

# SPRINGS

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All Equations and Figures From: Shigley & Mischke. Mechanical Engineering Design. Fifth Edition, 2002.



## Extension Springs

Note: Loading the Extension Spring creates a shear stress in the spring.  
**Extension Springs:** These must have a way of transferring load from a support to the body of the spring. Using Springs with a hooked end, stress concentration must be considered. This is shown below. The lower spring has a decreased stress concentration because the moment arm is smaller.

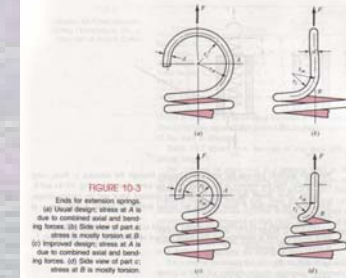


FIGURE 10-10  
Preferred Range of Torsional Stresses Due to Initial Torsion for Steel Helical Extension Springs

WIRE SIZE	STRESS RANGE
INCH	MPa
4	115-163
5	105-150
6	92-127
8	80-106
10	68-86
12	57-75
14	47-62
16	37-47
18	27-37

Source: Associated Spring-Black & Veatch, Chicago, Ill., 1963, p. 100.

Stress Concentration Factor:

$$K = \frac{r_m}{r_c}; \text{ eqn. 10-10}$$

## Fatigue Loading

**Spring Fatigue Loading:**  
 Some springs are subjected to fatigue loading. It must be determined whether the spring will need to have infinite life or finite life. Helical springs are never designed to be used in both compression and tension.

Alternating stress  $F_a$ :  

$$F_a = \frac{F_{max} - F_{min}}{2}; \text{ eqn. 10-26}$$

Mean stress  $F_m$ :  

$$F_m = \frac{F_{max} + F_{min}}{2}; \text{ eqn. 10-27}$$

Stress Amplitude  $\tau_a$ :  

$$\tau_a = \frac{K_b \cdot 8 \cdot F_m \cdot D}{\pi \cdot d^3}; \text{ eqn. 10-28}$$

Mean Stress  $\tau_m$ :  

$$\tau_m = \frac{K_s \cdot 8 \cdot F_m \cdot D}{\pi \cdot d^3}; \text{ eqn. 10-29}$$

## Helical Springs

- Compression
- Torsion
- Extension



## Torsion Springs

Note: Twisting the Torsion Spring creates a normal stress in the spring.

### Helical Torsion Springs:

Helical torsion springs are wound in the same manner as extension and compression springs. For torsion springs, the ends are designed to transmit torque.

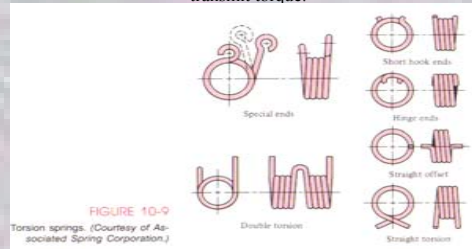


FIGURE 10-9  
Torsion springs. (Courtesy of Associated Spring Corporation.)

Stress Concentration Factor on inside of the Spring  $K_i$ :

$$K_i = \frac{4 \cdot C^2 - C - 1}{4 \cdot C \cdot (C - 1)}; \text{ eqn. 10-32}$$

Bending stress for a round wire torsion spring:

$$\sigma = \frac{K_i \cdot 32 \cdot F \cdot r}{\pi \cdot d^3}; \text{ eqn. 10-33}$$

Displacement in torsion springs is described in radians.

$$\theta = \frac{64 \cdot F \cdot r \cdot D \cdot N}{d^4 \cdot E}; \text{ 10-34}$$

The spring rate, taking into account the curvature of the wire:

$$k' = \frac{d^4 \cdot E}{10.8 \cdot D \cdot N}; \text{ 10-37}$$

## Compression Springs

Note: compression produces shear stress in the spring

Governing equation:  $F=k \cdot y$

$$y = \frac{8 \cdot F \cdot D^3 \cdot N}{d^4 \cdot G} \left[ 1 + \frac{1}{2 \cdot C^2} \right]; \text{ eqn. 10-8}$$

Spring Rate  $k$ :

$$k = \frac{d^4 \cdot G}{8 \cdot D^3 \cdot N}; \text{ eqn. 10-9}$$

End Conditions:

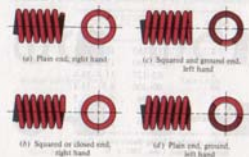


FIGURE 10-5

Types of ends for compression springs: (a) both ends plain; (b) both ends squared; (c) both ends squared and ground; (d) both ends plain and ground.

Critical Deflection:

**Critical deflection in a compression spring:** This is when the deflection becomes too large and the spring buckles. The equation is shown below.

$$y_{cr} = L_0 \cdot C_1 \cdot \left[ 1 - \left[ 1 - \left[ \frac{C_2}{\lambda \cdot e_{eff}^2} \right] \right] \right]^{\frac{1}{2}}; \text{ eqn. 10-11}$$

The effective slenderness ratio is given by:  
 $\lambda_{eff} = \alpha \cdot L/D$ ; eqn. 10-12

The chart below describes  $\alpha$ , which is the end condition constant.

TABLE 10-3	END CONDITION	CONST.
End-Condition Constants $\alpha$ for Helical Compression Springs*	Spring supported between flat parallel surfaces (fixed ends)	0.5
	One end supported by flat surface perpendicular to spring axis (fixed), other end pivoted (hinged)	0.7
	Both ends pivoted (hinged)	1
	One end clamped; other end free	2

\*Each supported by flat surface must be squared and ground.

$$C_1 = \frac{E}{2 \cdot (E - G)}; \text{ eqn. 10-13}$$

$$C_2 = \frac{2 \cdot \pi^2 \cdot (E - G)}{2 \cdot G + E}; \text{ eqn. 10-14}$$

$$L_0 < \frac{\pi \cdot D}{\alpha} \cdot \left[ \frac{2 \cdot (E - G)}{2 \cdot G + E} \right]^{\frac{1}{2}}; \text{ eqn. 10-15}$$

This is the free length of the spring.

