Mohr’s Circle for Moment of Inertia

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Many engineering students are introduced to the ideas and concepts of Mohr’s Circle when studying stress states due to various loading conditions on structures or machines. Generally this comes in Mechanics of Materials after the student has learned how to properly identify external loads, and internal reactions that take place at areas of interest. Everything that can be calculated using the graphical method of Mohr’s circle, can be calculated with a set of equations. The benefit that Mohr’s circle provides is that it creates a visual representation of what is happening, and the relative positioning of the stress states on an element relative to a set of coordinate axes.

When using Mohr’s circle to evaluate stress elements, the major things that are determined are; principal normal stresses, max shear stresses, and the angle of the plane that these stresses are on. This information along with material analysis is used to determine max loads and fatigue strengths of designs. This graphical method can also be used in the analysis of Moment of Inertia. It can be used for the following; **a.)** the determination of principal axes and principal moments of inertia of the cross section, and **b.)** the moments and products of inertia of the area with respect to a rotated configuration of the same cross section. As the cross section is rotated the ability of the member to resist bending is increased or decreased, and Mohr’s circle is used to see this graphically. The approach to the construction of the circle is very similar to those of the construction of the 2D stress element.

Before this is attempted, the student should have a good understanding of what moment of inertia is and how to calculate Ix, Ixy and Iy using tables and equations. Just as with the stress elements the circle will be based on the location of the center C, and the radius R of the circle. These values will be measured relative to a rectangular coordinate axis. The distance from the origin to the center represents the average moment of inertia (Iave). The average is calculated for a known orientation with equations or tables, as well as the radius. Point X (*variables are located on the figure*) is on the opposite side of the circle as point Y. *Point X is defined by (Ix, Ixy), and point Y is defined by (Iy,-Ixy).* Since the moment of Inertia must always be positive, it must always lie to the right of the Ixy axis, if it is not you have done something incorrectly. As the coordinate axes are rotated through an angle of θ, the associated rotation of the diameter of Mohr's circle is equal to 2θ, and is in the same direction. Make sure that all points are labeled on the circumference of the circle with capital letters.

An example of a possible use of Mohr's circle for moment of Inertia is on the anti-roll system of the Formula SAE car. This system has a set of "blades" that has rectangular, tapered cross sections that are used to adjust the amount of body roll that the car has. The adjustment is made by rotating the blade from a vertical configuration to a horizontal configuration, which changes the moment of inertia. This allows the blade to flex different amounts depending on the configuration, and give the car different handling characteristics for different race tracks and driving styles. With Mohr's circle this will give a great graphical representation of what the moment of inertia will be in a given configuration.