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ME325
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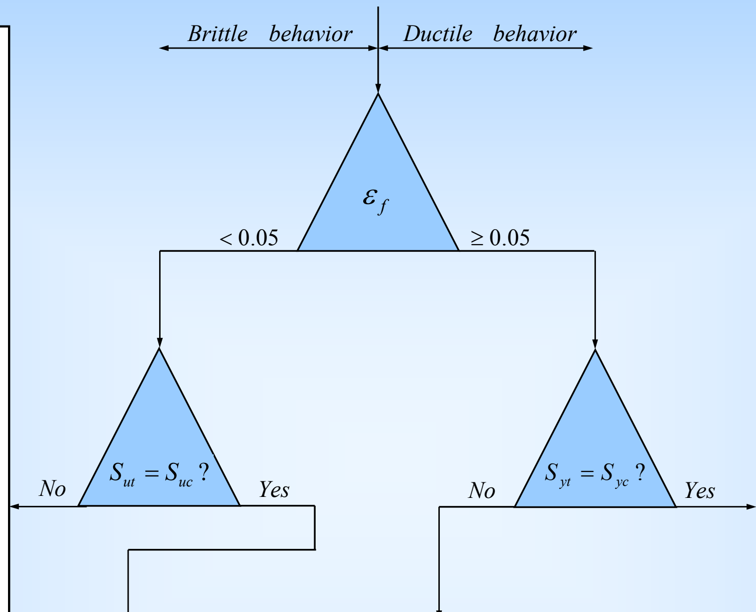
Failure Theories

The failure of a statically loaded member in uni-axial tension or compression is relatively easy to predict. One can simply compare the stress incurred with the strength of the material. However, when the loading conditions are less simple (i.e. biaxial loading, sheer stresses) then we must use some method to compare multiple stresses to a single strength value. Below are four common criteria for predicting failure and determining factors of safety as well as lists of some common materials for which each would be preferred.



Modified II-Mohr

This failure criteria is a modification of the Mohr Hypothesis that best accounts for the experimentally produced data from fracture tests of brittle materials (i.e. strain to failure < 0.05). It is applicable when the $|S_{UC}| > S_{UT}$ and σ_B is less than $-S_{UT}$.

Where

$$0 \le \sigma_A \le S_{ut} \qquad -S_{ut} \le \sigma_B \le S_{ut}$$

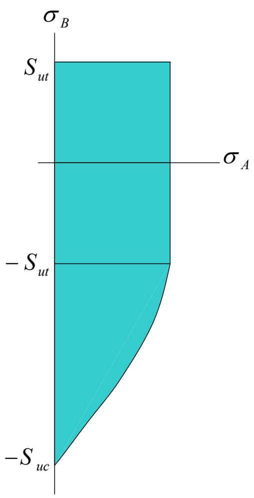
$$\sigma_A = \frac{S_{ut}}{n}$$

Where

$$0 \le \sigma_A \le S_{ut} \qquad -S_{uc} \le \sigma_B \le S_{ut}$$

$$\frac{n\sigma_A}{S_{ut}} = 1 - \left(\frac{n\sigma_B + S_{ut}}{-S_{uc} + S_{ut}}\right)^2$$

Applicable Material Examples:
- Gray Cast Iron
- Ceramics

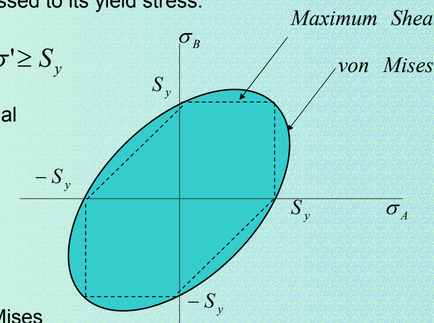


Distortion-Energy Hypothesis (DE)

The distortion-energy hypothesis predicts that failure will occur in a member when the distortion energy per unit volume of the member equals the distortion energy in the member when it is uni-axially stressed to its yield stress.

Failure when: $\sigma' \ge S_y$

Applicable Material Examples:
- Aluminum
  - Wrought,
  - Drawn,
  - or Rolled
- Most Steels



Where the von Mises stress is:

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{\frac{1}{2}}$$

The safety factor can be described by: $n = \dfrac{S_y}{\sigma'}$

Maximum-Normal-Stress (MNS) Hypothesis

The maximum-normal-stress theory states that failure occurs whenever one of the three principal stresses equals or exceeds the strength.

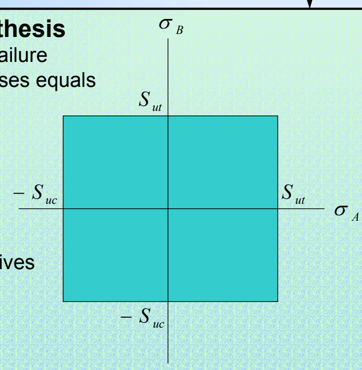
Using $\sigma_1 > \sigma_2 > \sigma_3$ failure occurs when

$$\sigma_1 = S_{ut} \quad \text{or} \quad \sigma_3 = -S_{uc}$$

Correlating the hypothesis to a factor of safety gives

$$n = \frac{S_{ut}}{\sigma_1} \quad \text{or} \quad n = \frac{-S_{uc}}{\sigma_3}$$

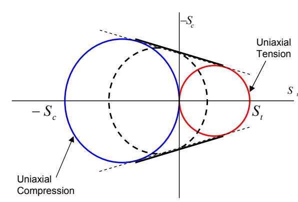
Applicable Material Example:
- Cast Aluminum

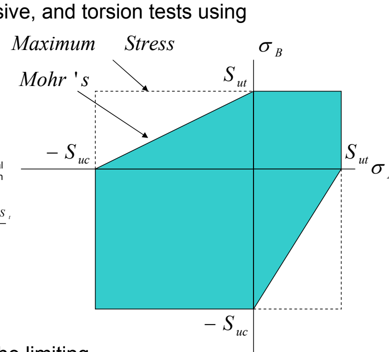


Coulomb Mohr Hypothesis

The Coulomb Mohr Hypothesis Predicts that failure will occur in a multiaxial state of stress when the larger Mohr circle associated with the state of stress at the critical location becomes tangent to, or exceeds the bounds of the failure envelope established by conditions of failure in simple tensile, compressive, and torsion tests using specimens of the same material and condition.

Case	Principle Stress	Theory Requirements
1 (1st Quadrant)	$\sigma_A > 0, \sigma_B > 0$	$\sigma_A < S_t, \sigma_B < S_t$
2 (3rd Quadrant)	$\sigma_A < 0, \sigma_B < 0$	$\sigma_A > -S_c, \sigma_B > -S_c$
3 (4th Quadrant)	$\sigma_A > 0, \sigma_B < 0$	$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} = \frac{1}{n}$
4 (2nd Quadrant)	$\sigma_A < 0, \sigma_B > 0$	$\frac{-\sigma_A}{S_t} + \frac{\sigma_B}{S_c} = \frac{1}{n}$





Applicable Material Examples:
- Iconel®
- Certain Titanium Alloys

The left circle is for uniaxial compression at the limiting compression stress S_c of the material. Likewise, the right circle is for uniaxial tension at the limiting tension stress S_t

Referenced Materials: 1. Shigley, J; Mischke, C; "Failure Prevention." Mechanical Engineering Design. 5th ed. Boston: McGraw Hill, 2002. 2. http://www.efunda.com/formulae/solid_mechanics/failure_criteria 3. www.matweb.com