathline is

$$a_t = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

where the first term is the local acceleration and the second term is the convective acceleration. The acceleration normal to the pathline is

$$a_n = \frac{V^2}{r}$$

where r is the local radius of curvature of the pathline.

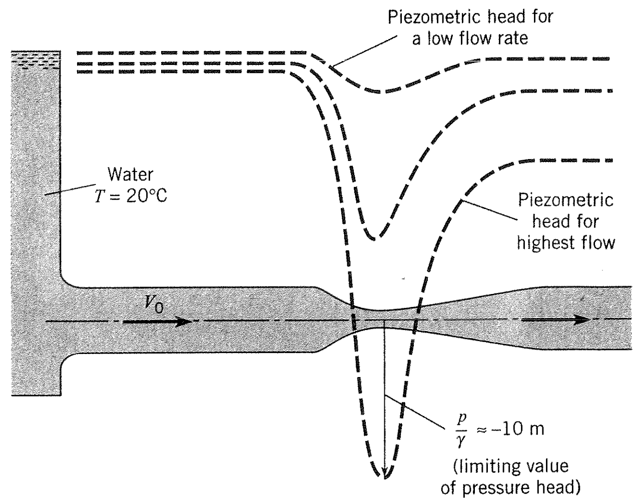
Applying Newton's second law to a fluid element in an incompressible, inviscid flow results in *Euler's equation,*

$$-\frac{\partial}{\partial \ell}(p + \gamma z) = \rho a_\ell$$

where ℓ is an arbitrary direction. Integrating Euler's equation along a streamline in steady flow results in the *Bernoulli equation,*

$$p + \gamma z + \rho\frac{V^2}{2} = C$$

FIGURE 5.10
Flow through pipe restriction: Variation of piezometric head.



Cavitation typically occurs at locations where the velocity is high. Consider the water flow through the pipe restriction shown in Fig. 5.10. The pipe area is reduced, so the velocity is increased according to the continuity equation and, in turn, the pressure is reduced as dictated by the Bernoulli equation. The physical configuration and the plots of piezometric head along the wall of the pipe are shown in Fig. 5.10. For low flow rates, there is a relatively small drop in pressure at the restriction, so the water remains well above the vapor pressure and boiling does not occur. This is indicated in Fig. 5.10 where the piezometric head lies above the centerline, indicating a positive pressure. However, as the flow rate increases, the pressure at the restriction can become sub-atmospheric. The pressure can drop no lower than the vapor pressure of the liquid because, at this point, the liquid will boil and cavitation ensues.

The formation of vapor bubbles at the restriction of a venturimeter is shown in Fig. 5.11*a*. The vapor bubbles form and then collapse as they move into a region of higher pressure and are swept downstream with the flow. When the flow velocity is increased further, the minimum pressure is still the local vapor pressure, but the zone of bubble formation is extended as shown in Fig. 5.11*b*. In this case, the entire vapor pocket may intermittently grow and collapse, producing serious vibration problems. Severe damage that occurred on a centrifugal pump impeller is shown in Fig. 5.12, and serious erosion produced by cavitation in a spillway tunnel of Hoover Dam is shown in Fig. 5.13. Obviously, cavitation should be avoided or minimized by proper design of equipment and structures and by proper operational procedures.

A video of cavitation occurring in the region of a marine propeller is available on the net at www.wiley.com/college/crowe. In this situation, the high liquid velocities produced by the rotating propeller cause a low pressure and cavitation. The bubbles observed in the video indicate cavitation.

index/negative of the/ index/2/

The cavitation ~~number~~, σ, is defined as the pressure coefficient where cavitation occurs. Ideally, the cavitation ~~number~~ is ~~the pressure coefficient~~ based on the vapor pressure,

$$\sigma = \frac{(p_v/g) - h_0}{V_0^2/2g} \longrightarrow \sigma = \frac{P_0 - P_\sigma}{\frac{1}{2}\rho V_0^2}$$

$$\frac{[(\rho u)_{x+(1/2)\Delta x}] - [(\rho u)_{x-(1/2)\Delta x}]}{\Delta x} + \frac{[(\rho v)_{y+(1/2)\Delta y}] - [(\rho v)_{y-(1/2)\Delta y}]}{\Delta y} + \frac{[(\rho w)_{z+(1/2)\Delta z}] - [(\rho w)_{z-(1/2)\Delta z}]}{\Delta z} + \frac{\partial \rho}{\partial t} = 0$$

Taking the limit as the volume approaches zero (that is, as Δx, Δy, and Δz uniformly approach zero) yields the differential form of the continuity equation

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = -\frac{\partial \rho}{\partial t} \tag{5.32}$$

If the flow is steady, we obtain

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{5.33}$$

And if the fluid is incompressible, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{5.34a}$$

for both steady and unsteady flow.

In vector notation, Eq. (5.34*a*) is given as

$$\nabla \cdot \mathbf{V} = 0 \tag{5.34b}$$

where ∇ is the del operator, defined as

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

example 5.13

The expression $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ is said to represent the velocity for a two-dimensional incompressible flow. Check it to see whether it satisfies continuity.

Solution

$$u = 10x \quad \text{so} \quad \frac{\partial u}{\partial x} = 10$$

$$v = -10y \quad \text{so} \quad \frac{\partial v}{\partial y} = -10$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10 - 10 = 0$$

Continuity is satisfied.

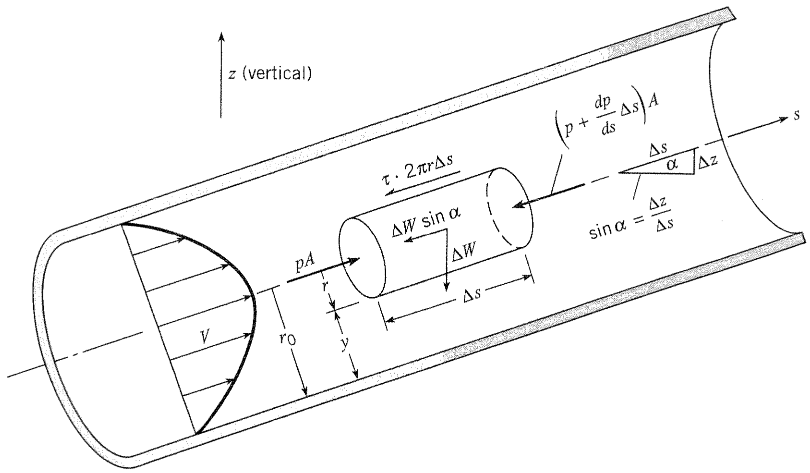


FIGURE 10.1
Flow in a pipe.

uniform flow, equilibrium between the pressure, gravity, and shearing forces acting on the fluid will prevail. Consequently, the momentum equation yields the following:

τ – tau
π – pi
γ – gamma

$$\sum F_s = 0$$

$$pA - \left(p + \frac{dp}{ds}\Delta s\right)A - \Delta W \sin a - \tau(2\pi r)\Delta s = 0 \tag{10.1}$$

In Eq. (10.1) $\Delta W = \gamma A \Delta s$ and $\sin a = dz/ds$. Therefore, Eq. (10.1) reduces to

$$-\frac{dp}{ds}\Delta s A - \gamma A \Delta s \frac{dz}{ds} - \tau(2\pi r)\Delta s = 0 \tag{10.2}$$

Then, when we divide Eq. (10.2) through by $\Delta s A$ and simplify, we obtain

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.3}$$

Since the gradient itself, $d/ds\,(p + \gamma z)$, is negative (see Section 7.4) and constant across the section for uniform flow,* it follows that $-d/ds\,(p + \gamma z)$ will be positive and constant across the pipe section. Thus τ in Eq. (10.3) will be zero at the center of the pipe and will increase linearly to a maximum at the pipe wall. We will use Eq. (10.3) in the following section to derive the velocity distribution for laminar flow.

* The combination $p + \gamma z$ is constant across the section because the streamlines are straight and parallel in uniform flow, and for this condition there will be no acceleration of the fluid normal to the streamline. Thus hydrostatic conditions prevail across the flow section. For a hydrostatic condition, $p/\gamma + z = \text{constant}$ or $p + \gamma z = \text{constant}$ as shown in Chapter 3.

10.2

Laminar Flow in Pipes

We determine how the velocity varies across the pipe by substituting for τ in Eq. (10.3) its equivalent $\mu dV/dy$ and integrating. First, making the substitution, we have

μ – mu

$$\mu \frac{dV}{dy} = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.4}$$

Because $dV/dy = -dV/dr$, Eq. (10.4) becomes

$$\frac{dV}{dr} = -\frac{r}{2\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.5}$$

When we separate variables and integrate across the section, we obtain

$$V = -\frac{r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] + C \tag{10.6}$$

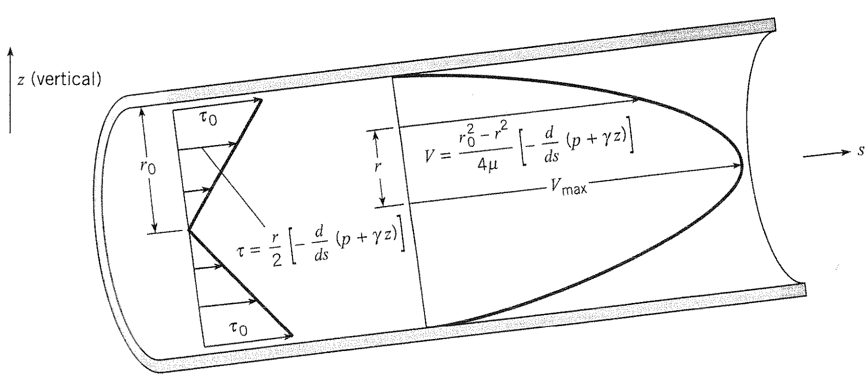
We can evaluate the constant of integration in Eq. (10.6) by noting that when $r = r_0$, the velocity $V = 0$. Therefore, the constant of integration is given by $C = (r_0^2/4\mu)[-d/ds(p + \gamma z)]$, and Eq. (10.6) then becomes

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.7}$$

Equation (10.7) indicates that the velocity distribution for laminar flow in a pipe is parabolic across the section with the maximum velocity at the center of the pipe. Figure 10.2 shows the variation of the shear stress and velocity in the pipe.

FIGURE 10.2

Distribution of shear stress and velocity for laminar flow in a pipe.



Laminar flow in a round pipe is known as *Hagen-Poiseuille flow,* named after a German, Hagen, and a Frenchman, Poiseuille, who studied low-speed flows in tubes in the 1840s.

example 10.1

Oil (S = 0.90; $\mu = 5 \times 10^{-1}\ \text{N}\cdot\text{s/m}^2$) flows steadily in a 3-cm pipe. The pipe is vertical, and the pressure at an elevation of 100 m is 200 kPa. If the pressure at an elevation of 85 m is 250 kPa, is the flow direction up or down? What is the velocity at the center of the pipe and at 6 mm from the center, assuming that the flow is laminar?

Solution First determine the rate of change of $p + \gamma z$. Taking s in the z direction,

$$\frac{d}{ds}(p + \gamma z) = \frac{(p_{100} + \gamma z_{100}) - (p_{85} + \gamma z_{85})}{15}$$

$$= \frac{[200 \times 10^3 + 8830(100)] - [250 \times 10^3 + 8830(85)]}{15}$$

$$= \frac{(1.083 \times 10^6 - 1.00 \times 10^6)\ \text{N/m}^2}{15\ \text{m}} = 5.53\ \text{kN/m}^3$$

The quantity $p + \gamma z$ is not constant with elevation—it increases upward (decreases downward). Therefore, the direction of flow is downward. This can be seen by substituting $d(p + \gamma z)/ds = 5.53\ \text{kN/m}^3$ into Eq. (10.7). When this is done, V is negative for all values of r in the flow. When $r = 0$ (center of the pipe), the velocity will be maximum. Thus

$$V_{\text{center}} = V_{\text{max}} = \frac{r_0^2}{4\mu}(-5.53\ \text{kN/m}^3)$$

$$= \frac{0.015^2\ \text{m}^2}{4(5 \times 10^{-1}\ \text{N}\cdot\text{s/m}^2)}(-5.53 \times 10^3\ \text{N/m}^3) = -0.622\ \text{m/s} \quad \triangleleft$$

At first it may seem strange that the velocity is in a direction opposite to the direction of decreasing pressure. However, it may not seem so peculiar if one realizes that in this example the pipe is vertical, so the gravitational force as well as pressure helps to establish the flow. What counts when flow is other than in the horizontal direction is how the combination $p + \gamma z$ changes with s. If $p + \gamma z$ is constant, then we have the equation of hydrostatics and no flow occurs. However, if $p + \gamma z$ is not constant, flow will occur in the direction of decreasing $p + \gamma z$.

Next determine the velocity at $r = 6\ \text{mm} = 0.006\ \text{m}$. Using Eq. (10.7), we find that

$$V = \frac{0.015^2\ \text{m}^2 - 0.006^2\ \text{m}^2}{4(5 \times 10^{-1}\ \text{N}\cdot\text{s/m}^2)}(-5.53 \times 10^3\ \text{N/m}^3) = -0.522\ \text{m/s} \quad \triangleleft$$

For many problems we wish to relate the pressure change to the rate of flow or mean velocity $\bar{V}$ in the conduit. Therefore, it is necessary to integrate $dQ = VdA$ over the cross-sectional area of flow. That is,

$$Q = \int V\,dA$$
$$= \int_0^{r_0} \frac{(r_0^2 - r^2)}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right](2\pi r\,dr) \tag{10.8}$$

The factor $\pi[d(p + gz)/ds]/4m$ is constant across the pipe section. Therefore, upon integration, we obtain

$$Q = \frac{\pi}{4\mu}\left[\frac{d}{ds}(p + \gamma z)\right]\frac{(r^2 - r_0^2)^2}{2}\bigg|_0^{r_0} \tag{10.9}$$

which reduces to

$$Q = \frac{\pi r_0^4}{8\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.10}$$

If we divide through by the cross-sectional area of the pipe, we have an expression for the mean velocity:

$$\bar{V} = \frac{r_0^2}{8\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.11}$$

Comparing Eqs. (10.11) and (10.7) reveals that $\bar{V} = V_{max}/2$. Also, by substituting $D/2$ for r_0, we have

$$\bar{V} = \frac{D^2}{32\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.12}$$

or

$$\frac{d}{ds}(p + \gamma z) = -\frac{32\mu\bar{V}}{D^2} \tag{10.13}$$

Integrating Eq. (10.13) along the pipe between sections 1 and 2, we obtain

$$p_2 - p_1 + \gamma(z_2 - z_1) = -\frac{32\mu\bar{V}}{D^2}(s_2 - s_1) \tag{10.14}$$

Here $s_2 - s_1$ is the length L of pipe between the two sections. Therefore, Eq. (10.14) can be rewritten as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \frac{32\mu L\bar{V}}{\gamma D^2} \tag{10.15}$$

It can be seen that when the general energy equation for incompressible flow in conduits, Eq. (7.24), is reduced to one for uniform flow in a constant-diameter pipe where $V_1 = V_2$, the result is

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_f \tag{10.16}$$

Here h_f is used instead of h_L to signify head loss due to frictional resistance of the pipe. Comparison of Eqs. (10.15) and (10.16) then shows that the head loss for laminar flow is given by

$$h_f = \frac{32\mu LV}{\gamma D^2} \tag{10.17}$$

Here the bar over the V has been omitted to conform to the standard practice of denoting the mean velocity in one-dimensional flow analyses by V without the bar.

10.3 Criterion for Laminar or Turbulent Flow in a Pipe

To predict whether flow will be laminar or turbulent, it is necessary to explore the characteristics of flow in both laminar and turbulent states. Although other scientists before him had sensed the marked physical difference between laminar and turbulent flow, it was Osborne Reynolds (1) who first developed the basic laws of turbulent flow. With his analytical and experimental work he showed that the Reynolds number was a basic parameter relating to laminar as well as turbulent flow. For example, using an experimental apparatus such as that shown in Fig. 10.3, he found that the onset of turbulence in a smooth pipe was related to the Reynolds number ($VD\rho/\mu$) in a very interesting way. If the fluid in the upstream reservoir was not completely still or if the pipe had some vibration in it, the flow in the pipe as it was gradually increased from a low rate to higher rates was initially laminar but then changed from laminar to turbulent flow at a Reynolds number in the neighborhood of 2100. However, Reynolds found that if the fluid was initially completely motionless and if there was no vibration in the equipment while the flow was increased, it was possible to reach a much higher Reynolds number before the flow became turbulent. He also found that, when going from high-velocity turbulent flow to low-velocity flow, the change from turbulent flow always occurred at a Reynolds number of about 2000.

These experiments of Reynolds indicate that under carefully controlled conditions it is possible to have laminar flow in pipes at Reynolds numbers much higher than 2000. However, the slightest disturbances will trigger the onset of turbulence at high values of Re. Because most engineering applications involve some vibration or flow disturbance, it is reasonable to expect that pipe flow will be laminar for Reynolds numbers less than 2000 and turbulent for Reynolds numbers greater than 3000. When Re is between 2000 and 3000, the type of flow is very unpredictable and often changes back and forth between laminar and turbulent states. Fortunately, however, most engineering applications either are not in this range or are not significantly affected by the unstable flow.

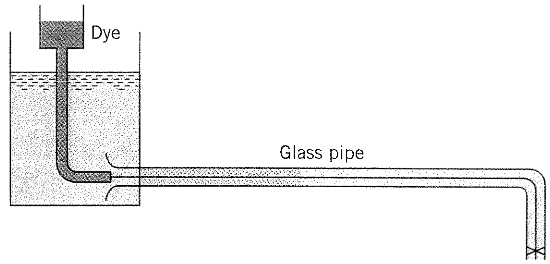


FIGURE 10.3

Schematic diagram of apparatus used by Reynolds to study laminar and turbulent flow.

example 10.2

Oil (S = 0.85) with a kinematic viscosity of $6 \times 10^{-4}\ \text{m}^2/\text{s}$ flows in a 15-cm pipe at a rate of 0.020 m^3/s. What is the head loss per 100-m length of pipe?

Solution First we determine whether the flow is laminar or turbulent by checking to see if the Reynolds number is below 2000 or above 3000.

$$V = \frac{Q}{A} = \frac{0.020\ \text{m}^3/\text{s}}{(\pi/4)D^2\ \text{m}^2} = \frac{0.020\ \text{m}^3/\text{s}}{0.785(0.15^2\ \text{m}^2)} = 1.13\ \text{m/s}$$

Then

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13\ \text{m/s})(0.15\ \text{m})}{6 \times 10^{-4}\ \text{m}^2/\text{s}} = 283$$

Since the Reynolds number is less than 2000, the flow is laminar. The head loss per 100 m is obtained from Eq. (10.17):

$$h_f = \frac{32\mu LV}{\gamma D^2}$$

Here $\mu/\gamma = \nu/g$; hence

$$h_f = \frac{32\nu LV}{gD^2}$$

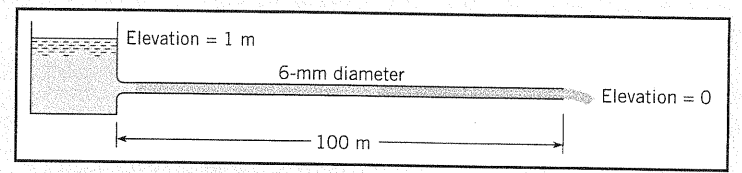
Then

$$h_f = \frac{32(6)(10^{-4}\ \text{m}^2/\text{s})(100\ \text{m})(1.13\ \text{m/s})}{(9.81\ \text{m/s}^2)(0.15^2\ \text{m}^2)} = 9.83\ \text{m}$$

The head loss is 9.83 m/100 m of length. ◁

example 10.3

Kerosene (0°C) flows under the action of gravity in the pipe shown, which is 6 mm in diameter and 100 m long. Determine the rate of flow in the pipe.



Solution Because the pipe diameter is small and because the head producing flow is also quite small, it is expected that the velocity in the pipe will be small. Hence it will be initially assumed that the flow is laminar and $V^2/2g$ is negligible. Then, to solve for the velocity, we apply the energy equation to the problem. We write this equation between a section at the upstream liquid surface and the outlet of the pipe. Thus we have

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + \frac{32\mu LV}{\gamma D^2}$$

With the assumption we have noted, this equation reduces to

$$0 + 0 + 1 = 0 + 0 + 0 + \frac{32mLV}{gD^2}$$

or

$$\frac{32\mu LV}{\gamma D^2} = 1$$

For 0°C the viscosity (from Figs. A.2 and A.3 in the Appendix) is

$$\mu = 3.2 \times 10^{-3}\ \mathrm{N \cdot s/m^2} \qquad \nu = 3.9 \times 10^{-6}\ \mathrm{m^2/s}$$

Then

$$V = \frac{1 \times \gamma D^2}{32 \mu L}$$

$$= \frac{1(8010\ \mathrm{N/m^3})(0.006^2\ \mathrm{m^2})}{32(3.2 \times 10^{-3}\ \mathrm{N \cdot s/m^2})(100\ \mathrm{m})} = 0.0282\ \mathrm{m/s} = 28.2\ \mathrm{mm/s}$$

Now check Re to see if the flow is laminar, and check $V^2/2g$ to see if it is indeed negligible:

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(0.0282\ \mathrm{m/s})(0.006\ \mathrm{m})}{3.9 \times 10^{-6}\ \mathrm{m^2/s}} = 43.4$$

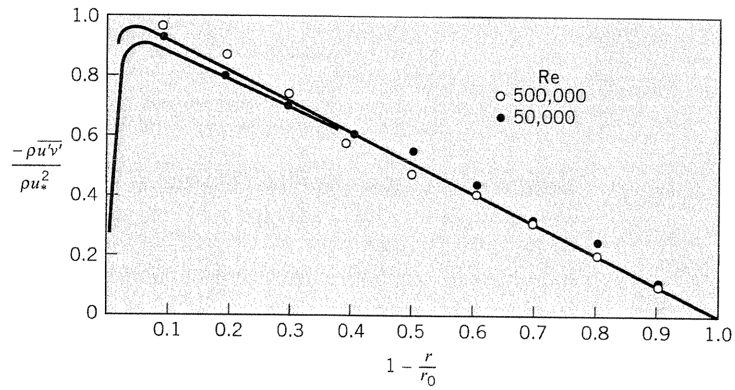
$$\frac{V^2}{2g} = \frac{(0.0282\ \mathrm{m/s})^2}{(2)(9.81\ \mathrm{m/s})} = 4.05 \times 10^{-5}\ \mathrm{m} \quad \text{(negligible)}$$

Therefore, the flow is laminar and the velocity is valid. The discharge is then calculated as follows:

$$Q = VA = (0.282\ \mathrm{m/s})\left(\frac{\pi}{4}\right)(0.006\ \mathrm{m})^2 = 7.97 \times 10^{-7}\ \mathrm{m^3/s} \qquad \triangleleft$$

FIGURE 10.4

Apparent shear stress in a pipe. [After Laufer (2)]



10.4

Turbulent Flow in Pipes

Turbulence and Its Influence in Pipe Flow

In the preceding section it was pointed out that pipe flow is turbulent when the Reynolds number is larger than approximately 3000. However, to say that the flow is turbulent is only a gross description of it. We can obtain a better "feel" for the flow by exploring the similarities between turbulent flow in a pipe and flow in a turbulent boundary layer and by relating the shear stress in the pipe to the level of turbulence. Once we understand these basic physical relationships, we will be better equipped to proceed to the development of equations for the velocity distribution and the resistance to turbulent flow in pipes.

The similarities between turbulent boundary-layer flow and turbulent flow in pipes are many. In fact, it is valid to think of turbulent flow in a pipe as a turbulent boundary layer that has become as thick as the radius of the pipe. With this perspective we realize that flow in a smooth pipe has a viscous sublayer just as a flat-plate boundary layer does. In addition, the velocity gradient in the viscous sublayer will be consistent with the shear stress, as given by $t = \mu du/dy$. However, outside the viscous sublayer the viscous shear stress is negligible compared with the resistance resulting from turbulence. We have already referred in Chapter 9 to the *apparent shear stress,* $t_{app} = -\rho \overline{u'v'}$, which involves an exchange of momentum, but its effect is like that of a true shear stress. It is zero at the pipe center and increases to a maximum near the wall, as shown in Fig. 10.4. Here it is seen that the apparent shear stress increases linearly almost to the edge of the pipe. This linear change in τ_{app} is in accordance with Eq. (10.3), which was developed in Section 10.1. Near the wall, in the viscous sublayer, τ_{app} reduces to zero because all of the shear stress there is in the form of viscous shear stress.

We have shown that there are indeed many analogies between turbulent boundary-layer flow and turbulent flow in pipes. The primary difference is that pipe flow is uniform and boundary-layer flow is not. Of course, this difference does not apply near the inlet of the pipe, where the flow is nonuniform.

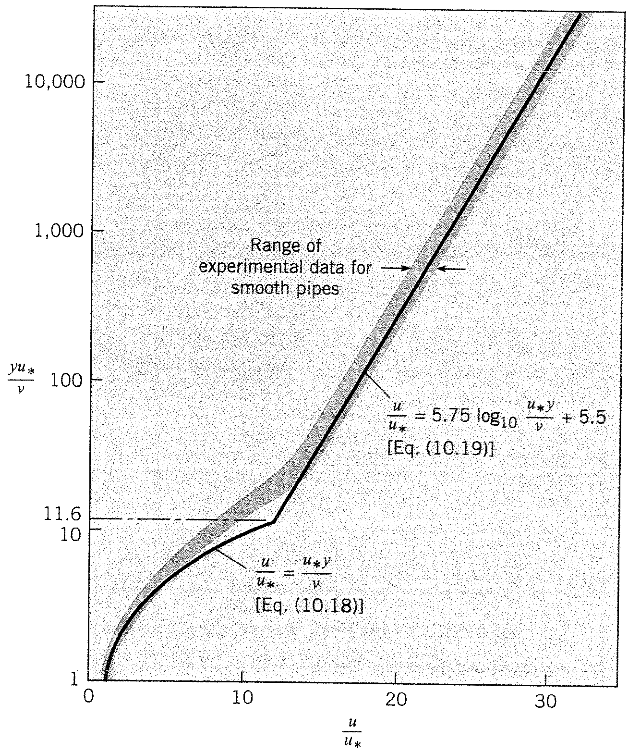


FIGURE 10.5

Velocity distribution for smooth pipes. [After Schlichting (3)]

Velocity Distribution and Resistance in Smooth Pipes

Experiments have shown that, in the viscous sublayer and in the turbulent zone near the wall, the velocity distribution equations are of the same form as those for the turbulent boundary layer. That is, for a smooth pipe,

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \quad \text{for } 0 < \frac{yu_*}{\nu} < 5 \tag{10.18}$$

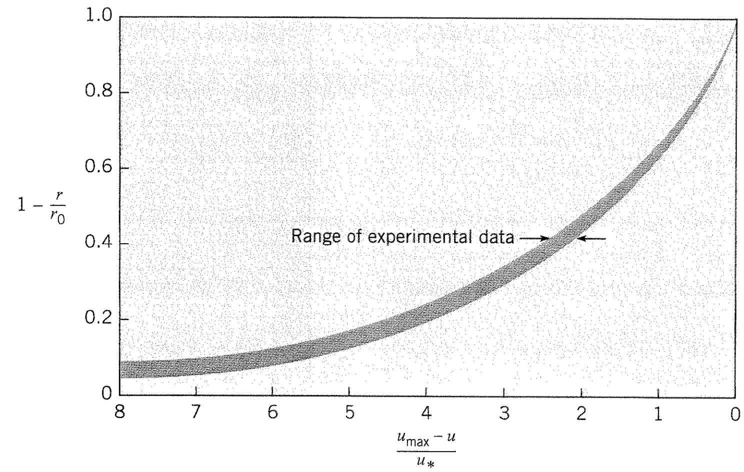
$$\frac{u}{u_*} = 5.75 \log \frac{u_* y}{\nu} + 5.5 \quad \text{for } 20 < \frac{yu_*}{\nu} \leq 10^5 \tag{10.19}$$

Figure 10.5 is a plot of Eqs. (10.18) and (10.19) as well as an indication of the range of experimental data from various sources. For flow near the center of the pipe, as for flow near the outer limit of the boundary layer, the velocity defect law is applicable, as shown in Fig. 10.6. Figure 10.6 also includes the range of experimental velocity data obtained from flow in rough conduits. Again, a power-law formula like that for the turbulent boundary layer is applicable everywhere except close to the wall. This formula is

$$\frac{u}{u_{max}} = \left(\frac{y}{r_0}\right)^m \tag{10.20}$$

FIGURE 10.6

Velocity defect law for turbulent flow in smooth and rough pipes. [After Schlichting (3)]



Here y is the distance from the wall and m is an empirically determined quantity. Some references indicate that m has a value of $1/7$ for turbulent flow. However, Schlichting (3) shows that m varies from $1/6$ to $1/10$ depending on the Reynolds number. His values for m are given in Table 10.1.

In Chapter 9 the local shear stress on a flat plate was expressed as

$$\tau_0 = c_f \rho \frac{V_0^2}{2}$$

where c_f is a function of the character of flow (laminar or turbulent) and the Reynolds number. For pipe flow it is customary to express τ_0 in a similar manner; however, we use the mean velocity as the reference velocity, and the coefficient of proportionality is given as $f/4$ instead of c_f. Here f is called the *resistance coefficient* or *friction factor* of the pipe. Thus we have

TABLE 10.1 EXPONENTS FOR POWER-LAW EQUATION AND RATIO OF MEAN TO MAXIMUM VELOCITY

Re→	4×10^3	2.3×10^4	1.1×10^5	1.1×10^6	3.2×10^6
m→	$\frac{1}{6.0}$	$\frac{1}{6.6}$	$\frac{1}{7.0}$	$\frac{1}{8.8}$	$\frac{1}{10.0}$
$\bar{V}/V_{max}$→	0.791	0.807	0.817	0.850	0.865

SOURCE: Schlichting (3). Used with permission of the McGraw-Hill Companies.

$$t_0 = \frac{f}{4}\rho\frac{V^2}{2} \tag{10.21}$$

Or, because $\sqrt{t_0/\rho} = u_*$, we have

$$\frac{u_*}{V} = \sqrt{\frac{f}{8}}$$

Noting that $\tau = \tau_0$ when $r = r_0$ in Eq. (10.3), we can eliminate τ_0 between Eqs. (10.3) and (10.21). Then, by integrating between two sections along the pipe, we obtain

$$h_1 - h_2 = f\frac{L}{D}\frac{V^2}{2g}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{10.22}$$

where h_f is the head loss created by viscous effects and is equal to the change in piezometric head along the pipe. Equation (10.22) is called the *Darcy-Weisbach equation.* It is named after Henry Darcy, a French engineer of the nineteenth century, and Julius Weisbach, a German engineer and scientist of the same era. Weisbach first proposed the use of the nondimensional resistance coefficient, and Darcy carried out numerous tests on water pipes. Brief accounts of their work are given by Rouse and Ince (4). It can be easily shown by a simultaneous solution of Eqs. (10.17) and (10.22) that the resistance coefficient for *laminar flow* is given by

$$f = \frac{64}{\text{Re}} \tag{10.23}$$

For turbulent flow, analytical and empirical results on smooth pipes yield the following approximate relation for f:

$$\frac{1}{\sqrt{f}} = 2\log(\text{Re}\sqrt{f}) - 0.8 \quad \text{for Re} > 3000 \tag{10.24}$$

Equation (10.24) was first developed by Prandtl.

Velocity Distribution and Resistance—Rough Pipes

Numerous tests on flow in rough pipes all show that a semilogarithmic velocity distribution is valid over most of the pipe section (5, 4). This relationship is given in the following form:

$$\frac{u}{u_*} = 5.75\log\frac{y}{k} + B \tag{10.25}$$

Here y is the distance from the rough wall, k is a measure of the height of the roughness elements, and B is a function of the character of roughness. That is, B is a function of the type, concentration, and size variation of the roughness. Research by Roberson and Chen proportional to V^2; thus f becomes constant for these conditions. The effect of roughness can be summarized by (13)

$$\left(\frac{k_s}{D}\right) \text{Re} < 10 \qquad \text{roughness unimportant, pipe considered smooth}$$

$$\left(\frac{k_s}{D}\right) \text{Re} > 1000 \qquad \text{fully rough, } f \text{ independent of Reynolds number}$$

The region between these limits is the transitional roughness regime.

The uniform character of the sand grains used in Nikuradse's tests produced a dip in the f-versus-Re curve (Fig. 10.7) before the curve reached a constant value of f. However, tests on commercial pipes where the roughness is somewhat random reveal that no such dip occurs. Using data from commercial pipes, Colebrook (14) in 1939 developed an empirical equation, called the Colebrook-White formula, for the friction factor. Moody (4) used the Colebrook-White formula to generate a design chart similar to that shown in Fig. 10.8. This chart is now known as the *Moody diagram* for commercial pipes.

TABLE 10.2 EQUIVALENT SAND GRAIN ROUGHNESS, k_s, FOR VARIOUS PIPE MATERIALS

Boundary Material	**k_s, millimeters**	**k_s, inches**
Glass, plastic	Smooth	Smooth
Copper or brass tubing	0.0015	6×10^{-5}
Wrought iron, steel	0.046	0.002
Asphalted cast iron	0.12	0.005
Galvanized iron	0.15	0.006
Cast iron	0.26	0.010
Concrete	0.3 to 3.0	0.012–0.12
Riveted steel	0.9–9	0.035–0.35
Rubber pipe (straight)	0.025	0.001

In Fig. 10.8 the variable k_s is the symbol used to denote the *equivalent sand roughness.* That is, a pipe that has the same resistance characteristics at high Re values as a sand-roughened pipe of the same size is said to have a size of roughness equivalent to that of the sand-roughened pipe. Table 10.2 gives the equivalent sand roughness for various kinds of pipes. This table can be used to calculate the relative roughness for a given pipe diameter, which, in turn, is used in Fig. 10.8 to find the friction factor.

In Fig. 10.8 the abscissa (labeled at the bottom) is the Reynolds number Re, and the ordinate (labeled at the left) is the resistance coefficient f. Each blue curve is for a constant relative roughness k_s/D, and the values of k_s/D are given on the right at the end of each curve. To find f, given Re and k_s/D, one goes to the right to find the correct relative roughness curve. Then one looks at the bottom of the chart to find the given value of Re and, with this value of Re, moves vertically upward until the given k_s/D curve is reached. Finally, from this point one moves horizontally to the left scale to read the value of f. If the curve for the given value of k_s/D is not plotted in Fig. 10.8, then one simply

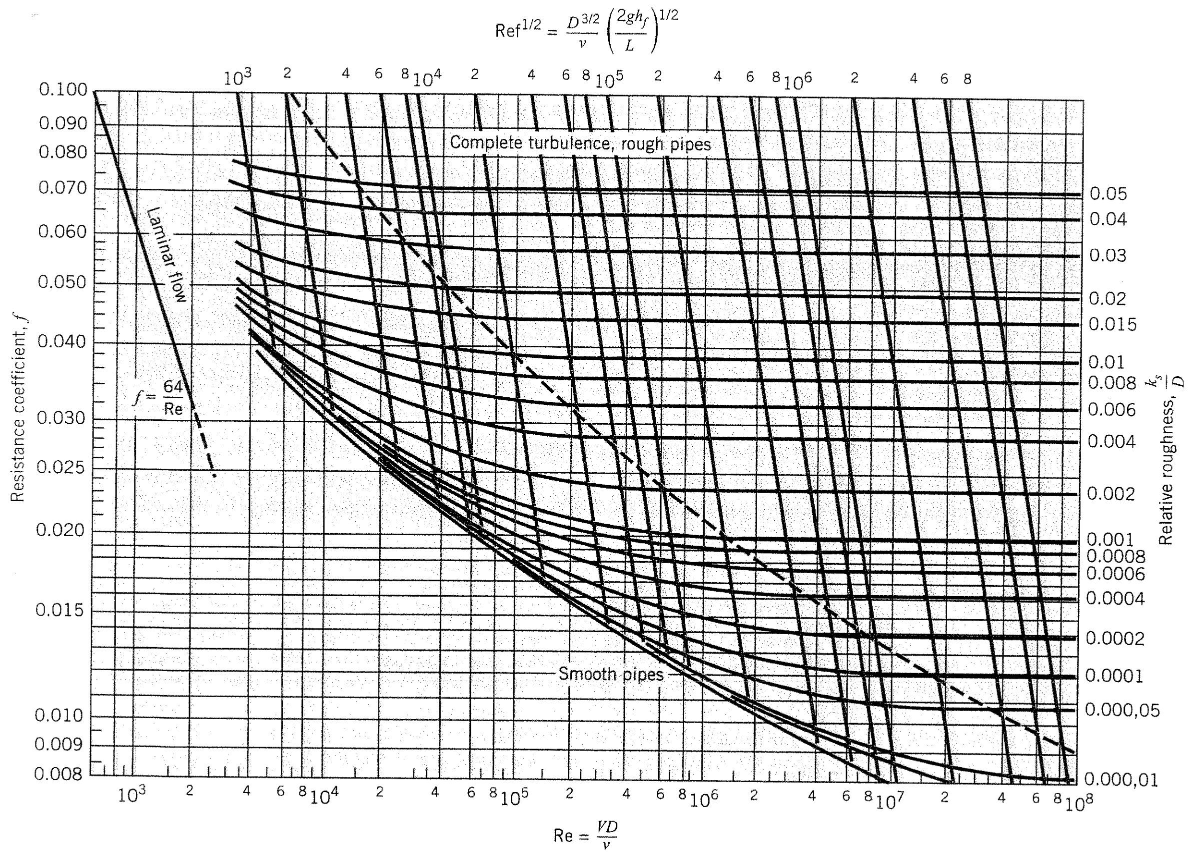


FIGURE 10.8

Resistance coefficient f versus Re. Reprinted with minor variations. [After Moody (5). Reprinted with permission from the A.S.M.E.]

finds the proper position on the graph by interpolation between the k_s/D curves that bracket the given k_s/D.

For some problems it is convenient to enter Fig. 10.8 using a value of the parameter $\text{Re}\,f^{1/2}$. This parameter is useful when h_f and k_s/D are known but the velocity V is not. Without V the Reynolds number cannot be computed, so f cannot be read by entering the chart with Re and k_s/D. But from $h_f = f(L/D)V^2/2g$ [or $V = (2gh_f/L)^{1/2}(D/f)^{1/2}$] and $\text{Re} = VD/\nu$, one can see that Re can be given as

$$\text{Re} = \frac{D^{3/2}}{f^{1/2}\nu}\left(\frac{2gh_f}{L}\right)^{1/2}$$

Upon multiplying both sides of the above equation by $f^{1/2}$, we get

$$\text{Re}\,f^{1/2} = \frac{D^{3/2}}{\nu}(2gh_f/L)^{1/2}$$

Thus a value of $\mathrm{Re} f^{1/2}$ can be calculated for this type of flow problem, which, in turn, enables us to determine f directly, using Fig. 10.8, where curves of constant $\mathrm{Re} f^{1/2}$ are plotted slanting from the upper left to lower right and the values of $\mathrm{Re} f^{1/2}$ for each line are given at the top of the chart.

When using computers to carry out pipe flow calculations, it is much more convenient to have an equation for the friction factor as a function of Reynolds number and relative roughness. By using the Colebrook-White formula, Swamee and Jain (15) developed an explicit equation for friction factor, namely

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{10.26}$$

It is reported that this equation predicts friction factors that differ by less than 3% from those on the Moody diagram for $4 \times 10^3 < \mathrm{Re} < 10^8$ and $10^{-5} < k_s/D < 2 \times 10^{-2}$.

There are basically three types of problems involved with uniform flow in a single pipe. These are

1. Determine the head loss, given the kind and size of pipe and the flow rate.
2. Determine the flow rate, given the head loss, kind, and size of pipe.
3. Determine the size of pipe needed to carry the flow, given the kind of pipe, head, and flow rate.

In the first type of problem, the Reynolds number and k_s/D are first computed and then f is read from Fig. 10.8, after which the head loss is obtained by the use of Eq. (10.22).

example 10.4

Water ($T = 20°\mathrm{C}$) flows at a rate of 0.05 m^3/s in a 20-cm asphalted cast-iron pipe. What is the head loss per kilometer of pipe?

Solution First compute the Reynolds number, VD/ν. Here $V = Q/A$. Thus

$$V = \frac{0.05\ \mathrm{m}^3/\mathrm{s}}{(p/4)(0.20^2\ \mathrm{m}^2)} = 1.59\ \mathrm{m/s}$$

$$\nu = 1.0 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s} \quad \text{(from Table A.5)}$$

Then

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.59\ \mathrm{m/s})(0.20\ \mathrm{m})}{10^{-6}\ \mathrm{m}^2/\mathrm{s}} = 3.18 \times 10^5$$

From Table 10.2, the roughness for asphalted cast-iron pipe is 0.12 mm, so the relative roughness (k_s/D) is 0.0006. Then from Fig. 10.8, using the values obtained for k_s/D and Re, we find $f = 0.019$. Finally, the head loss is computed from the Darcy-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.019\left(\frac{1000\text{ m}}{0.20\text{ m}}\right)\left(\frac{1.59^2\text{ m}^2/\text{s}^2}{2(9.81\text{ m/s}^2)}\right) = 12.2\text{ m} \quad \triangleleft$$

The head loss per kilometer is 12.2 m.

In the second type of problem, k_s/D and the value of $(D^{3/2}/\nu)\sqrt{2gh_f/L}$ are computed so that the top scale can be used to enter the chart of Fig. 10.8. Then, once f is read from the chart, the velocity from Eq. (10.22) is solved for and the discharge is computed from $Q = VA$.

example 10.5

The head loss per kilometer of 20-cm asphalted cast-iron pipe is 12.2 m. What is the discharge of water?

Solution First compute the parameter $D^{3/2}\sqrt{2gh_f/L}/\nu$. Assume $T = 20°\text{C}$, so that

$$D^{3/2}\frac{\sqrt{2gh_f/L}}{\nu} = (0.20\text{ m})^{3/2}\frac{[2(9.81\text{ m/s}^2)(12.2\text{ m}/1000\text{ m})]^{1/2}}{1.0\times10^{-6}\text{ m}^2/\text{s}}$$

$$= 4.38\times10^4$$

From Table 10.2, the roughness for asphalted cast-iron pipe is 0.12 mm, so the relative roughness (k_s/D) is 0.0006. Using Fig. 10.8, we read $f = 0.019$. We use this f in the Darcy-Weisbach equation to solve for V:

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$12.2\text{ m} = \frac{0.019(1000\text{ m})}{0.20\text{ m}}\frac{V^2}{2(9.81\text{ m/s}^2)}$$

$$V^2 = 2.52\text{ m}^2/\text{s}^2$$

$$V = 1.59\text{ m/s}$$

Finally we compute the discharge:

$$Q = VA = V\frac{\pi}{4}D^2$$

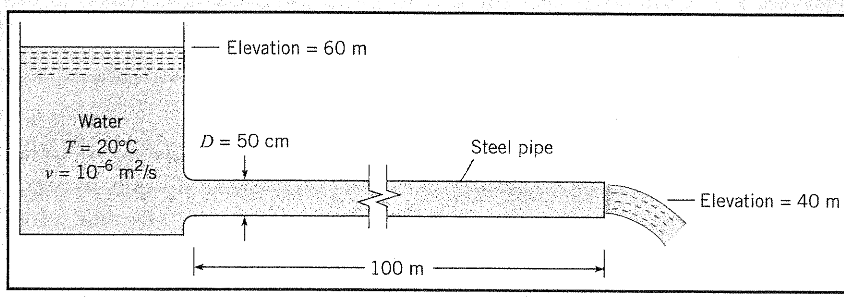
$$= (1.59\text{ m/s})(0.785)(0.20^2\text{ m}^2) = 0.050\text{ m}^3/\text{s} \quad \triangleleft$$

Examples 10.4 and 10.5 are good checks on the validity of the methods of solution because their basic data are exactly the same—in one case, the head loss is unknown; in the other, the discharge is unknown.

In Example 10.5 the head loss in the pipe was given. Therefore, it was possible to obtain a direct solution by entering Fig. 10.8 with a value of Re$f^{1/2}$. However, many problems for which the discharge Q is desired cannot be solved directly. For example, a problem in which water flows from a reservoir through a pipe and into the atmosphere cannot be solved directly. Here part of the available head is lost to friction in the pipe, and part of the head remains as kinetic energy in the jet as it leaves the pipe. Therefore, at the outset one does not know how much head loss occurs in the pipe itself. To effect a solution, one must iterate on f. The energy equation is written and an initial value for f is guessed. Because f tends to a constant value at high values of Re, an "educated" first guess is to use this limiting value of f. Next one solves for the velocity V. With this value of V, one then computes a Reynolds number that makes it possible to determine a better value of f using Fig. 10.8, and so on. This type of solution usually converges quite rapidly because f changes more slowly than Re. Once f and V have been determined, one calculates the discharge by using the continuity equation.

example 10.6

Determine the discharge of water through the 50-cm steel pipe shown in the figure.



Solution From Table 10.2, the roughness for steel pipe is 0.046 mm, so the relative roughness (k_s/D) is 9.2×10^{-5}. Now write the energy equation from the reservoir water surface to the free jet at the end of the pipe:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 60 = 0 + \frac{V_2^2}{2g} + 40 + f\frac{L}{D}\frac{V_2^2}{2g}$$

or

$$V = \left(\frac{2g \times 20}{1 + 200f}\right)^{1/2}$$

First trial: Assume $f = 0.020$; then $V = 8.86$ m/s and Re $= 4.43 \times 10^6$. With Re $= 4.43 \times 10^6$ and $k_s/D = 9.2 \times 10^{-5}$, then $f = 0.012$ (from Fig. 10.8). This f then yields $V = 10.7$ m/s.

Second trial: For $V = 10.7$ m/s, Re $= 5.35 \times 10^6$ and $f = 0.012$,

$$Q = VA = 10.7 \text{ m/s} \times (\pi/4) \times (0.50)^2 \text{ m}^2 = 2.10 \text{ m}^3/\text{s} \quad \triangleleft$$

This example can also be solved using a programmable calculator and the explicit equation for the friction factor, Eq. (10.26). The procedure is to write a program that, given a velocity, calculates the Reynolds number, then calculates the friction factor from Eq. (10.26) and, finally, calculates the velocity from the above equation. This velocity is then entered again and a new velocity is calculated until the velocity no longer changes. The following table gives the velocities calculated starting with an initial guess of 20 m/s and finishing when the velocity difference between iterations is less than 0.01.

1st iteration	20.0
2nd iteration	10.73
3rd iteration	10.67

The convergence is fast because the friction factor is a very weak function of Reynolds number.

In the third type of problem, it is usually best to first assume a value of f and then to solve for D, after which a better value of f is computed based on the first estimate of D. This iterative procedure is continued until a valid solution is obtained. A trial-and-error procedure is necessary because without D one cannot compute k_s/D or Re to enter Fig. 10.8.

example 10.7

What size of asphalted cast-iron pipe is required to carry water at a discharge of 3 cfs and with a head loss of 4 ft per 1000 ft of pipe?

Solution First assume $f = 0.015$. Then

$$h_f = \frac{fLV^2}{D2g} = \frac{fLQ^2/A^2}{D\ \ 2g} = \frac{fLQ^2}{2g(\pi/4)^2D^5}$$

or

$$D^5 = \frac{fLQ^2}{0.785^2(2gh_f)}$$

For this example,

$$D^5 = \frac{0.015(1000\text{ ft})(3\text{ ft}^3/\text{s})^2}{0.615(64.4\text{ ft/s}^2)(4\text{ ft})} = 0.852\text{ ft}^5$$

$$D = 0.97\text{ ft}$$

Now compute a more accurate value of f:

$$\frac{k_s}{D} = 0.0004 \qquad V = \frac{Q}{A} = \frac{3\text{ ft}^3/\text{s}}{0.785(0.94\text{ ft}^2)} = 4.07\text{ ft/s}$$

Then

$$\text{Re} = \frac{VD}{\nu} = \frac{(4.07\text{ ft/s})(0.97\text{ ft})}{1.21(10^{-5}\text{ ft}^2/\text{s})} = 3.26 \times 10^5$$

From Fig. 10.8, $f = 0.0175$. Now recompute D by applying the ratio of f's to previous calculations for D^5:

$$D^5 = \frac{0.0175}{0.015}(0.852\text{ ft}^5) = 0.994\text{ ft}^5$$

$$D = 0.999\text{ ft}$$

Use a pipe with a 12-in. diameter. ◁

Note: If a size that is not available commercially is calculated during design, it is customary practice to choose the next larger available size. The cost will be less than that for odd-sized pipe, and the pipe will be more than large enough to carry the flow.

Explicit Equations for Q and D

In the foregoing discussion, methods were presented by which Q and D can be calculated. All of these methods involve the use of the Moody diagram (Fig. 10.8).

To provide an alternative to the Moody diagram, Swamee and Jain (15) developed an explicit equation for discharge:

$$Q = -2.22 D^{5/2}\sqrt{gh_f/L}\,\log\left(\frac{k_s}{3.7D} + \frac{1.78\nu}{D^{3/2}\sqrt{gh_f/L}}\right) \tag{10.27}$$

They also developed a formula for the explicit determination of D. A modified version of that formula, given by Streeter and Wylie (16), is

$$D = 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \tag{10.28}$$

If you want to solve for head loss given Q, L, D, k_s, and ν, simply solve for f by Eq. (10.26) and compute h_f with the Darcy-Weisbach equation, Eq. (10.22). Straightforward calculations for Q and D can also be made if h_f is known. However, for problems involving head losses in addition to h_f, an iterative solution is required. For computing Q, you can assume an f and solve for Q from the energy equation after substituting Q/A in that equation. Then compute Re and use the result in Eq. (10.26) to get a better estimate of f, and so on, until Q converges analogous to the procedure for determining Q using the Moody diagram. In this case, however, Eq. (10.26) is substituted for the Moody diagram. Similarly, you can determine D if you are given Q, ν, the change in pressure or head, and the geometric configuration.

example 10.8

Solve Example 10.5 using Eq. (10.27).

Solution From Table 10.2, k_s for asphalted cast-iron pipe is given as 1.2×10^{-4} m. From the given conditions, $h_f/L = 0.0122$. Assume $T = 20°\text{C}$, so $\nu = (10^{-6}\ \text{m}^2/\text{s})$. Then, using Eq. (10.27), we have

$$Q = -2.22(0.20\ \text{m})^{5/2}\sqrt{9.81\ \text{m/s}^2 \times 0.0122}$$
$$\times \log\left(\frac{1.2\times 10^{-4}\ \text{m}}{3.7\times 0.20\ \text{m}} + \frac{1.78\times 10^{-6}\ \text{m}^2/\text{s}}{(0.20\ \text{m})^{3/2}\sqrt{9.81\ \text{m/s}^2\times 0.0122}}\right)$$
$$= 0.050\ \text{m}^3/\text{s}$$

◁

10.5

Flow at Pipe Inlets and Losses from Fittings

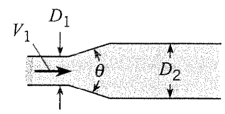
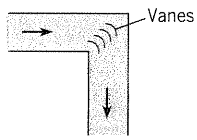
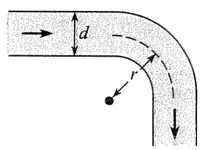
In the preceding section, formulas were presented that are used to determine the head loss for uniform flow in a pipe. However, pipe systems also include inlets, outlets, bends, and other appurtenances that create additional head losses. The resulting flow separation and the generation of additional turbulence force usually cause these head losses. In this section we will consider the flow patterns and resulting head losses for some of these flow transitions.

Flow in a Pipe Inlet

If the inlet to a pipe is well rounded, as shown in Fig. 10.9, the boundary layer will develop from the inlet and grow in thickness until it extends to the center of the pipe. After that point, the flow in the pipe will be uniform. The length L_e of the developing region at the entrance is equal to approximately $0.05D\text{Re}$ for laminar flow and approximately $50D$ for turbulent flow. Velocity and pressure distribution for the inlet region of a pipe with turbulent flow are shown in Fig. 10.10. The head loss that is produced by inlets, outlets, or fittings is expressed by the equation

$$h_L = K\frac{V^2}{2g} \qquad (10.29)$$

TABLE 10.3 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS (CONTINUED)

Description	Sketch	Additional Data		K	Source
Expansion		D_1/D_2	K_E $u = 20°$	K_E $u = 180°$	(17)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend		Without vanes		$K_b = 1.1$	(23)
		With vanes		$K_b = 0.2$	(23)
90° smooth bend		r/d			(24) and (17)
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings	Globe valve—wide open	$K_v = 10.0$			(23)
	Angle valve—wide open	$K_v = 5.0$			
	Gate valve—wide open	$K_v = 0.2$			
	Gate valve—half open	$K_v = 5.6$			
	Return bend	$K_b = 2.2$			
	Tee				
	straight-through flow	$K_t = 0.4$			
	side-outlet flow	$K_t = 1.8$			
	90° elbow	$K_b = 0.9$			
	45° elbow	$K_b = 0.4$			

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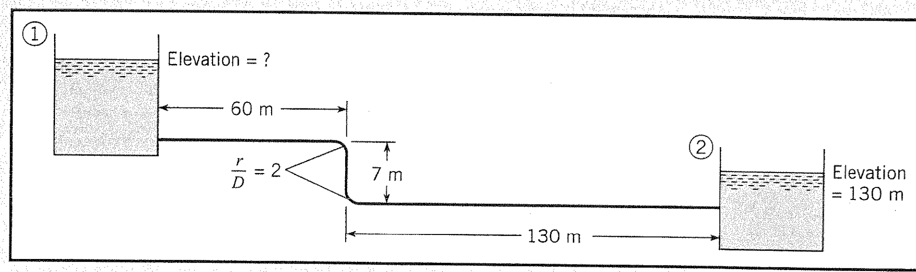
example 10.9

If oil ($\nu = 4 \times 10^{-5}$ m^2/s, S = 0.9) flows from the upper to the lower reservoir at a rate of 0.028 m^3/s in the 15-cm smooth pipe, what is the elevation of the oil surface in the upper reservoir?



Solution Apply the energy equation between the surfaces of the upper and lower reservoirs:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\sum h_L$$

$$0+0+z_1=0+0+130\text{ m}+\frac{fL}{D}\frac{V^2}{2g}+2K_b\frac{V^2}{2g}+K_e\frac{V^2}{2g}+K_E\frac{V^2}{2g}$$

Here K_b, K_e, and K_E are loss coefficients for bend, entrance, and outlet, respectively. These have values of 0.19, 0.5, and 1.0 (Table 10.3). To determine f, we get Re in order to enter Fig. 10.8:

$$\text{Re}=\frac{VD}{\nu}$$

But

$$V=\frac{Q}{A}=\frac{(0.028\text{ m}^3/\text{s})}{0.785(0.15\text{ m})^2}=1.58\text{ m/s}$$

Then

$$\text{Re}=\frac{1.58\text{ m/s}(0.15\text{ m})}{4\times10^{-5}\text{ m}^2/\text{s}}=5.93\times10^3$$

Now we read f from Fig. 10.8 (smooth pipe curve): $f=0.035$. Then

$$z_1=130\text{ m}+\frac{V^2}{2g}\left[\frac{0.035(197\text{ m})}{0.15\text{ m}}+2(0.19)+0.5+1\right]$$

$$=130\text{ m}+\left[\frac{(1.58\text{ m/s})^2}{2(9.81\text{ m/s}^2)}\right](46+0.38+0.5+1)$$

$$=130\text{ m}+6.1\text{ m}=136.1\text{ m}$$

◁

example 10.10

Refer to Fig. 10.14. The difference in water-surface elevation between the reservoirs is 5.0 m, and the horizontal distance between them is 300 m. Using the explicit formula for f, Eq. (10.26), determine the size of steel pipe needed for a discharge of 2 m^3/s when the gate valve is wide open.

Solution First write the energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$5 \text{ m} = \sum h_L$$

$$= \frac{V^2}{2g}\left(\frac{fL}{D} + K_e + K_v + K_E\right)$$

$$= \frac{Q^2}{2gA^2}(f \times 300/D + 0.5 + 0.2 + 1.0)$$

$$= \frac{Q^2}{2g(\pi/4)^2 D^4}\left[300\left(\frac{f}{D}\right) + 1.7\right]$$

$$= \frac{(2 \text{ m}^3/\text{s})^2}{2 \times 9.81 \text{ m/s}^2 \times (\pi/4)^2 \times D^4}\left[300\left(\frac{f}{D}\right) + 1.7\right]$$

$$= \frac{0.33 \text{ m}^5}{D^4}\left(300\frac{f}{D} + 1.7\right)$$

The other equations for solving this problem are Eq. (10.26), $V = Q/A$, and $\text{Re} = VD/\nu$. As we did when using the Moody diagram, we make an initial assumption for D. Next we compute V and Re, after which we compute f from Eq. (10.26). Then we compute D from the energy equation (above). With this calculated value of D we go through the process again to get a better estimate of D, and so on, until the change in D is negligibly small. In this example, $\nu = 10^{-6}$ m^2/s and $k_s = 4.6 \times 10^{-5}$ m (from Table 10.2). We assume an initial value for D of 1 m. Then

$$V = Q/A = \frac{2 \text{ m}^3/\text{s}}{(p/4) \times (1 \text{ m})^2} = 2.55 \text{ m/s}$$

$$\text{Re} = \frac{VD}{\nu} = 2.55 \times \frac{1}{10^{-6}} = 2.55 \times 10^6$$

$$f = 0.25\left[\log\left(\frac{k_s}{3.70} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^{-2}$$

$$= 0.25\left[\log\left(\frac{4.6 \times 10^{-5} \text{ m}}{3.7 \times 1 \text{ m}} + \frac{5.74}{(2.55 \times 10^6)^{0.9}}\right)\right]^{-2} = 0.0116$$

With this value of f, we solve the energy equation for D to obtain

$$D = 0.79 \text{ m}$$

With $D = 0.79$ m we repeat the computational procedure again and again, until convergence. The next iteration yields $D = 0.79$ m. Since there is no significant change, we have a solution: $D = 0.79$ m.

10.6

Pipe Systems

Simple Pump in a Pipeline

We have considered a number of pipe flow problems in which the head for producing the flow was explicitly given. Now we shall consider flow in which the head is developed by a pump. However, the head produced by a centrifugal pump is a function of the discharge. Hence a direct solution is usually not immediately available. The solution (that is, the flow rate for a given system) is obtained when the system equation (or curve) of head versus discharge is solved simultaneously with the pump equation (or curve) of head versus discharge. The solution of these two equations (or the point where the two curves intersect) yields the operating condition for the system. Consider flow of water in the system of Fig. 10.15. When the energy equation is written from the reservoir water surface to the outlet stream, we obtain the following equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum K_L \frac{V^2}{2g} + \sum \frac{fL}{D}\frac{V^2}{2g}$$

For a system with one size of pipe, this simplifies to

$$h_p = (z_2 - z_1) + \frac{V^2}{2g}\left(1 + \sum K_L + \frac{fL}{D}\right) \qquad (10.30)$$

Hence, for any given discharge, a certain head h_p must be supplied to maintain that flow. Thus we can construct a head-versus-discharge curve, as shown in Fig. 10.16. Such a curve is called the *system curve*. Any given centrifugal pump has a head-versus-discharge curve that is characteristic of that pump at a given pump speed. Such curves are supplied by the pump manufacturer; a typical one for a centrifugal pump is shown in Fig. 10.16.

Figure 10.16 reveals that, as the discharge increases in a pipe, the head required for flow also increases. However, the head that is produced by the pump decreases as the discharge increases. Consequently, the two curves intersect, and the operating point is at the point of intersection—that point where the head produced by the pump is just the amount needed to overcome the head loss in the pipe.

Solution First we write the energy equation from water surface to water surface:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\sum h_L$$

$$0+0+200+h_p=0+0+230+\left(\frac{fL}{D}+K_e+K_b+K_E\right)\frac{V^2}{2g}$$

Here $K_e = 0.5$, $K_b = 0.35$, and $K_E = 1.0$. Hence

$$h_p = 30+\frac{Q^2}{2gA^2}\left[\frac{0.015(1000)}{0.40}+0.5+0.35+1\right]$$

$$= 30+\frac{Q^2}{2\times 9.81\times[(p/4)\times 0.4^2]^2}(39.3) = 30\text{ m}+127Q^2\text{ m}$$

Now we make a table of Q versus h_p (see below) to give values to produce a system curve that will be plotted with the pump curve. When the system curve is plotted on the same graph as the pump curve, it is seen (Fig. 10.16) that the operating condition occurs at $Q = 0.27\text{ m}^3/\text{s}$. ◁

Q, m^3/s	Q^2, m^6/s^2	$127Q^2$	$h_p = 30\text{ m} + 127Q^2\text{ m}$
0	0	0	30
0.1	1×10^{-2}	1.3	31.3
0.2	4×10^{-2}	5.1	35.1
0.3	9×10^{-2}	11.4	41.4

Pipes in Parallel

Consider a pipe that branches into two parallel pipes and then rejoins, as shown in Fig. 10.17. A problem involving this configuration might be to determine the division of flow in each pipe, given the total flow rate.

No matter which pipe is involved, the pressure difference between the two junction points is the same. Also, the elevation difference between the two junction points is the same. Because $h_L = (p_1/\gamma + z_1) - (p_2/\gamma + z_2)$, it follows that h_L between the two junction points is the same in both of the pipes of the parallel pipe system. Thus we can write

$$h_{L_1} = h_{L_2}$$

$$f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$$

Then

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{f_2L_2D_1}{f_1L_1D_2} \quad\text{or}\quad \frac{V_1}{V_2} = \left(\frac{f_2L_2D_1}{f_1L_1D_2}\right)^{1/2}$$

and $Q_{0,A} = 0.0346$ and $Q_{0,C} = 0.0693$. These values are substituted into the matrix equation to solve for the ΔQ's. The discharges are corrected by $Q_0^{\text{new}} = Q_0^{\text{old}} + \Delta Q$ and substituted into the matrix equation again to yield new ΔQ's. The iterations are continued until sufficient accuracy is obtained. The accuracy is judged by how close the column matrix on the right approaches zero. A table with the results of iterations for this example is shown below.

		Iteration			
	Initial	**1**	**2**	**3**	**4**
Q_A	0.0346	0.0328	0.0305	0.0293	0.0287
Q_B	0.0346	0.0393	0.0384	0.0394	0.0384
Q_C	0.0693	0.0721	0.0689	0.0687	0.0671

This solution technique is called the Newton-Raphson method for nonlinear systems of algebraic equations. It can be implemented easily on a computer. The solution procedure for more complex systems is the same.

10.7

Turbulent Flow in Noncircular Conduits

Basic Development

Earlier in this chapter (Section 10.4), τ_0 was eliminated between Eqs. (10.3) and (10.21) to yield the Darcy-Weisbach equation, Eq. (10.22). It should be noted that Eq. (10.3) was derived by writing the equation of equilibrium in the longitudinal direction for an element of fluid with a circular cross section. If one derives an equation analogous to Eq. (10.3) for flow in a noncircular conduit in which the shear stress acts on the conduit surface having a perimeter P (such as the perimeter of a rectangular conduit) instead of perimeter $2\pi r$, then τ_0 is given by

$$t_0 = \frac{A}{P}\left[-\frac{d}{ds}(p + \gamma z)\right] \tag{10.35}$$

In Eq. (10.3), to which Eq. (10.35) is analogous, the shear stress τ_0 was everywhere constant around the perimeter of the cylindrical element. In Eq. (10.35) for the noncircular conduit, the shear stress is not constant over the perimeter. However, we can still use Eq. (10.21) to relate τ_0 and V, where τ_0 is now the average shear stress on the boundary. Eliminating τ_0 between Eqs. (10.21) and (10.35) and integrating along the pipe yields

$$h_f = \frac{f}{4A/P}\frac{L}{}\frac{V^2}{2g} \qquad (10.36)$$

In Eq. (10.36), h_f is the head loss between two points in the conduit, L is the length between the points, and P is the wetted perimeter of the conduit. Thus Eq. (10.36) is the same as the Darcy-Weisbach equation except that D is replaced by $4A/P$. The ratio of the cross-sectional area A to the wetted perimeter P is defined as the *hydraulic radius* R_h. Obviously, for flow of a gas the wetted perimeter P is the perimeter of the duct. Experiments have shown that we can solve flow problems involving noncircular conduits, such as rectangular ducts, if we apply the same methods and equations that we did for pipes but use $4R_h$ in place of D. Consequently, the relative roughness is $k_s/4R_h$, and the Reynolds number is defined as $V(4R_h)/\nu$.

example 10.13

Air ($T = 20°\text{C}$ and $p = 101$ kPa absolute) flows at a rate of 2.5 m^3/s in a commercial steel rectangular duct 30 cm by 60 cm. What is the pressure drop per 50 m of duct?

Solution First compute Re and $k_s/4R_h$:

$$\text{Re} = \frac{V(4R_h)}{\nu}$$

Here

$$V = \frac{Q}{A} = \frac{2.5 \text{ m}^3/\text{s}}{0.18 \text{ m}^2} = 13.9 \text{ m/s}$$

The hydraulic radius is given by

$$R_h = \frac{A}{P} = \frac{0.18 \text{ m}^2}{1.8 \text{ m}} = 0.10 \text{ m}$$

$$4R_h = 4(0.10 \text{ m}) = 0.40 \text{ m}$$

$$\nu = 1.51 \times 10^{-5} \text{ m}^2/\text{s}$$

Hence

$$\text{Re} = \frac{13.9 \times 0.40}{1.51 \times 10^{-5}} = 3.68 \times 10^5$$

$$\frac{k_s}{4R_h} = \frac{4.6 \times 10^{-5} \text{ m}}{0.40 \text{ m}} = 1.15 \times 10^{-4}$$

Then from Fig. 10.8, $f = 0.015$. Thus

$$h_f = \frac{fL}{4R_h}\frac{V^2}{2g}$$

or

$$gh_f = \Delta p_f = \frac{fL}{4R_h}\rho\frac{V^2}{2}$$

$$\rho = 1.2\ \text{kg/m}^3$$

Finally,

$$\Delta p_f \text{ per } 50\ \text{m} = \frac{0.015(50\ \text{m})}{0.40\ \text{m}}(1.2\ \text{N}\cdot\text{s}^2/\text{m}^4)\frac{13.9^2}{2}\text{m}^2/\text{s}^2 = 217\ \text{Pa} \quad \triangleleft$$

Uniform Free-Surface Flows

A free-surface flow signifies the flow in a channel or duct with the surface open to the atmosphere. These flows are also known as open-channel flows. Such flows are encountered in culverts and irrigation canals. In this case there is no pressure gradient in the flow direction, so the change in piezometric pressure or head is due solely to elevation changes (gravity effects). Also, the hydraulic radius is based on the wetted area, so the portion of the perimeter on the free surface is not included. A uniform flow requires that the velocity be constant in the flow direction, so the shape of the channel and the depth of fluid will be the same from section to section. Figure 10.20 is an example of uniform free-surface flow in a channel with a rectangular cross section. Note here that the velocity varies across the section but does not change in the flow direction. A more extensive discussion of open channels with nonuniform flows is provided in Chapter 15.

The criterion for determining whether the flow in open channels will be laminar or turbulent is similar to that for flow in pipes. Recall that in pipe flow, if the Reynolds number ($VD\rho/\mu = VD/\nu$) is less than 2000, the flow will be laminar, and if it is greater than about 3000, one can expect the flow to be turbulent. The Reynolds number criterion for open-channel flow is the same if we replace D in the Reynolds number by $4R_h$, as we did in the Darcy-Weisbach equation in the preceding subsection. Thus, we can expect laminar flow to occur in open channels if $V(4R_h)/\nu < 2000$. The Reynolds number for open channels is usually defined as VR_h/ν. Therefore, in open channels, if the Reynolds number is less than 500, we will have laminar flow, and if it is greater than about 750, we can expect to have turbulent flow. A brief analysis of this turbulent criterion will show that water flow in channels will usually be turbulent unless the velocity and/or the depth is very small. Example 10.14 illustrates this point.

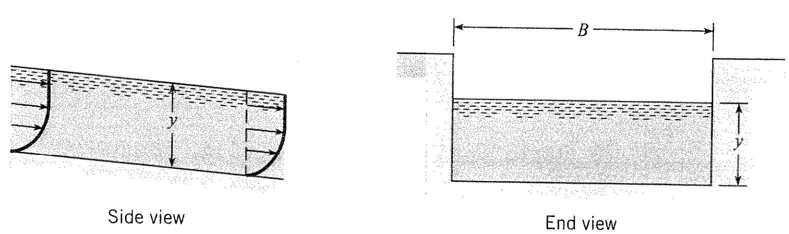


FIGURE 10.20

Open-channel relations.

It should be noted that the wetted perimeter used for calculating the hydraulic radius is the perimeter of the channel that is actually in contact with the flowing liquid. For example, in Fig. 10.20 the hydraulic radius of this channel of rectangular cross section is

$$R_h = \frac{A}{P} = \frac{By}{B+2y} \tag{10.37}$$

One can see that for very wide, shallow channels the hydraulic radius approaches the depth y.

example 10.14

Water (60°F) flows in a 10-ft-wide rectangular channel at a depth of 6 ft. What is the Reynolds number if the mean velocity is 0.1 ft/s? With this velocity, at what maximum depth can we be assured of having laminar flow?

Solution $\text{Re} = VR_h/\nu$

where

$$V = 0.1 \text{ ft/s}$$

$$R_h = A/P = By/(B+2y)$$

$$= (10\times 6)/(10+2\times 6)$$

$$= 2.73 \text{ ft}$$

$$\nu = 1.22\times 10^{-5} \text{ ft}^2/\text{s} \quad \text{(from Table A.5)}$$

then $\text{Re} = (0.1 \text{ ft/s})(2.73 \text{ ft})/(1.22\times 10^{-5} \text{ ft}^2/\text{s}) = 22{,}377$ ◁

The maximum Reynolds number at which we can expect to have laminar flow in open channels is 500. Thus, for this limit of Re and for a water velocity of 0.10 ft/s, we can solve for the depth at which this condition will prevail:

$$\text{Re} = VR_h/\nu = (0.10 \text{ ft/s})R_h/(1.22\times 10^{-5} \text{ ft}^2/\text{s}) = 500$$

Solving for R_h yields

$$R_h = (500)(1.22\times 10^{-5} \text{ ft}^2/\text{s})/(0.10 \text{ ft/s}) = 0.061 \text{ ft}$$

For rectangular channels

$$R_h = (By)/(B+2y)$$

Thus

$$(By)/(B+2y) = (10y)/(10+2y) = 0.061 \text{ ft}$$

$$y = 0.062 \text{ ft}$$ ◁

Example 10.14 shows that indeed the velocity and/or depth must be very small to yield laminar flow of water in a channel. Note also, the depth and hydraulic radius are virtually the same for this case, where the depth is very small relative to the width of the channel.

To determine the head loss for uniform flow in open channels, we utilize Eq. (10.36). That is, the Darcy-Weisbach equation with D replaced by $4R_h$ is used.

example 10.15

Estimate the discharge of water that a concrete channel 10 ft wide can carry if the depth of flow is 6 ft and the slope of the channel is 0.0016.

Solution We use the Darcy-Weisbach equation:

$$h_f = \frac{fL}{4R_h}\frac{V^2}{2g} \quad \text{or} \quad \frac{h_f}{L} = \frac{f}{4R_h}\frac{V^2}{2g}$$

When we have uniform open-channel flow, the slope of the EGL, $S_f = h_f/L$, is the same as the channel slope S_0. Therefore, $h_f/L = S_0$. The foregoing equation then reduces to $V^2/2g = 4R_hS_0/f$, or

$$V = \sqrt{\frac{8g}{f}R_hS_0}$$

Assume $k_s = 0.005$ ft. Then the relative roughness is

$$\frac{k_s}{4R_h} = \frac{0.005\text{ ft}}{4(60\text{ ft}^2/22\text{ ft})} = \frac{0.005\text{ ft}}{4(2.73\text{ ft})} = 0.00046$$

Using $k_s/4R_h = 0.00046$ as a guide and referring to Fig. 10.8, we assume that $f = 0.016$. Thus

$$V = \sqrt{\frac{8(32.2\text{ ft/s}^2)(2.73\text{ ft})(0.0016)}{0.016}} = \sqrt{70.6\text{ ft}^2/\text{s}^2} = 8.39\text{ ft/s}$$

Then

$$\text{Re} = V\frac{4R_h}{\nu} = \frac{8.39\text{ ft/s}(10.9\text{ ft})}{1.2(10^{-5}\text{ ft}^2/\text{s})} = 7.62 \times 10^6$$

Using this new value of Re and with $k_s/4R_h = 0.00046$, we read f as 0.016. Our initial guess was good; and now that the velocity is known, we can compute Q:

$$Q = VA = 8.39\text{ ft/s}(60\text{ ft}^2) = 503\text{ cfs}$$ ◁

For rock-bedded channels such as those in some natural streams or unlined canals, the larger rocks produce most of the resistance to flow, and essentially none of this resistance is due to viscous effects. Thus, the friction factor is independent of the Reynolds number. For this type of channel, Limerinos (28) has shown that the resistance coefficient f can be given in terms of the size of rock in the stream bed as where $Q = 110\ \text{ft}^3/\text{s}$

$n = 0.013$

$S_0 = 0.006$ (assume atmospheric pressure along the pipe)

Then $$AR^{2/3} = \frac{(110\ \text{ft}^3/\text{s})(0.013)}{(1.49)(0.006)^{1/2}} = 12.39\ \text{ft}^{8/3}$$

But $$R = \frac{A}{P} \qquad R^{2/3} = \left(\frac{A}{P}\right)^{2/3}$$

Then $$AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = 12.39\ \text{ft}^{8/3}$$

For a pipe flowing full, $A = \pi D^2/4$ and $P = \pi D$, or

$$\frac{(\pi D^2/4)^{5/3}}{(pD)^{2/3}} = 12.39\ \text{ft}^{8/3}$$

Solving for diameter yields $D = 3.98$ ft $= 47.8$ in. Use the next commercial size larger, which is $D = 48$ in. ◁

$$A = \frac{\pi D^2}{4} = 50.3\ \text{ft}^2 \text{ (for pipe flowing full)}$$

Thus $$V = \frac{Q}{A} = \frac{(110\ \text{ft}^3/\text{s})}{(50.3\ \text{ft}^2)} = 2.19\ \text{ft/s}$$ ◁

A culvert is a conduit placed under a fill such as a highway embankment. It is used to convey streamflow from the uphill side of the fill to the downhill side. Figure 10.21 shows the essential features of a culvert. Culverts are designed to pass the design discharge without adverse effects on the fill. That is, the culvert should be able to convey runoff from a design storm without overtopping the fill and without erosion of the fill at either the upstream or downstream end of the culvert. The design storm, for example, might be the maximum storm that could be expected to occur once in 50 years at the particular site.

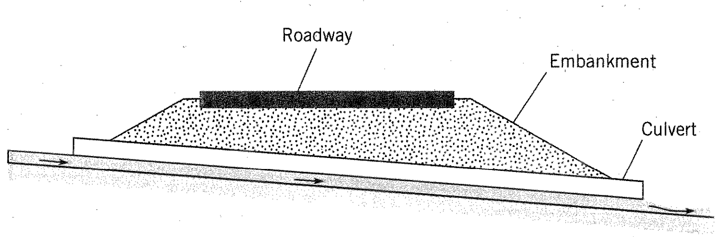


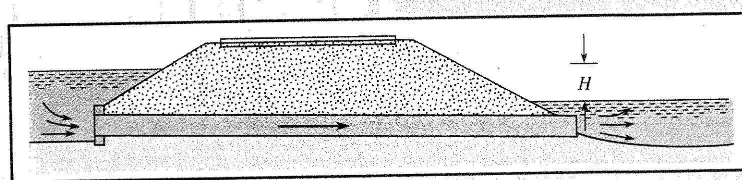
FIGURE 10.21

Culvert under a highway embankment.

The flow in a culvert is a function of many variables, including cross-sectional shape (circular or rectangular), slope, length, roughness, entrance design, and exit design. Flow in a culvert may occur as an open channel throughout its length, it may occur as a completely full pipe, or it may occur as a combination of both. The complete design and analysis of culverts are beyond the scope of this text; therefore, only a simple example is included here. For more extensive treatment of culverts, please refer to Chow (32), Henderson (33), and American Concrete Pipe Assoc. (34).

example 10.21

A 54-in.-diameter culvert laid under a highway embankment has a length of 200 ft and a slope of 0.01. This was designed to pass a 50-year flood flow of 225 cfs under full flow conditions (see the figure below). For these conditions, what head H is required? When the discharge is only 50 cfs, what will be the uniform flow depth in the culvert? Assume $n = 0.012$.



Solution For the flood flow of 225 cfs, one must consider the entrance and exit head losses as well as the head loss in the pipe itself; therefore, use the energy equation to solve this example.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

Let points 1 and 2 be at the upstream and downstream water surfaces, respectively.

Thus, $$p_1 = p_2 = 0 \text{ gage} \quad \text{and} \quad V_1 = V_2 = 0$$

Also, $$z_1 - z_2 = H$$

Then we have $$H = \sum h_L$$

$$H = \text{pipe head loss} + \text{entrance head loss} + \text{exit head loss}$$

$$H = \frac{V^2}{2g}(K_e + K_E) + \text{pipe head loss}$$

Assume

$$K_e = 0.50 \text{ (from Table 10.3)}$$

$$K_E = 1.00 \text{ (from Table 10.3)}$$

For the pipe head loss, use Eq. (10.45):

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2} \tag{10.45}$$

where

$$Q = 225\ \text{ft}^3/\text{s}$$
$$A = \frac{\pi D^2}{4} = 15.90\ \text{ft}^2$$
$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = 1.125\ \text{ft}$$
$$R_h^{2/3} = (1.125\ \text{ft})^{2/3} = 1.0817\ \text{ft}^{2/3}$$
$$S_0 = \frac{h_f}{L}$$

Then Eq. (10.45) is written as

$$225 = \frac{1.49}{0.012}(15.90\ \text{ft}^2)(1.0817\ \text{ft}^{2/3})\left(\frac{h_f}{200}\right)^{1/2}$$
$$h_f = 2.22\ \text{ft}$$
$$V = \frac{Q}{A} = \frac{225\ \text{ft}^3/\text{s}}{15.90\ \text{ft}^2} = 14.15\ \text{ft/s}$$

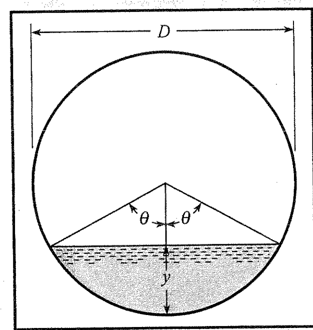
Solving for H,

$$H = \frac{14.15^2}{64.4}(0.50 + 1.0) + 2.22$$
$$H = 4.66\ \text{ft} + 2.22\ \text{ft} = 6.88\ \text{ft}$$ ◁

For $Q = 50$ cfs, we need to use Eq. (10.45):

$$50 = \frac{1.49}{0.012} A R_h^{2/3} (0.01)^{1/2}$$

However, this culvert will flow only partly full with a Q of 50 cfs. Therefore, the physical relationship will be as shown below.



Thus, if the angle θ is given in degrees, the cross-sectional flow area will be given as

$$A = \left[\left(\frac{\pi D^2}{4}\right)\left(\frac{2\theta}{360°}\right)\right] - \left(\frac{D}{2}\right)^2 (\sin u \ \cos u)$$

The wetted perimeter will be $P = \pi D(u/180°)$, or

$$R_h = \frac{A}{P} = \left(\frac{D}{4}\right)\left[1 - \left(\frac{\sin u \ \cos u}{p(\theta/180°)}\right)\right]$$

Substituting these relations for A and R_h into the discharge equation and solving for θ yields

$$u = 70°$$

Depth of flow:

$$y = \frac{D}{2} - \frac{D}{2}\cos u = \left(\frac{54 \text{ in.}}{2}\right)(1 - 0.342) = 17.8 \text{ in.}$$ ◁

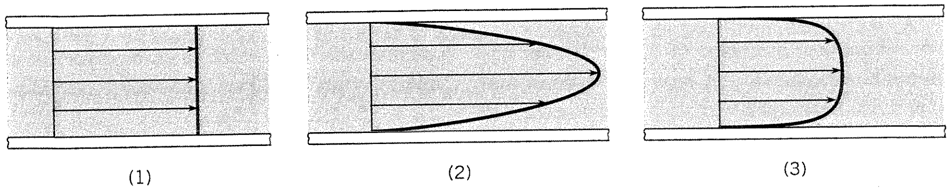
10.8 Summary

Flow in conduits is important to a wide variety of industries. As noted in Chapter 7, the analysis of a piping system requires information on the head (or pressure) loss to predict flow rates, power delivery, or power requirements for system operation. The head loss is given by the general equation

$$h_L = \sum f \frac{L}{D} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

where f is the Darcy-Weisbach friction factor, L is the pipe length, D is the diameter, V is the mean velocity and K is a loss coefficient. The first group of terms represents the loss

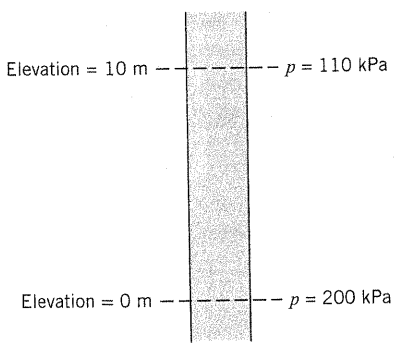
Problems



PROBLEM 10.1

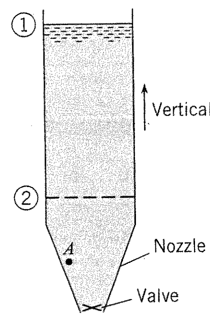
10.1 Consider the mean-velocity profiles for flow in the pipes shown. Match the profiles with the following: (a) turbulent flow, (b) obviously a case of hypothetical flow (zero viscosity), (c) laminar case, (d) $\alpha = 1.0$, (e) $\alpha = 1.05$, (f) $\alpha = 2.00$.

10.2 Liquid in the pipe shown in the figure has a specific weight of 8 kN/m^3. The acceleration of the liquid is zero. Is the liquid stationary, moving upward, or moving downward in the pipe? If the pipe diameter is 1 cm and the liquid viscosity is 3.0×10^{-3} $N \cdot s/m^2$, what is the magnitude of the mean velocity in the pipe?



PROBLEM 10.2

10.3 A viscous oil is contained in this cylinder/nozzle system that has a vertical orientation. A valve is instantaneously opened to let the oil drain out of the cylinder. Below are listed words that might characterize the flow at point *A*. Which ones are valid characterizations at the time when the oil surface reaches the level of section 2? (a) unsteady, (b) steady, (c) irrotational, (d) rotational, (e) nonuniform, (f) uniform.



PROBLEM 10.3

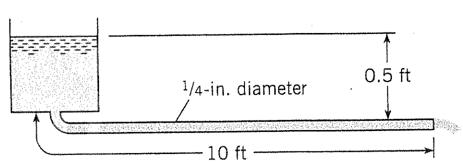
10.4 Oil (S = 0.97, $\mu = 10^{-2}$ lbf-s/ft^2) is pumped through a 1-in. pipe at the rate of 0.05 cfs. What is the pressure drop per 100 ft of level pipe?

FLUID SOLUTIONS **10.5** Liquid flows downward in a 1-cm, vertical, smooth pipe with a mean velocity of 2.0 m/s. The liquid has a density of 1000 kg/m^3 and a viscosity of 0.06 $N \cdot s/m^2$. If the pressure at a given section is 600 kPa, what will be the pressure at a section 10 m below that section?

10.6 A liquid ($\rho = 1000$ kg/m3, $\mu = 10^{-1}$ N · s/2 m^2, $\nu = 10^{-4}$ m^2/s) flows uniformly with a mean velocity of 1 m/s in a pipe with a diameter of 8 mm. For this condition, will the velocity distribution be logarithmic or parabolic? What will be the ratio of the shear stress at 1 mm from the wall to the shear stress on the wall?

10.7 Glycerine at a temperature of 30°C flows at a rate of 8×10^{-6} m^3/s through a horizontal tube with a 30-mm diameter. What is the pressure drop in pascals per 10 m?

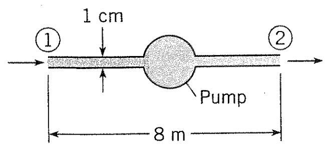
10.8 Kerosene (S = 0.80 and T = 68°F) flows from the tank shown and through the 1/4-in.-diameter (ID) tube. Determine the mean velocity in the tube and the discharge. Assume the only head loss is in the tube. FLUID SOLUTIONS



PROBLEM 10.8

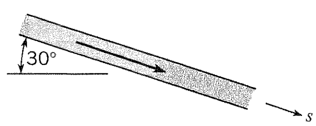
10.9 Oil (S = 0.94, μ = 0.048 N · s/m^2) is pumped through a horizontal 5-cm pipe. Mean velocity is 0.7 m/s. What is the pressure drop per 10 m of pipe?

FLUID SOLUTIONS **10.10** SAE 10W-30 oil is pumped through an 8-m length of 1-cm-diameter drawn tubing at a discharge of 7.85×10^{-4} m^3/s. There is a pump in the line as shown. The pipe is horizontal, and the pressures at points 1 and 2 are equal. Find the power necessary to operate the pump, assuming the pump has an efficiency of 100%. Properties of SAE 10W-30 oil: kinematic viscosity = 7.6×10^{-5} m^2/s, specific weight = 8630 N/m^3.



PROBLEM 10.10

10.11 Oil (S = 0.9; $\mu = 10^{-2}$ lbf-s/ft^2; ν = 0.0057 ft^2/s) flows downward in the pipe, which is 0.10 ft in diameter and has a slope of 30° with the horizontal. Mean velocity is 2 ft/s. What is the pressure gradient (dp/ds) along the pipe?



PROBLEM 10.11

10.12 A fluid ($\mu = 10^{-2}$ N · s/m^2; ρ = 800 kg/m^3) flows with a mean velocity of 5 cm/s in a 10-cm smooth pipe. Answer the following questions relating to the given flow conditions.

a. What is the magnitude of the maximum velocity in the pipe?

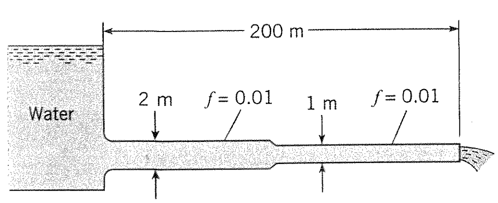
b. What is the magnitude of the resistance coefficient f?

c. What is the shear velocity for these flow conditions?

d. What is the shear stress at a radial distance of 25 mm from the center of the pipe?

10.13 Kerosene (20°C) flows at a rate of 0.04 m^3/s in a 25-cm pipe. Would you expect the flow to be laminar or turbulent?

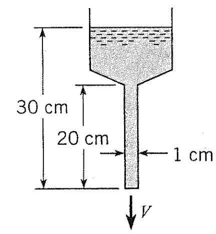
10.14 In the pipe system for a given discharge, the ratio of the head loss in a given length of the 1-m pipe to the head loss in the same length of the 2-m pipe is (a) 2, (b) 4, (c) 16, (d) 32.



PROBLEM 10.14

10.15 Glycerine (T = 68°F) flows in a pipe with a 1/2-ft diameter at a mean velocity of 2 ft/s Is the flow laminar or turbulent? Plot the velocity distribution across the flow section.

10.16 Glycerine (T = 20°C) flows through a funnel as shown. Calculate the mean velocity of the glycerine exiting the tube. Assume the only head loss is due to friction in the tube.



PROBLEM 10.16

10.17 What size of steel pipe should be used to carry 0.2 cfs of castor oil at 90°F a distance of 0.5 mi with an allowable pressure drop of 10 psi (μ = 0.085 lbf-s/ft^2)? Assume S = 0.85.

10.18 Mercury at 20°C flows downward in a long circular tube that is open to the atmosphere at the top and bottom. The tube is vertically oriented. Find the tube diameter for which the flow would just become turbulent (Re = 2000). FLUID SOLUTIONS

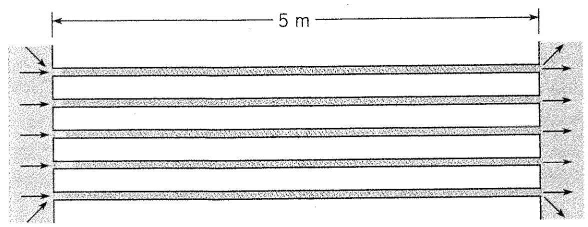
10.19 Glycerine (20°C) flows in a 4-cm steel tube with a mean velocity of 40 cm/s. Is the flow laminar or turbulent? What is the shear stress at the center of the tube and at the wall? If the tube is vertical and the flow is downward, will the pressure increase or decrease in the direction of flow? At what rate?

10.20 A small tank with a tube connected to it is to be used as a viscometer for liquids. Design a viscometer utilizing such equipment. State all assumptions and describe the procedure for the viscosity measurements. Assume that liquids to be measured will range from kerosene to glycerine.

10.21 Velocity measurements are made across a 1-ft pipe. The velocity at the center is found to be 2 fps, and the velocity distribution is seen to be parabolic. If the pressure drop is found to be 15 psf per 100 ft of pipe, what is the kinematic viscosity ν of the fluid? Assume that the fluid's specific gravity is 0.90.

10.22 Velocity measurements are made in a 30-cm pipe. The velocity at the center is found to be 1.5 m/s, and the velocity distribution is observed to be parabolic. If the pressure drop is found to be 1.9 kPa per 100 m of pipe, what is the kinematic viscosity ν of the fluid? Assume that the fluid's specific gravity is 0.80.

10.23 Water is pumped through a heat exchanger consisting of tubes 5 mm in diameter and 5 m long. The velocity in each tube is 12 cm/s. The water temperature increases from 20°C at the entrance to 30°C at the exit. Calculate the pressure difference across the heat exchanger, neglecting entrance losses but accounting for the effect of temperature change by using properties at average temperatures.



PROBLEM 10.23

10.24 The velocity of oil (S = 0.8) through the 2-in. smooth pipe is 5 ft/s. Here L = 30 ft, z_1 = 2 ft, z_2 = 4 ft, and the manometer deflection is 4 in. Determine the flow direction, the resistance coefficient f, whether the flow is laminar or turbulent, and the viscosity of the oil.

10.25 The velocity of oil (S = 0.8) through the 5-cm smooth pipe is 1.2 m/s. Here L = 12 m, z_1 = 1 m, z_2 = 2 m, and the manometer deflection is 10 cm. Determine the flow direction, the resistance coefficient f, whether the flow is laminar or turbulent, and the viscosity of the oil.

10.26 Flow of a liquid in a smooth 3-cm pipe is thought to yield a head loss of 2 m per meter of pipe when the mean velocity is 1 m/s. If the rate of flow was doubled, would the head loss also be doubled? Explain.

10.27 In a 12-in. smooth pipe, f is 0.017 when oil having a specific gravity of 0.82 flows with a mean velocity of 6 ft/s. What is the viscous shear stress on the wall?

FLUID SOLUTIONS **10.28** Consider the flow of oil (ρ = 900 kg/m3; r μ = 10^{-1} N · s/m^2) in a 10-cm smooth pipe and the flow of a gas (ρ = 1.0 kg/m^3; μ = 10^{-5} N · s/m^2) in a 10-cm smooth pipe. Both the oil and the gas flow with a mean velocity of 1 m/s. Will the ratio of the maximum velocity in the oil to the maximum velocity in the gas ($V_{max,oil}/V_{max,gas}$) be (a) greater than 1, (b) equal to 1, or (c) less than 1?

10.29 Water (50°F) flows with a speed of 5 ft/s through a horizontal run of PVC pipe. The length of the pipe is 100 ft, and the pipe is schedule 40 with a nominal diameter of 2.5 inches. Calculate (a) the pressure drop in psi, (b) the head loss in feet, and (c) the power in horsepower needed to overcome the head loss. (*Note:* PVC is a type of plastic. A 2.5-in. schedule 40 pipe has an inside diameter of 2.45 in.)

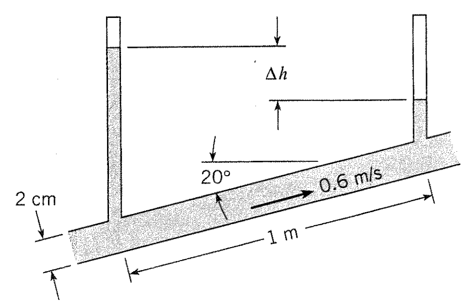
10.30 Water (10°C) flows with a speed of 2 m/s through a horizontal run of PVC pipe. The length of the pipe is 50 m and the pipe is schedule 40 with a nominal diameter of 2.5 inches. Calculate (a) the pressure drop in kilopascals, (b) the head loss in meters, and (c) the power in watts needed to overcome the head loss. (*Note:* PVC is a type of plastic. A 2.5-in. schedule 40 pipe has an inside diameter of 62.2 mm.)

10.31 Water (70°F) flows through a 6-in. smooth pipe at the rate of 2 cfs. What is the resistance coefficient f ?

10.32 Water (10°C) flows through a 25-cm smooth pipe at a rate of 0.06 m^3/s. What is the resistance coefficient f?

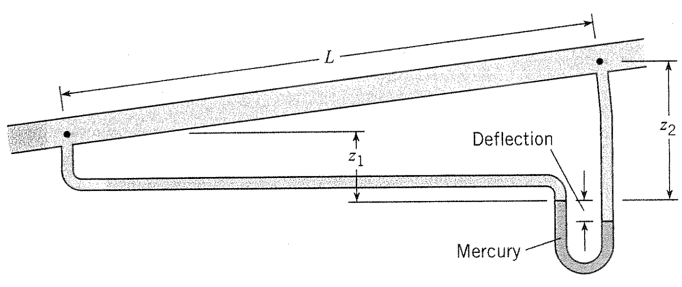
10.33 Air flows in a 3-cm smooth tube at a rate of 0.015 m^3/s. If T = 20°C and p = 110 kPa absolute, what is the pressure drop per meter of length of tube? FLUID SOLUTIONS

10.34 Glycerine at 20°C flows at 0.6 m/s in the 2-cm commercial steel pipe. Two piezometers are used as shown to measure the piezometric head. The distance along the pipe between the standpipes is 1 m. The inclination of the pipe is 20°. What is the height difference Δh between the glycerine in the two standpipes?



PROBLEM 10.34

10.35 Air flows in a 1-in. smooth tube at a rate of 30 cfm. If T = 80°F and p = 15 psia, what is the pressure drop per foot of length of tube?



PROBLEMS 10.24, 10.25

due to friction in straight lengths of pipe. The second group of terms represents the head loss associated with components such as valves, elbows, bends, and transition sections.

The Darcy-Weisbach friction factor is a function of the pipe Reynolds number,

$$\mathrm{Re} = \frac{VD}{\nu}$$

For Reynolds numbers less than 2000, the pipe flow is laminar. An analytic solution shows the velocity distribution is parabolic (Hagen-Poiseuille flow), and the Darcy-Weisbach friction factor is

$$f = \frac{64}{\mathrm{Re}}$$

For Reynolds numbers greater than 3000, the flow is turbulent and characterized by a near uniform velocity profile with high velocity gradients near the pipe wall. The friction factor depends on the Reynolds number and the relative roughness:

$$f = f\left(\mathrm{Re}, \frac{k_s}{D}\right)$$

where k_s is the equivalent sand grain roughness. The values for friction factor can be obtained from the Moody diagram or empirical equations. For a smooth pipe, the friction factor is independent of the relative roughness and depends only on the Reynolds number. For the fully rough condition, the friction factor is independent of the Reynolds number and depends only on the relative roughness.

The head loss coefficients may be obtained from tables and other sources of information for various flow components.

Noncircular pipes can be analyzed using the hydraulic radius, which is defined as

$$R_h = \frac{A}{P}$$

where A is the cross-sectional area of the conduit and P is the wetted perimeter. To analyze noncircular ducts, the diameter in the equations for circular pipes is replaced by $4R_h$.

The analysis of pipe networks is based on the continuity equation being satisfied at each junction and the head loss between any two junctions being independent of pipe path between the two junctions. A series of equations based on these principles are solved iteratively to obtain the flow rate in each pipe and the pressure at each junction in the network.

In an open-channel flow, the head loss corresponds to the potential energy change associated with the slope of the channel. The discharge in an open channel is given by the Chezy equation:

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

where A is the flow area, S_0 is the slope of the channel, and n is the resistance coefficient (Manning's n), which has been tabulated for different surfaces.