A BALLISTIC GAS COMPRESSOR FOR AUTOMOTIVE AIRBAG INITIATOR RESEARCH AND AUTOMOTIVE ENGINE TESTING AND DEVELOPMENT

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Abstract
Dynamic pressure transducers are used in the testing of energetic components due to the transient characteristics typical of detonations and rapid deflagrations. Quasi-static tests do not ensure transducer dynamic accuracy and there is currently no standard method for dynamic calibration. A promising apparatus to quantify the dynamic accuracy of pressure transducers is a Ballistic Gas Compressor (BGC). The BGC consists of two pressure reservoirs - one containing an initially high pressure gas, the other containing an initially low pressure gas - connected by a tube containing a heavy metal piston providing a seal between the two reservoirs. When released the alternating differential pressure in the system causes the piston to oscillate freely between the two reservoirs. In this manner, the Ballistic Gas Compressor reliably produces repeatable dynamic pressure environments whose characteristics can be controlled through selection of initial gas pressures, piston mass, and gas composition. In this research, design failure mode effects analysis (DFMEA) is used to define and prioritize ballistic compressor failure modes. Structural analysis with Algor and machine design calculations based on standard mechanics of materials reveals that the limiting component is the high pressure plug at a maximum operating pressure of 25 ksi.
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EXECUTIVE SUMMARY

Performance acceptance criteria for pyrotechnic initiators rely upon accurate measurement of the dynamic pressures produced when these devices are discharged into closed vessels. Therefore, it is important that the pressure transducers used to monitor these events be accurately calibrated. However, there currently is no accepted standard technique for calibration of dynamic pressure transducers although a number of different techniques have been developed for this purpose. Ballistic piston gas compression apparatuses are commonly used to study the thermodynamic behavior of gases at extreme pressures and temperatures, and the versatility of these devices suggests they may be successfully applied to the calibration of dynamic pressure transducers. This report describes work-in-progress at the University of Idaho to develop a new ballistic piston gas compression apparatus for this purpose. A simple mathematical model of the apparatus has been developed to allow prediction of the pressures generated within the device as a function of important design variables such as the piston mass, barrel length, and gas properties. Based on this model, a ballistic compressor prototype has been designed and fabricated. Due to the potentially hazardous modes of operation of the compressor, a comprehensive analysis tool (Design for Failure Mode and Effects Analysis - DFMEA) was used to define and prioritize ballistic compressor failure modes. Components with the highest risk priority number (RPN) were the high pressure plug, piston seals, and trigger toggle block. Structural analysis with Algor®, machine design calculations based on standard mechanics of materials, and a literature review on O-ring performance revealed that the limiting component was the high pressure plug at a maximum operating pressure of 25 ksi, yet the trigger is not robust enough to operate at these pressures. With some redesign and modification of the trigger mechanism the ballistic compressor should be able to safely withstand pressures of 25 ksi (excluding a safety factor) and by following the guidelines in this work; the ballistic compressor can be operated for up to 4000 cycles with minor maintenance.

DESCRIPTION OF PROBLEM

Bridge-wire initiators are widely used in the aerospace and automotive industries to ignite pyrotechnic, propellant, and other explosive materials. The products of combustion of these
energetic materials are used to perform an assortment of work functions including actuation of mechanisms, explosive separation of components, and deployment of supplemental restraint or other protective systems. In all of these applications, it is important the initiators perform reliably over a wide range of temperatures, often after prolonged exposure to harsh environmental conditions of humidity, temperature extremes, and vibration.

The most common method of measuring initiator performance involves discharging these devices into constant volume closed vessels and monitoring the rate and magnitude of pressure increase. Typically, the maximum pressures generated by the discharge of initiators into closed vessels are used as acceptance criteria for lot acceptance tests of these devices.\(^1\) Alternatively, the maximum dynamic pressures may be used to infer the energy liberated by the combustion process, and the derived values of energy release may be used as an input for models used to describe the ignition and combustion of additional gas generating materials and reactive gas mixtures.\(^2\) Therefore, it is important to be able to measure the maximum dynamic pressure as accurately as possible.

Earlier work at the University of Idaho focused on closed vessel design, fabrication, and testing, and understanding how the cumulative effects of uncertainties in instrumentation, data acquisition, and signal processing influence the derived values of initiator energy release.\(^3,4\) To characterize their energy release, initiators were discharged into high pressure inert gas environments and subsequent maximum internal pressures exceeded 60 MPa in less than 5 milliseconds. When initiators were discharged into environments containing reactive gases and liquids, maximum pressures approached 200 MPa in time frames of less than 1 millisecond. Given repetitive tests under these harsh environments, the need was recognized immediately for a method to validate the accuracy of the dynamic pressure transducers. The goal of this research is to design, fabricate, and test an apparatus for calibration of dynamic pressure sensors used in closed vessel initiator testing.

**DYNAMIC PRESSURE CALIBRATION METHODS**

Selection of a method for dynamic calibration of pressure transducers is problematic because there is currently no recognized standard for this purpose, although a number of different
Ballistic Compressor

Techniques are currently employed. Five types of devices commonly used have the potential to produce dynamic pressure responses characteristic of those observed in our ballistic research: a drop tube, a shock tube, an Aronson tester (a type of fast-acting mechanical valve), a pyrotechnically-actuated valve, and a ballistic piston gas compressor. Each of these apparatuses is an aperiodic device that produces a step or peak pressure and each has its own merits and limitations.

Currently, we rely upon the transducer manufacturer to provide the initial calibration data appropriate for the specified operating range of a transducer. The static responses of the transducers are routinely checked using a standard dead weight calibration apparatus, but this does not ensure proper calibration for dynamic conditions. In an attempt to verify dynamic calibration, we initially designed and constructed a drop tube tester similar to those offered commercially by PCB Piezotronics. The drop tube tester uses a mass dropped from a predetermined height to strike a plunger. The plunger, in turn, compresses a fluid (Dow Corning 200 silicon oil) contained in a manifold housing two transducers, one of which must be considered of known calibration. The output of the two transducers is then compared to determine if the unknown transducer can be considered within specification. While the drop tube system is simple and robust, we are concerned over the accuracy of the technique and would like to develop an alternative method of dynamic calibration so that results from the two techniques can be compared.

Shock tubes are also commonly used for the dynamic calibration of pressure transducers. These devices basically consist of a long tube partitioned by a thin diaphragm or burst disk separating a high pressure (driving-gas) region from a low pressure (driven gas) region. The characteristics of the driving and driven gases can be tailored so that upon rupture of the diaphragm the high pressure gas flows into the low pressure side producing a well-defined step pressure response. This transient pressure change is precisely known so a transducer can be calibrated independently from other transducers. As the name implies a shock wave is typically formed in the process, and therefore the characteristics (interior dimensions, inside surface finish, diaphragm characteristics, etc.) of the apparatus must be determined before it can be used for calibration purposes. While shock tubes are relatively simple devices, there
are disadvantages in their use as calibration devices. Unless reactive gas mixtures are used, the tube must be designed to contain the desired test pressure ratios statically, and a compression system must be available to supply gases over the range of desired pressure ratios. While reactive mixtures can certainly be used, their use introduces significant safety and operational concerns in testing. Another disadvantage to shock tubes is that the process of the diaphragm rupture must be well-understood and repeatable.

Fast acting mechanical valves and pyrotechnically actuated valves also represent possible solutions to the dynamic calibration problem. A pyrotechnically actuated valve is similar to a shock tube in that a partition is used to separate two volumes; one pressurized to a known higher value while the other volume is pressured to a lower pressure and houses the transducer to be calibrated. The valve is actuated by gases generated by appropriate-sized pyrotechnic charge which rapidly displaces the partition, thereby allowing the high pressure gas to flow into the lower pressure chamber, providing a pressure step similar to that of the shock tube. While pyrotechnically operated valves can be designed to be quite versatile and provide a wide-range of pressure responses, they also must be designed to store compressed gases at high differential static pressures. In addition, there is some concern over the anticipated life of the device as well as the need to regularly replace consumable seals and partitions.

Ballistic piston gas compressors have historically been used to study the P-v-T behavior of high pressure ionized gases and plasmas.\textsuperscript{8,9,10} These devices feature a piston inside a long barrel. In a manner somewhat analogous to a shock tube, the piston separates a reservoir containing a high pressure gas upstream of the piston from the low pressure gas in the barrel. The system is designed so that upon actuation the gas from the high pressure reservoir causes the piston to be propelled into the low pressure gas contained in the barrel. Through control of design parameters including the types of gases used in the barrel, the pressure differential across the piston, the mass of the piston, and the area of the barrel, a wide-range of dynamic pressure responses can be achieved in the barrel. Because of its inherent versatility and ability to replicate transient pressures produced in closed volume initiator tests, it is our
intention to develop a ballistic piston compression apparatus as a primary means of calibrating dynamic pressure transducers.

**BASICS OF BALLISTIC GAS COMPRESSORS**

In the mid-twentieth century, a need had arisen for data on the P-v-T relationships of gasses at temperatures and pressures well above the capabilities of steady-state compression and heating devices. The free-piston ballistic compressor met that need in the early 1950’s and was used to study these gas properties for small periods of time.\textsuperscript{10} Since then, the ballistic compressor has been used by numerous universities\textsuperscript{11,12} and the Naval Ordnance Laboratory\textsuperscript{13} to study P-v-T relationships of gasses as well as other phenomenon.

The ballistic compressor features two pressure vessels connected by a tube in which a sealed piston is free to move. One pressure cylinder is very large, compared to volume changes during operation, and contains an assumed constant pressure gas (driving gas) while the other cylinder has a very small comparative volume and contains the gas to be compressed (driven gas), shown in Figure 1.

![Figure 1: Ballistic compressor gas chambers.](image)

The piston is initially locked into position so the driven volume is at its greatest value (the first position in Figure 2). In this position, the driving gas is pressurized to its operating point creating a pressure differential across the piston. This operating pressure is well within the limits of steady-state compressors and is substantially higher than that of the driven gas, which is typically at atmospheric or sub-atmospheric pressure. When the piston is released the pressure difference between the two cylinders drives the piston towards the driven gas. When the piston is within fractions of an inch to the end of the tube, the pressure of the
driven gas rapidly increases to its maximum value, reversing the pressure differential on the piston. This adverse pressure difference accelerates the piston in the opposite direction, reversing its movement and sending it back towards the driving gas. The system acts as an under-damped, second-order system where the piston oscillates several times, shown in Figure 2, before friction leads to the dissipation of the energy of the piston and the equalization of the pressures of the gasses.

Figure 2: Progression of piston during operation.
Although the ballistic compressor is defined by its free piston separating two different gas reservoirs, it also has three other complex subsystems that can vary greatly from design-to-design. These subsystems are required to usefully operate the device and are as follows: the trigger mechanism, gas management system, and the instrumentation system.

The triggering system holds the piston in place while the driving end is being pressurized. Once the driving end is pressurized, the trigger mechanism must be able to release the piston, allowing it to move freely towards the driven end of the compressor. The gas management system fills the driving and driven gas pressure chambers. This system can be as simple as an air compressor and a valve to pressurize the driving gas or complex enough to introduce different gas mixtures as driven gas, catch exhaust gasses, and many other operations.

The last subsystem is the instrumentation system. Although the instrumentation subsystem is not essential to the operation of the ballistic compressor, it is necessary to record testing data. The instrumentation system is the most complex system of all three subsystems, because of the speed of measurements and the diversity of sensors that are implemented. The instrumentation system needs to be robust and versatile so that pressure, temperature, position, and many other different measurements may be recorded. Gunter has designed, fabricated, and tested an instrumentation system suitable for a ballistic gas compression apparatus that is capable of monitoring the rapid response of multiple sensor types.

The design of the ballistic compressor described above was first seen in the early 1950’s. Two different ballistic compressors were built at relatively the same time. One was built at the Naval Ordnance Laboratory (NOL) and the other at the California Institute of Technology. Longwell et al. were the first to publish a paper on the construction of a ballistic compressor that described their compressor at the California Institute of Technology. This compressor was first used to compress helium and carbon dioxide and expanded to include compression of combustible gasses. Their paper describes the structure of the compressor, the instrumentation incorporated into testing, and a math model to predict the response of the system. This paper laid out the basic framework for ballistic compressor design and is referenced by most work pertaining to ballistic compressor design.
Figure 3: Simplified schematic of California Institute of Technology’s Compressor.

Figure 3 shows the schematic of the ballistic compressor built at the California Institute of Technology. The compressor was triggered by turning the handle on the left that sheared a pin inside of the driving gas chamber. When the pin was sheared, the piston was released for the test. Although simple in design, the compressor had to be disassembled and the pin replaced in order to prepare for the next test. The gas management system had the ability to insert different mixtures of gasses into the driven end of the compressor, and the instrumentation system was set up to record position as a function of time, pressure, and heat flux data.

The second ballistic compressor of the time was being operated at NOL and was published by Price and Lalos. This compressor was built much like the one in California but for the specific purpose of studying equation-of-state relationships. Price and Lalos discussed the proper operation of the ballistic compressor and also explained that initially the piston did not have piston seals, a topic referred to later in this thesis.

Figure 4 is a schematic of the compressor at NOL. The trigger mechanism used a shear pin much like the compressor at the California Institute of Technology. The trigger system was the only subsystem that was common to the two ballistic compressors. The instrumentation on this ballistic compressor allowed recording of position as a function of time, pressure, and temperature data using more accurate sensors with faster response times. The containment system had all of the abilities that the compressor at the California Institute of Technology, as well as the ability to capture gasses that were purged from inside the compressor.
Lalos continued work and research using ballistic compressors at the NOL researching optical properties, equation of state, and many other properties of high temperature and pressure gasses. This work included construction of other ballistic compressors with more advanced instrumentation to take measurements such as spectrum emissions, temperatures, pressures, and densities. Published works focused on gas behavior and not on design of ballistic compressors themselves, even though there were multiple generations built.

Work with ballistic compressors has continued at several universities since the first compressors in the 1950’s. In the late 1960’s, researchers at the University of Oregon studied the gas leakage and the friction assumptions of earlier work. Studies at Princeton in the mid 1970’s in metal corrosion supported the study and publication of the use of a ballistic compressor and the performance of a ballistic compressor’s piston blow-by, valve losses, friction of the piston, shock waves, and heat losses. Germany’s Kiel University constructed a ballistic compressor to reach pressures up to 1 GPa in the mid 1980’s for the study of gas properties. While this is not an exhaustive list of ballistic compressors, the material above describes the structure and capabilities of ballistic compressors.

MATHEMATICAL MODEL

In order to properly design a ballistic compressor for this application, it is necessary to develop a mathematical model for the apparatus. The model should replicate the range of dynamic pressures expected to be produced during initiator testing through varying important design parameters such as the piston mass, barrel diameter and length, and gas pressure ratio.
Performance information derived from this model will be used to guide the mechanical design of the apparatus, and allow us to determine the component geometry and material requirements necessary to withstand the expected loads generated within the device.

Since the objective of the model is to generate a prediction of a dynamic signal, a State Space Model is considered the best approach to analyze the behavior of ballistic piston gas compressor. This approach closely resembles that of a classical mass-spring-damper system, with the piston representing the mass, the gas being compressed acting as the spring, and the friction at the piston barrel interface the damper. This approach closely follows those of previous researchers.\textsuperscript{14}

There are a number of assumptions in our analysis. First, the system is considered closed with no leakage of gas from the test section past the piston. Second, the gas is considered to behave as an Ideal Gas. This assumption is made for convenience with the understanding that a more accurate equation of state can be introduced into the model at a later date if one is deemed appropriate. Third, the process will initially be considered as isentropic. Earlier studies have generally introduced this same assumption,\textsuperscript{12,14} although some researchers have shown that some heat transfer does occur.\textsuperscript{15} Ultimately, experimentally derived values of pressure and specific volume will be used to determine the characteristic value of the polytropic exponent as a means to validate the isentropic assumption. An additional assumption in the analysis is that the driving pressure on the piston remains approximately constant throughout the duration of piston travel.

The equation of motion for the piston can be written as

\[ m\ddot{x}(t) + \beta \dot{x}(t) = A_p(P_{\text{exit}}(t) - P_{\text{in}}(t)) \]  

(1)

Since the processes is considered as isentropic and the mass of gas is constant

\[ P \nu^k = \text{constant} \]  

(2)

which leads to the relations
A significant simplification in the analysis results when the driving gas reservoir volume is chosen large enough so that the change in volume due to the piston travel is negligible. In this case, the upstream pressure acting on the piston ($P_{Ut}$) can be considered as constant. Substituting equations (3) and (4) into equation (1) yields

$$m\ddot{x}(t) + \beta \dot{x}(t) = A_P \left[ P_{Ut} \left[ \frac{(V_r)}{V_r + (A_p)\dot{x}(t))} \right]^{k_U} \right] \left[ P_{Df} \left[ \frac{(A_p)(L_b)}{(A_p)(L_b - x(t))} \right]^{k_D} \right]$$

If the volume change due to piston travel is negligible, equation (5) reduces to

$$m\ddot{x}(t) + \beta \dot{x}(t) = A_P \left[ P_{Ut} \left[ \frac{(A_p)(L_b)}{(A_p)(L_b - x(t))} \right]^{k_U} \right] \left[ P_{Df} \left[ \frac{(A_p)(L_b)}{(A_p)(L_b - x(t))} \right]^{k_D} \right]$$

Since the pressure within the barrel ($P_{Df}$) is a function of the length of travel of the piston, equation (6) is solved numerically using Mathematica 5.2® given substitution of appropriate values for the gas properties, initial pressures, and the apparatus geometry. Estimates for the damping factor ($\beta$) can be inferred through modeling and consideration of performance data from prior ballistic compression systems.\textsuperscript{16,17}

**DFMEA**

A design failure mode effects analysis (DFMEA) approach was used to aid in the design analysis of the ballistic compressor. This failure analysis quantified initial findings of the severity, frequency, and detection of component failures. By ranking each one of these categories and evaluating them together, high risk components were identified. The
information from the DFMEA helps allocate design and analysis resources to components that limit maximum system operation. This approach is typically used during the design stage to identify and eliminate problems before production but is also beneficial after production as a continually improved document. In this work, the DFMEA was used to identify the failure modes of components for the establishment of maximum load limits. Analysis results from identified failure modes given those pressure limits.

The DFMEA of the ballistic compressor began by examining the individual components of the ballistic compressor and how they affect its overall operation. By resolving the compressor into its individual parts, failure modes that stem from components could be traced back through their subsystems and to the ballistic compressor as a whole. This information created a link between component failure and system operation, making the DFMEA a diagnostic tool for troubleshooting incorrect operation of the ballistic compressor.

Identification of the failure modes was necessary before they could be scored in the DFMEA. This was ambiguous because the compressor has not been operated yet, so potential failures had to be postulated. Failure modes of the individual components were easiest to identify and ultimately lead to the identification of global compressor failures. Once the failure mode of a component was identified the effects of that failure were found for the subsystems and the ballistic compressor as a whole.

After the failure modes of the ballistic compressor were identified, they were rated in three different categories from one to five as shown in Table 1. The first category, severity, was based on the effects of a failure. If a failure was easily fixed and requires little cost, the severity was low; but if a failure causes extensive damage that would require high costs and time, then it received a high score. The second category was frequency of failure. This category assessed how often a failure would occur. If a failure was predicted never to occur within the design limits then it received a low score; but if it occurred during regular operation after a given amount of time, the rating was higher. The last rating scores how easily a failure is detected before it occurs. If a failure mode’s limiting factors are known and those factors are monitored, then it is unlikely that a failure would occur and the score is low.
If the failure mode required intensive disassembly and inspection then the detection score is high.

Table 1: DFMEA Rating Scale

<table>
<thead>
<tr>
<th>SEVERITY RATING</th>
<th>FREQUENCY RATING</th>
<th>DETECTION RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MINOR DISASSEMBLY AND MAINTENANCE</td>
<td>FAILS AT LOADS GREATER THAN DESIGN LIMITS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FAILURE CONDITIONS ARE KNOWN AND MONITORED</td>
</tr>
<tr>
<td>2</td>
<td>MAJOR DISASSEMBLY AND MAINTENANCE</td>
<td>FAILS NEAR DESIGN LIMITS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FAILURE TIMELINE IS WELL CALCULATED</td>
</tr>
<tr>
<td>3</td>
<td>MINOR DAMAGE TO COMPRESSOR</td>
<td>FAILS WITHIN DESIGN LIMITS AFTER MANY CYCLES</td>
</tr>
<tr>
<td></td>
<td></td>
<td>QUICK INSPECTION IS REQUIRED</td>
</tr>
<tr>
<td>4</td>
<td>MAJOR DAMAGE TO COMPRESSOR</td>
<td>FAILS WITHIN DESIGN LIMITS AFTER FEW CYCLES</td>
</tr>
<tr>
<td></td>
<td></td>
<td>INTENSIVE INSPECTION IS REQUIRED</td>
</tr>
<tr>
<td>5</td>
<td>MAJOR DAMAGE TO COMPRESSOR AND POSSIBLE BODILY HARM</td>
<td>FAILS WITHOUT WARNING AT ANY TIME</td>
</tr>
</tbody>
</table>

A sample of the ballistic compressor DFMEA is shown in Table 2. As shown in this table, once a failure mode had ratings for all three categories, it then received a Risk Priority Number (RPN). The RPN was the final value in the DFMEA. This final value was the product of severity, occurrence, and detection ratings. The RPN identified components that were relatively high risks to compressor operation.
The components that were identified as high risk through the DFMEA were the trigger toggle block, piston o-rings, and the high pressure plug. Failure of piston o-rings and the plug were high risk components because of the potential to cause major damage to the compressor. The toggle block was high risk because failure was guaranteed to occur during regular operating conditions but was not easily detectable. A complete description of the analysis of each of these three components is beyond the scope of this report. Therefore, a description of the analysis of only one component – the high pressure plug – will be described in the following sections.
COMPONENT DESIGN – THE HIGH PRESSURE PLUG

The high pressure plug (shown in Figure 5) holds the pressure transducers in the high pressure end (driven end) of the compressor. It is made out of an annealed 4340 steel alloy that is ductile enough to deform to a relatively large extent before breaking. It has buttress-type threads that are designed specifically for axial loads in only one direction.

![Figure 5: High pressure plug](image)

A basic mechanics of materials analysis was used to find the maximum pressure and the life expectancy of the plug. To verify the analytical model, a static FEA model was used to verify stresses found in the threads. The FEA model did not solve for the life expectancy of the plug and was only used to compare maximum stresses in the threads.

The high pressure plug received a high RPN because the failure load was unknown. Failure of the plug would cause massive damage to the high pressure plenum and cause pieces of the ballistic compressor to be propelled into the surrounding area. This would not only require costly repairs to the compressor but could potentially cause bodily harm. Being the first
failure point of the high pressure end, the plug is the limiting component of the compressor. Analysis of the high pressure plug gives the design limits of the plug and therefore the design limits of the compressor.

The mechanics model followed closely to an approach for power threads given by Collins. The threads were assumed to be a single cantilever beam that was wound around the plug. This cantilever beam supported the axial load created by the pressure on the plug by a single distributed load running its length. Figure 6 shows a diagram of the thread as a straight cantilever beam. The figure does not clearly show the length of the beam in order for the cross section to be larger.

Classic bending theory was applied to the cantilever beam in Figure 6. The stress state of the thread is given by the following equations:

\[
\sigma_r = \frac{M_t c}{I} - \frac{F_r}{A_{rh}} \tag{7}
\]

\[
\sigma_a = \frac{F_a}{A_r} \tag{8}
\]

\[
\tau_{ar} = \frac{VQ}{Ib_t} \tag{9}
\]

where \(\sigma_r\) is the radial stress, \(\sigma_a\) is the axial stress, and \(\tau_{ar}\) is the shear stress at the base of the threads. \(M_t, F_r, F_a,\) and \(V\) are associated with the loading condition and all other variables are geometry based.
The stress state was dependent on the location being analyzed. $M$ was dependent on the radial position. $Q$ and $c$ were dependant on the axial position. The moment reached its maximum value closest to the plug body and the bending stresses and shear stresses reached their maximums through the thickness. Based on these maximums the following three positions were then analyzed; the top, middle, and bottom of the thread closest to the plug body.

Once the stress state at each location was found it was then evaluated using a von Mises stress criteria using:\textsuperscript{19}
\[ \sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_a - \sigma_r)^2 + \sigma_d^2 + \sigma_t^2 + 6\tau^2} \] (10)

The von Mises stress criteria discards hydrostatic stresses and only takes into account the distortional stresses that cause failure. This final value for the von Mises stress was evaluated against the strength of the material to determine failure. Although the von Mises stress was the final stress it could only be found if the pressure acting on the plug was known. In order to find the pressure, the von Mises stress was set equal to the yield stress of the plug and the equations were back solved for pressure at each of the three locations. Then by comparing the results, the limiting pressure was found.

Fatigue life of the threads was then calculated using the maximum pressure found. In order to analyze the fatigue life, the von Mises stress was broken into mean \((\sigma_{vm, a})\) and alternating \((\sigma_{vm, m})\) components. The von Mises stress, that was solved for in equation (10) was the maximum stress of the plug during the initial peak pressure of each test. There are subsequent peak pressures after the initial one but their magnitude was unknown; and thus neglected for this part of the analysis, but was accounted for by an applied factor of safety. The mean and the alternating stress were both equal to half the maximum von Mises stress because the secondary peak pressures were neglected.

Using the mean and alternating stresses in a modified Goodman relation\(^{19}\), equation (11) shown below, was solved for the required fatigue strength, \(S_F\), of the plug.

\[ \frac{\sigma_{vm, a}}{S_F} + \frac{\sigma_{vm, m}}{S_{ut}} = 1 \] (11)

Once the fatigue strength was known the number of cycles, \(N\), was solved for directly using:
\[ S_F = a(N)^b \]

\[ a = \frac{(0.9S_{ut})^2}{S_e} \]  
(12)

\[ b = -\left(\frac{1}{3}\right)\log\frac{0.9S_{ut}}{S_e} \]

To verify the mechanics of materials approach, the von Mises stress given by a 30 ksi pressure on the plug was compared to the von Mises stress in a FEA model of the plug exposed to the same 30 ksi pressure: 30 ksi is the ideal operating pressure of the ballistic compressor so it was chosen to be the induced pressure.

The FEA model involved several geometry modifications. These modifications were to simplify the model and decrease the calculation time needed to solve. These assumptions are given in Table 3.

**Table 3: Geometry Changes for FEA**

<table>
<thead>
<tr>
<th>MODIFICATION</th>
<th>REASONING</th>
</tr>
</thead>
<tbody>
<tr>
<td>RADIUS REMOVED FROM TOP HEX</td>
<td>RADIUS BECOMES JAGGED AND INCREASES COMPUTATIONAL TIME</td>
</tr>
<tr>
<td>RADIUS REMOVED FROM BOTTOM</td>
<td>RADIUS BECOMES JAGGED AND INCREASES COMPUTATIONAL TIME</td>
</tr>
<tr>
<td>HELIX OF THREADS REMOVED SO THAT THREADS WERE NOT CONNECTED BUT SPACING REMAINED THE SAME</td>
<td>ELIMINATES BEGINNING AND ENDING DISCONTINUITIES OF THE THREAD THAT CAUSE POOR MESHING</td>
</tr>
<tr>
<td>2 AXIS OF SYMMETRY SO THAT ONLY A QUARTER OF THE PLUG WAS ANALYZED</td>
<td>GREATLY DECREASES COMPUTATION TIME BY REDUCING NUMBER OF ELEMENTS AND NODES</td>
</tr>
</tbody>
</table>
The model of the plug and the mesh generated for analysis is shown in Figure 7. The mesh was generated and solved multiple times to obtain converging results. Mesh refinement around only the first two threads provided faster solution times and a focus on the area to be analyzed.

Only a few simple constraints on the plug were needed. The constraints came from planes of symmetry and the contact face of the thread. Both surfaces that were created by symmetry were fixed in their normal direction as well as no rotation allowed about the two axes parallel to the surface. The final constraint on the plug prevented the load bearing thread face from moving in the axial direction.
RESULTS AND DISCUSSION

The mechanics of materials analysis of the plug threads showed that the limiting stress in the threads is at the top of the thread at the root diameter. The middle of the threads has only the shear stress and the associated axial stress which produce a von Mises stress of half that of the top of the thread. The bottom has the lowest von Mises stress due to having only compressive forces acting on it creating a close to hydrostatic stress state.

Results from the analysis at the top of the threads concluded that the ballistic compressor should not create pressures exceeding 25 ksi. At higher pressures than this, the plug threads are likely to see localized yielding. Yielding in the top of the thread causes the threads to deform making the plug difficult to remove and install.

At this limited produced pressure, the plug will be able to run for 4000 load cycles before fatigue failure. As the produced pressures decrease, fatigue life expectancy increases. Unlimited life of the plug is achieved if the operating pressure is always below 10 ksi.

Figure 8 shows that the life of the plug decreases as the pressure increases. This analysis has a factor of safety of one on the loads and fatigue life. Although the compressor’s generated pressures will be extremely consistent, a factor of safety should be applied to the number of cycles to run the plug. This analysis takes into account only the first oscillation of the piston that produces the extreme pressures to be measured. After the first peak pressure, the piston is sent back towards the driving end of the piston and pressure reduces causing yet another change in direction of movement. Once again, the piston moves towards the driving end and causes another high pressure. This secondary pressure should be much smaller than the initial pressure but it may still be significant. If the pressure is below the endurance limit pressure of 10 ksi the plug’s life will be unaffected, otherwise the life of the plug will decrease.
The multiple pressure cycles that will be seen during each test are of an unknown magnitude at this time. An accurate math model of the ballistic compressor will predict these pressures, providing insight into the needed factor of safety. Pressure readings from the conducted experiments should also give an idea as to the magnitude of the secondary pressures.

The results of the FEA show that the basic mechanics of materials approach is a valid approach to analyzing the stresses in the threads. At a produced pressure of 30 ksi the maximum stresses in the threads were calculated to be 80 ksi. Figure 9 shows the results from the finite element analysis when the plug is exposed to a 30 ksi produced pressure. The analysis shows a stress gradient of the threads that indicates that the first thread has a higher maximum stress at the top of the thread than any others do. This maximum stress is 130 ksi where the stress at the top of the next root is only 70 ksi.
Figure 10 shows a detailed view of the top of the first thread to show the distribution of von Mises stress in the plug. As can be seen, the stress decreases rapidly from the maximum of 130 ksi and by .01 inches the stress has decreased to the 80 ksi that was calculated to be seen by the mechanics of materials approach.

Although there is some discrepancy as to what the maximum stress is, this result does confirm the validity of the mechanics of materials analysis. The first reason is that although the maximum stress at the top of the first thread is 130 ksi, the rest of the threads are well below that stress. In the mechanics of materials analysis, the maximum stress was assumed to be on all threads and came out to be 80 ksi. The FEA was a static elastic only loading case but as the plug is exposed to multiple load cycles, the threads will deform slightly and share the load more evenly.
A second reason to believe that the FEA verifies the mechanics of materials assumptions is that the 130 ksi is in a region of less than .01 of an inch, which is no more than the size of one element in the analysis. This area borders a mathematical discontinuity in the corner of the thread, which is known to cause questionable results in FEA. Next to these elements the stress decreases rapidly down to near 80 ksi which matches the mechanics of materials analysis.

CONCLUSIONS AND DESIGN RECOMMENDATIONS

Once the high pressure plug is used for 4000 cycles not only will the plug need to be discarded and replaced but the female threads on the high pressure plenum will be near failure. Four thousand cycles is a long life for the compressor so immediate redesign and
reconstruction is not needed but redesign consideration should be given for the next generation design.

Higher pressure generation will most likely be desired for later testing. To reach higher pressures, there are two possible solutions the first being a material solution. The high pressure plenum and plug could both be made out of a higher strength material than the 4340 steel alloy or both could be heat treated to reach higher pressures sacrificing ductility. The second option is a redesign of the high pressure end. Redesign has several advantages over material changes and is recommended after the current design reaches its maximum life expectancy.

After reviewing the schematics and designs of other ballistic compressors, a design for much higher pressures is proposed as shown below in Figure 11.

Figure 11: High pressure plug redesign.
REFERENCES


