Research Article

Likelihood Cross-Validation Versus Least Squares Cross-Validation for Choosing the Smoothing Parameter in Kernel Home-Range Analysis

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Abstract

Fixed kernel density analysis with least squares cross-validation (LSCVh) choice of the smoothing parameter is currently recommended for home-range estimation. However, LSCVh has several drawbacks, including high variability, a tendency to undersmooth data, and multiple local minima in the LSCVh function. An alternative to LSCVh is likelihood cross-validation (CVh). We used computer simulations to compare estimated home ranges using fixed kernel density with CVh and LSCVh to true underlying distributions. Likelihood cross-validation generally performed better than LSCVh, producing estimates with better fit and less variability, and it was especially beneficial at sample sizes <~50. Because CVh is based on minimizing the Kullback-Leibler distance and LSCVh the integrated squared error, for each of these measures of discrepancy, we discussed their foundation and general use, statistical properties as they relate to home-range analysis, and the biological or practical interpretation of these statistical properties. We found 2 important problems related to computation of kernel home-range estimates, including multiple minima in the LSCVh and CVh functions and discrepancies among estimates from current home-range software. Choosing an appropriate smoothing parameter is critical when using kernel methods to estimate animal home ranges, and our study provides useful guidelines when making this decision. (JOURNAL OF WILDLIFE MANAGEMENT 70(3):641–648; 2006)

Key words

home range, kernel methods, Kullback-Leibler distance, least squares cross-validation, likelihood cross-validation, smoothing parameter, utilization distribution.

Home ranges are usually modeled from discrete observations, with the resulting estimate often described quantitatively as the utilization distribution (Kernohan et al. 2001). The utilization distribution is an estimate of the probability of an animal occurring in an area during a specified time (Worton 1995) and can be used to predict where an animal occurred but was not observed. Several methods have been introduced to estimate the utilization distribution, including the bivariate normal (Jennrich and Turner 1969), harmonic mean (Dixon and Chapman 1980), and Fourier series smoothing (Anderson 1982). The most recent and perhaps most commonly used method is kernel smoothing (Worton 1989). The kernel density at any point in space is an estimate of the amount of time spent there (Seaman and Powell 1996) and can be interpreted as the probability of an animal being in any part of its home range (Powell 2000:75). Kernel methods for home-range estimation are used widely and have been suggested as the best available nonmechanistic home-range estimator (Kernohan et al. 2001).

Kernel density estimation is a statistical technique for estimating an underlying probability density function (e.g., utilization distribution) from data (Silverman 1986). A bump (i.e., kernel) is placed over each observation, and the value of the probability density at any point in space is estimated by summing the contribution from each kernel at that point. The width of each kernel is called the smoothing parameter (h), window width, or bandwidth. The smoothing parameter must be specified and has a dramatic effect on the resulting estimate. The shape of the kernel must also be specified but has little effect on the resulting estimate compared to the choice of h (Silverman 1986). In general, Silverman (1986) suggested using any non-negative, symmetric unimodal kernel.

Various methods of choosing the appropriate value of h have been proposed based on statistical properties of the data (Silverman 1986, Jones et al. 1996). Two methods used extensively for home-range analysis include LSCVh and a method that determines the optimal h (h_{opt}) for a standard multivariate normal distribution (Worton 1989, 1995; Seaman and Powell 1996; Seaman et al. 1999). The latter, h_{opt}, also called h_{ref} in Worton (1995), is calculated

$$h_{opt} = \sqrt{\sigma^2} \times n^{-1/6}$$

where σ^2 is the average marginal covariance estimated from the x-y coordinates of the samples (i.e., locations), and n is the number of samples (Silverman 1986, Worton 1995). Because h_{opt} was developed for normal unimodal distributions, it oversmooths multimodal data (Worton 1995). Seaman et al. (1999) compared h_{opt} and LSCVh and concluded h_{opt} generally performed poorly compared to LSCVh.

Least squares cross-validation is based on minimizing the integrated square error between the estimated distribution $\hat{f}$ and the true distribution

$$\int(\hat{f}-f)^2 = \int\hat{f}^2 - 2\int\hat{f}f + \int f^2.$$

From this, Silverman (1986:48–49) derived the score function

$$\text{LSCVh} = \int\hat{f}^2 - 2n^{-1}\sum \hat{f}_{-i}(X_i)$$

where n is the number of observations, and $\hat{f}_{-i}$ is the density estimate without the data point X_i. The smoothing parameter is

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chosen by minimizing LSCVh. Worton (1995:795) provided a more-explicit version of LSCVh when a bivariate normal kernel was used

$$\mathrm{LSCVh} = \frac{1}{\pi h^2 n} + \frac{1}{4\pi h^2 n^2} \times \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\exp\left[\frac{-d_{ij}{}^2}{4h^2}\right] - 4\exp\left[\frac{-d_{ij}{}^2}{2h^2}\right]\right)$$

where d_{ij} is the distance between the ith and jth locations.

Because h_{opt} is unreasonable for multimodal distributions (i.e., most animal home ranges), LSCVh has become the most popular and recommended method for choosing h in home-range analysis (Kernohan et al. 2001). However, LSCVh has several drawbacks, including high variability (Park and Marron 1990, Jones et al. 1996), a tendency to undersmooth data (Sain et al. 1994), and multiple local minima in the LSCVh function (Sain et al. 1994). Therefore, it is important to investigate alternative methods for choosing h (Seaman et al. 1999).

One alternative to LSCVh is CVh (Silverman 1986:52–55). Unlike LSCVh that is based on minimizing the integrated squared error, CVh is based on minimizing the Kullback-Leibler distance between the true underlying and the estimated distribution. Likelihood cross-validation has general applicability beyond choosing h in kernel density estimation, having been used for both parameter estimation and model selection (e.g., Stone 1974, 1976). The CVh smoothing parameter is chosen by minimizing the score function

$$CVh = -n^{-1}\sum_{i=1}^{n}\log\hat{f}_{-i}(X_i)$$

over possible values of h (Silverman 1986:53).

Our goal was to evaluate the performance of CVh versus LSCVh methods for choosing the smoothing parameter in fixed kernel home-range analysis. In particular, we were interested in measuring the fit of estimated utilization distributions using CVh and LSCVh to a known underlying distribution. Because of the tradeoff between bias and variance in data-based bandwidth selectors (Jones et al. 1996), we were also interested in determining whether CVh or LSCVh consistently chose values for h that were too large or too small or whether either was an excessively variable estimator. Last, we investigated computational considerations such as the presence of multiple minima in the CVh and LSCVh functions and consistency of computation among current home-range software.

Methods

Simulations

We used simulated utilization distributions to represent an animal's true home range similar to Seaman et al. (1999). Simulated utilization distributions were unimodal and multimodal bivariate normal mixtures. By using simulated distributions, we were able to directly compare kernel estimates using LSCVh and CVh to a known underlying utilization distribution. We simulated 6 types of home ranges, including a circular normal and 1, 2, 4, 8, and 16 mode bivariate normal mixes. The circular normal had a mean X and Y at (0,0), standard deviations $sd_x = sd_y = 1$, and covariance $\rho = 0$. Parameter values for the bivariate normal mixes were randomly selected from uniform distributions with means (X, Y) ranging 0–20, standard deviations ranging 1.5–6, covariance ranging −1 to 1, and mixing proportions selected from 0 to 1 with the constraint that the sum of the proportions equaled 1.

For the 5 types of bivariate normal mixes, we simulated 30 realizations by selecting different parameter values for each realization. Therefore, we simulated 151 different home ranges (i.e., 150 bivariate normal mixes and 1 circular normal). To simulate animal-location data, we drew random samples of 10, 15, 20, 40, 80, 150, and 300 points. We simulated 100 replicate samples for each home range and sample size.

We calculated fixed kernel density estimates for each sample of n locations using a standard bivariate normal kernel by

$$\hat{f}(x,y) = \frac{1}{2\pi n h^2}\sum_{i=1}^{n}\exp\left(-\frac{d_i^2}{2h^2}\right)$$

where h is the smoothing parameter, and d_i is the distance of the ith observation from the x, y-coordinate. We determined values of h by numerical optimization for each method (Press et al. 1986) with a fractional precision of 0.01.

Surface Fit

We were interested in whether a better fit was obtained at a given sample size over a wide range of possible home-range shapes using LSCVh or CVh. We measured fit as the discrepancy between a simulated utilization distribution and the estimated utilization distribution. Therefore, for a given sample size, we took the mean difference between the LSCVh discrepancy and the CVh discrepancy.

We used 2 measures of discrepancy, integrated squared error (ISE) and Kullback-Leibler Distance (KL), because they are purportedly minimized by LSCVh and CVh, respectively (Silverman 1986). We assessed each measure of discrepancy over a regular grid of points bounded by the 99.9% contour of the simulated utilization distribution. We approximated integrated squared error by a discrete set of grid cells using

$$ISE(f,\hat{f}) = \sum_{i=1}^{n}\left[f(x)_i - \hat{f}(x)_i\right]^2$$

where n = the number of grid cells, x = vector of grid coordinates, $\hat{f}$ = estimated probability for the grid cell, and f = the true simulated probability for the grid cell.

We approximated Kullback-Leibler distance using

$$KL(f,\hat{f}) = \sum_{i=1}^{n} f(x)_i \times \left(\ln f(x)_i - \ln\hat{f}(x)_i\right).$$

We examined the relative improvement of surface fit due to choice of smoothing parameter versus increasing sample size by plotting median discrepancies using CVh and LSCVh across sample sizes 15–300. Because of possible differences in variability, we also plotted the 97.5% quantile at each sample size. We used medians and quantiles because the distributions of discrepancies for KL and ISE were positively skewed.

Bias

We assessed whether LSCVh or CVh generally selected too large (i.e., oversmoothed) or too small (i.e., undersmoothed) an h by

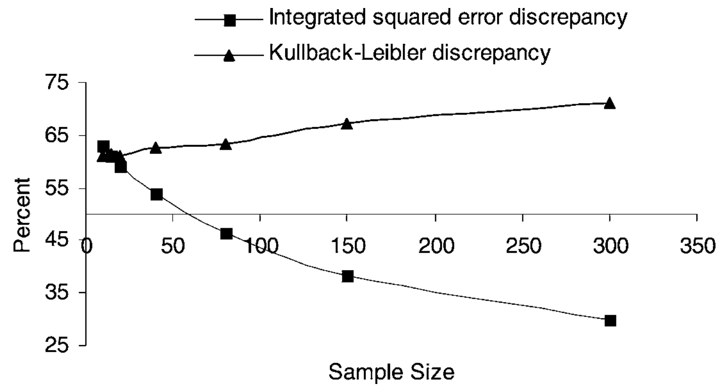

Figure 1. Percent of simulations where the estimated distribution using CVh was closer (i.e., smaller discrepancy) to the true utilization distribution than LSCVh. If both methods were similar, we would expect CVh to be closer 50% of the time.

comparing the value of the estimated smoothing parameter ($\hat{h}$) using LSCVh and CVh to the best-possible smoothing parameter for a given sample of locations. We determined the best-possible smoothing parameter (h_{best}) by finding the value of h that minimized the discrepancy between the true underlying utilization distribution and the estimated utilization distribution using ISE (ISEh) and KL (KLh). To measure whether too small or too large a value was chosen, we calculated a percent bias for each sample of locations using

$$(\hat{h} - h_{best})/h_{best} \times 100$$

where $\hat{h}$ was selected using CVh and LSCVh, and h_{best} was selected using KLh and ISEh. For this analysis, we simulated 10 realizations of bivariate normal mixes (i.e., 1, 2, 4, 8, 16 mode) and generated 50 replicate samples from each distribution. Sample sizes were 10, 15, 25, 50, 75, 100, and 300. Therefore, for each sample size, we compared 2,550 values of CVh, LSCVh, KLh, and ISEh.

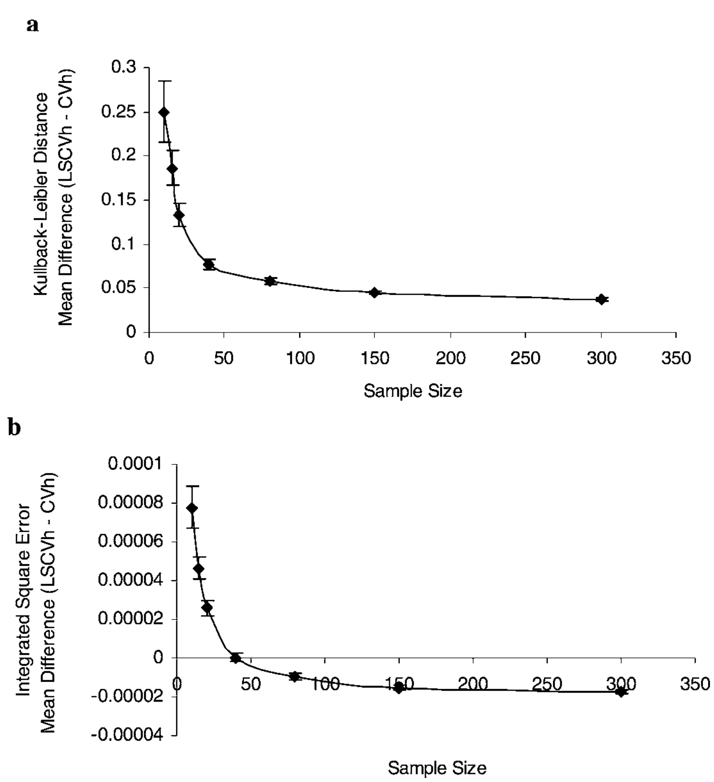

Figure 2. Mean difference in (a) Kullback-Leibler distance, and (b) integrated squared error between the true underlying distribution and the distributions estimated using CVh and LSCVh. Vertical bars represent 95% confidence intervals.

Computational Considerations

We searched for multiple minima in the LSCVh or CVh functions by generating 25 utilization distributions (i.e., 5 replicates of 1, 2, 4, 8, and 16 mode distributions) and generating 35 sets of samples from each distribution (i.e., 5 replicates for each sample of size 10, 15, 20, 40, 80, 150, and 300). These simulations generated 875 sets of samples from which we calculated h using CVh and LSCVh. Because multiple minima only become evident by viewing a plot of the CVh and LSCVh functions versus a range of h values, we narrowed our search by taking the 10 samples with the largest percent difference between h chosen by CVh and LSCVh. We then graphed the CVh and LSCVh functions over values of h.

To determine consistency among home-range software in computing LSCVh, we used the test data associated with KERNELHR and calculated the 90% contour, with LSCVh smoothing, using our algorithm and 3 common home-range programs, Animal Movement, KERNEL HR, and HOME RANGE.

Results

Kullback-Leibler Discrepancy

When KL was used as the measure of discrepancy, home-range estimates using CVh fit better, had less bias, and were less variable than estimates using LSCVh. If CVh and LSCVh performed similarly, we expected CVh to produce a better fit in half of the simulations, and LSCVh to produce a better fit in the other half. However, we found a better fit was obtained using CVh in 64.0% of the simulations (Fig. 1). Mean KL between the estimated and true distribution was smaller when CVh was used than LSCVh (Fig. 2a). Plots of median KL revealed little difference between CVh and LSCVh when compared to the effect of sample size (Fig. 3a). However, the 97.5% quantiles used to depict rare but possible estimates showed that if LSCVh was used, sample sizes would have to increase dramatically to obtain the same fit as when CVh was used (Fig. 4a). When comparing CVh and LSCVh to the best possible h found by minimizing KL, we found CVh oversmoothed the location data by 2–5%, whereas LSCVh oversmoothed at sample sizes $<\sim 20$ and undersmoothed the data by as much as 10–15% at sample sizes $>\sim 50$ (Fig. 5a).

Integrated Squared Error Discrepancy

When ISE was used as the measure of discrepancy, performance (i.e., fit, variability, bias) of CVh versus LSCVh was dependent on sample size with CVh generally outperforming LSCVh at sample sizes $<\sim 50$, whereas LSCVh generally outperformed CVh at sample sizes $>\sim 50$. A better fit was obtained in more simulations when CVh was used at sample sizes $<\sim 60$, while a better fit was obtained in a greater percentage of simulations using LSCVh when sample sizes were $>\sim 60$ (Fig. 1). Mean ISE between the

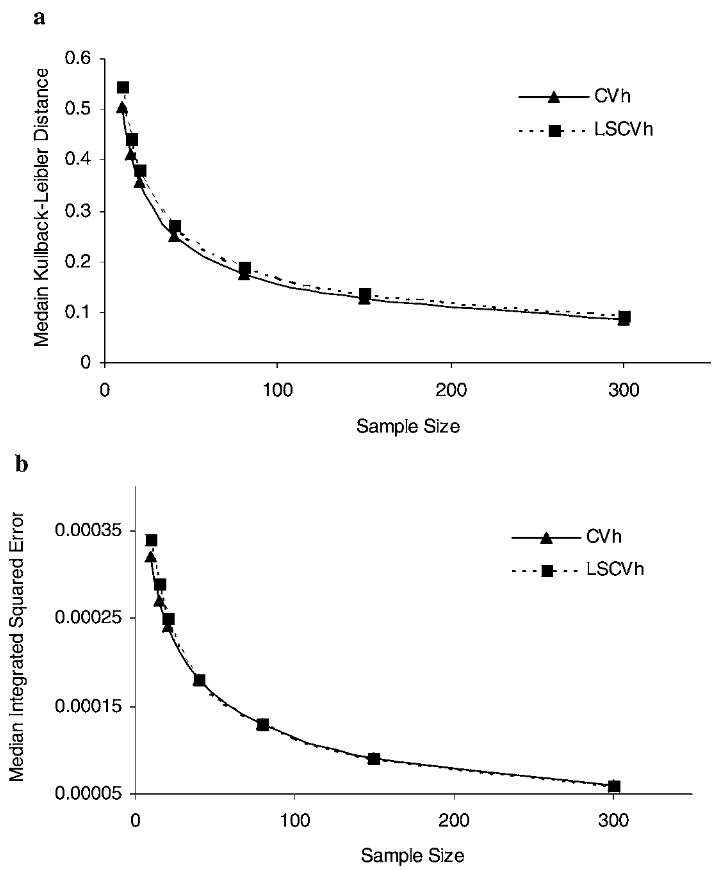

Figure 3. (a) Median Kullback-Leibler distance, and (b) integrated squared error between known underlying distributions and distributions estimated using CVh and LSCVh. Median values were used to determine the relative influence of sample size vs. method for choosing the smoothing parameter.

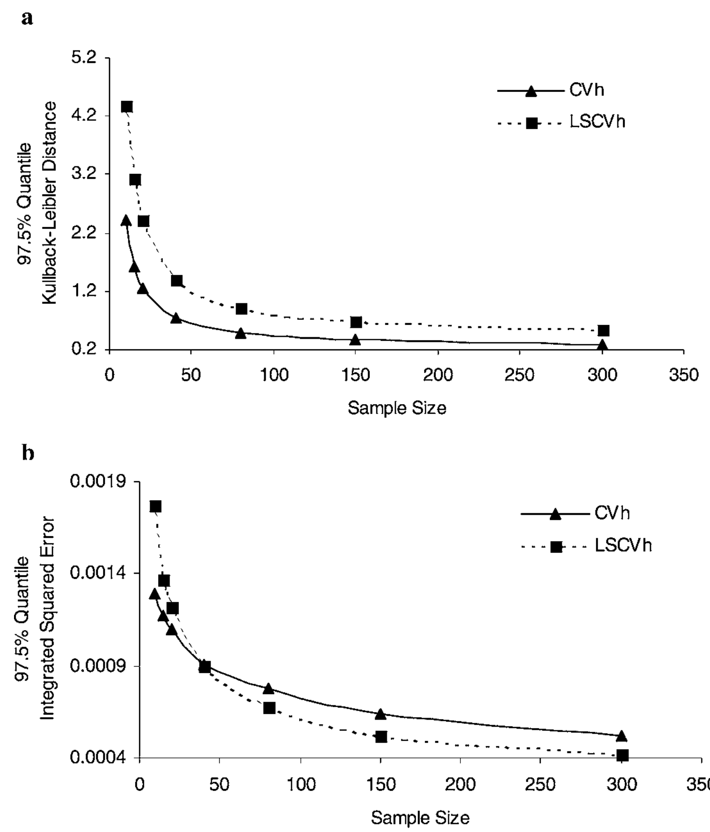

Figure 4. The 97.5% quantile ranges for (a) Kullback-Leibler distance, and (b) integrated squared error between known underlying distributions and distributions estimated using CVh and LSCVh. Quantiles were used to determine the relative influence of sample size vs. method for choosing the smoothing parameter.

estimated and true distribution was smaller when CVh was used than LSCVh at sample sizes $<\sim 40$, whereas LSCVh produced better estimates at sample sizes $>\sim 40$ (Fig. 2b). There was little difference in median ISE between CVh and LSCVh when compared to the effect of sample size (Fig. 3b). However, similar to when KL was used as the measure of discrepancy, larger differences became evident in the 97.5% quantiles used to depict rare but possible combinations of utilization distributions and locations. In these cases, choice of smoothing parameter was important when compared to the effect of sample size, especially at larger sample sizes (Fig. 4b). When comparing the CVh and LSCVh to the best possible h found by minimizing ISE, both CVh and LSCVh oversmoothed the location data (Fig. 5b).

Computational Considerations

Plots of the LSCVh and CVh functions versus h revealed that 2 of the 10 sets of sample locations produced multiple minima in the LSCVh function, and 1 of the 10 sets of samples produced multiple minima in the CVh function (Fig. 6). In both cases, our algorithms for CVh and LSCVh found the global minima.

Home-range boundaries were substantially different between program Animal Movement and the other programs (Fig. 7). The only consistency was between our program and HOME RANGE. KERNELHR selected LSCVh in each dimension (i.e., x and y), and Animal Movement substantially oversmoothed the data.

Discussion

Worton (1995) suggested that choosing the appropriate level of smoothing is the most important factor when using the kernel method for home-range analysis. Estimated distributions can vary greatly depending on which method is used to select the smoothing parameter (Fig. 8). Thus far, 2 methods for choosing the smoothing parameter in kernel home-range analysis have been investigated, with LSCVh performing better than h_{opt} (Seaman and Powell 1996, Seaman et al. 1999). We found CVh generally outperformed LSCVh and was especially beneficial at sample sizes $<\sim 50$ (Table 1).

We generally recommend using CVh for choosing the smoothing parameter in home-range analysis, but in some cases, performance depended on which measure of discrepancy was used. Linhart and Zucchini (1986:16) discussed both ISE and KL but only suggested selecting the measure of discrepancy, "...that is appropriate for the problem in hand." Therefore, it is important to understand the statistical properties of each measure of discrepancy and the biological or practical interpretation of these properties as they relate to home-range estimation.

Kullback-Leibler distance was derived from information theory (Kullback 1959) and has been described as the "information lost when $\hat{f}(x)$ is used to approximate $f(x)$" (Burnham and Anderson 1998:37). Kullback-Leibler distance is widely accepted as a

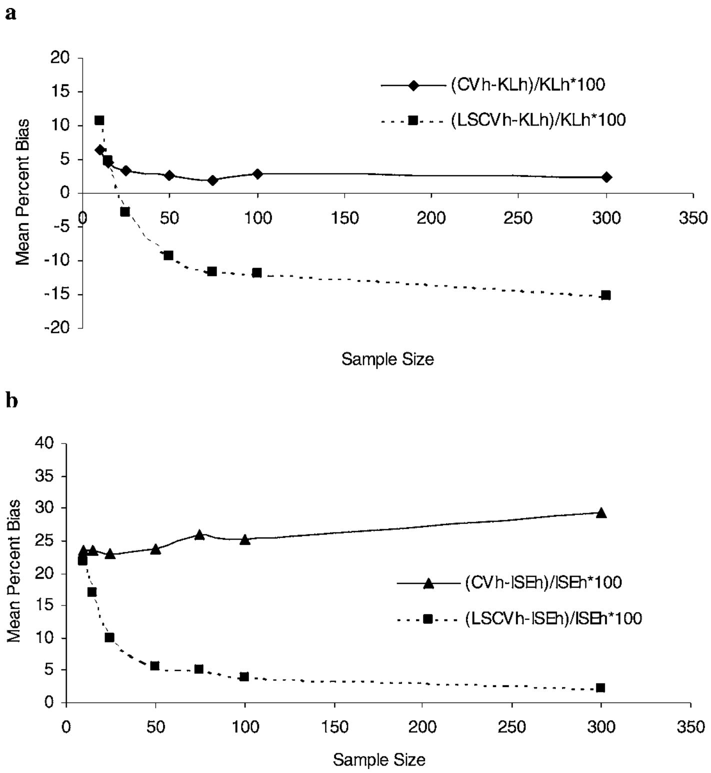

Figure 5. Percent relative bias of CVh and LSCVh when compared to the best smoothing parameter found by minimizing the (a) Kullback-Leibler distance, and (b) integrated squared error between the true and estimated utilization distributions.

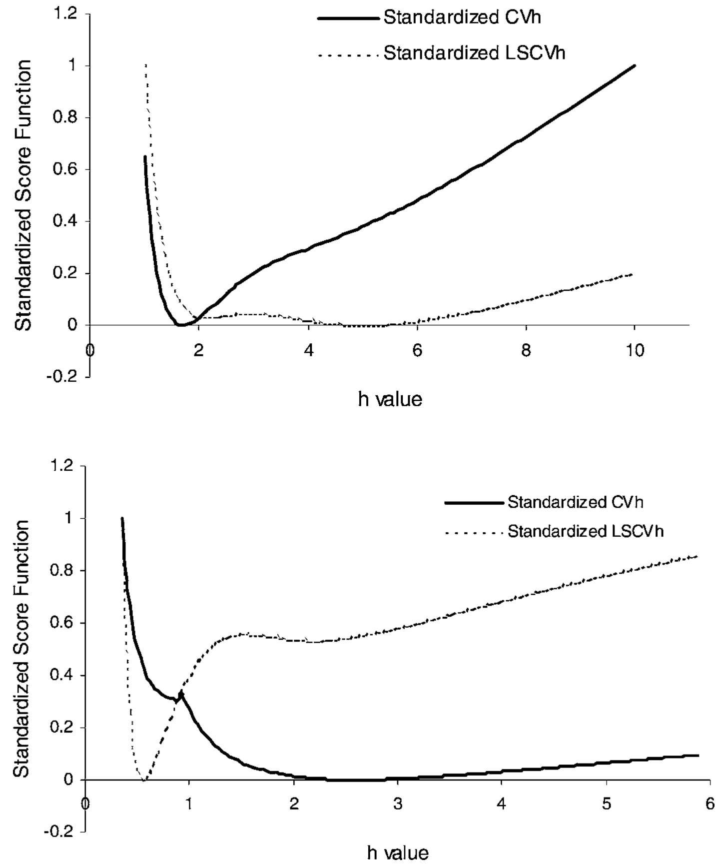

Figure 6. Plots of standardized (i.e., 0 to 1) LSCVh and CVh functions over a range of smoothing parameter (*h*) values. Plots illustrate presence of multiple minima in the functions.

measure of discrepancy because of its connection with information theory and has increasingly been suggested as the basis for model selection in wildlife science (Burnham and Anderson 1998; Anderson et al. 2000, 2001; Garton et al. 2005). The technique of squaring errors was originally used to provide unbiased parameter estimates because of its connection to the variance of the estimator (Berger 1980). It is likely a popular measure of discrepancy because of familiarity gained from its relationship to classical least-squares theory and because calculations are relatively straightforward and simple (Berger 1980).

We suggest 2 major differences between KL and ISE as measures of discrepancy important for home-range analysis. First is their sensitivity to extreme values. Because ISE is based on squared differences and KL is based on log differences, ISE penalizes large discrepancies more severely than KL (Fig. 9). The second difference is their sensitivity to the location of the discrepancy in the sample space. Integrated squared error treats discrepancies equally across the sample space, but because the log differences of KL are then multiplied by $f(x)$, KL penalizes discrepancies in high use areas more severely than low use areas.

Because of the sensitivity to outliers, minimizing ISE is susceptible to giving too much weight to clusters of locations or missing locations uncharacteristic of the true utilization distribution. This property of squared error has caused some to suggest that it may be overly sensitive to outliers (Berger 1980). Any problems estimating the home range due to uncharacteristic locations would be magnified by minimizing ISE. Practical aspects to obtaining locations on animals that are likely to result in these uncharacteristic locations include serial autocorrelation (Swihart and Slade 1995) and biased observation rates (Johnson et al. 1998).

Table 1. Summary conclusions[a] of the best method[b] for choosing the smoothing parameter in fixed kernel home-range analysis.

	Sample size $\leq$ 50			Sample size $>$50			
	Surface fit	**Bias**	**Variablilty**	**Surface fit**	**Bias**	**Variability**	**Overall conclusion**
KL	CVh	CVh	CVh	CVh	CVh	CVh	*CVh*
ISE	CVh	LSCVh	CVh	LSCVh	LSCVh	LSCVh	*LSCVh*
Overall conclusion	CVh	none	CVh	none	none	none	

[a] Conclusions were based on 3 measures of performance, including surface fit, bias, and variability, and 2 measures of discrepancy, including Kullback-Leibler distance (KL) and integrated squared error (ISE).
[b] The 2 methods evaluated for choosing the smoothing parameter included likelihood cross-validation (CVh) and least squares cross-validation (LSCVh).

Because KL penalizes discrepancies in high use areas more severely than low use areas, we suggest home-range analysis should seek to minimize KL when interest is in the high use areas of the utilization distribution. There are several practical aspects of home-range estimation that suggests researchers should concentrate on high use areas. Among these are statistical problems associated with estimating the tails of a distribution as well as ecological reasons. For example, Seaman et al. (1999) recommended that future home-range studies focus on areas of high use because of unreliable kernel estimates in the outer contours of the utilization distribution. Samuel et al. (1985) and Powell (2000) suggested that estimating the core home range (i.e., most heavily used areas) is an important component in understanding the ecological factors affecting an animal's space use. And recently, Marzluff et al. (2004) suggested a new analytic technique that used areas of high use estimated from kernel density to determine resource preferences for Steller's jays (*Cyanocitta stelleri*).

If the goal is to minimize KL between the true and the estimated distribution, then CVh performed better than LSCVh at all sample sizes and across all measures of performance (i.e., surface fit, variance, bias). Although CVh did perform better than LSCVh at a given sample size, our results supported the conclusions of Seaman et al. (1999) that the fit of estimated distributions using the kernel method was usually more sensitive to sample size than choice of smoothing parameter. For most utilization distributions and most samples of locations from these distributions, the choice of smoothing parameter had little effect on the resulting estimate when compared to increasing sample size. Differences between smoothing methods became evident only for certain combinations of utilization distributions and samples when the estimates differed substantially even when compared to the effect of sample size.

If the goal is to minimize ISE between the true and the estimated distribution, we found LSCVh produced estimated distributions with smaller integrated square errors than CVh but only at larger (i.e., $>\sim 50$) sample sizes. Seaman et al. (1999) reported very poor performance of LSCVh at sample sizes ranging 10–30. We found CVh produced better estimates at these smaller sample sizes even when ISE was used as the measure of discrepancy.

Bias versus Variance

Data-based methods for choosing the smoothing parameter seem to have an intrinsic tradeoff between minimizing bias versus variance (Jones et al. 1996). A biased method would consistently produce estimated distributions that either oversmooth or undersmooth the data. Highly variable methods may be unbiased over multiple samples from a single distribution, but a single sample may produce an estimate far from the true distribution.

The tendency of an estimator to select biased values of h will affect home-range estimates differently depending on whether h is biased too small (i.e., undersmoothed) or to large (i.e., oversmoothed). Undersmoothed data would tend to show structure where there is none and would result in smaller and more disjunct home-range contours. Oversmoothed data would result in larger, more contiguous home ranges. Our results generally supported the conclusions of Sain et al. (1994) that LSCVh is likely to undersmooth data especially when compared to CVh. In addition

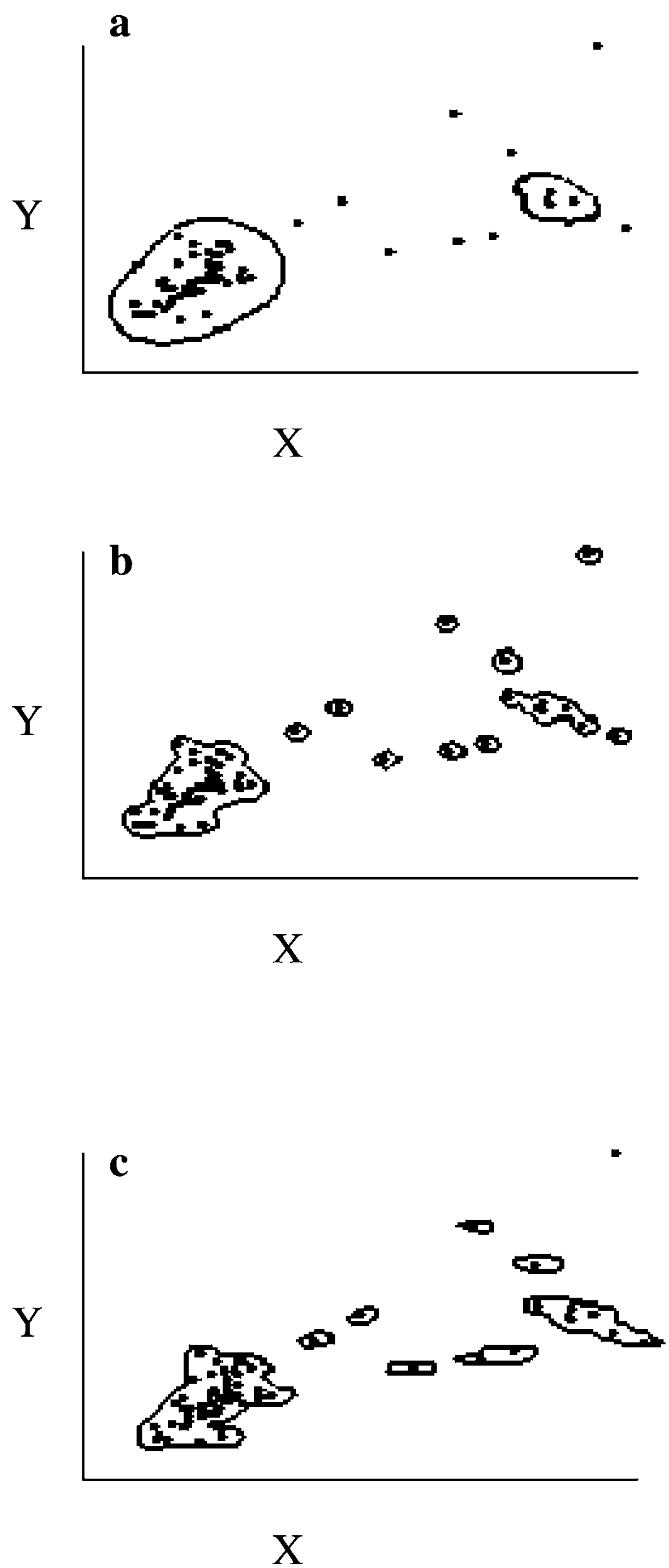

Figure 7. Fixed kernel estimates of the 90% contour using 4 home-range programs: (a) Animal Movement, (b) our algorithm and program HOME RANGE, and (c) KERNEL HR. For all estimates and programs, LSCVh was selected to choose the smoothing parameter.

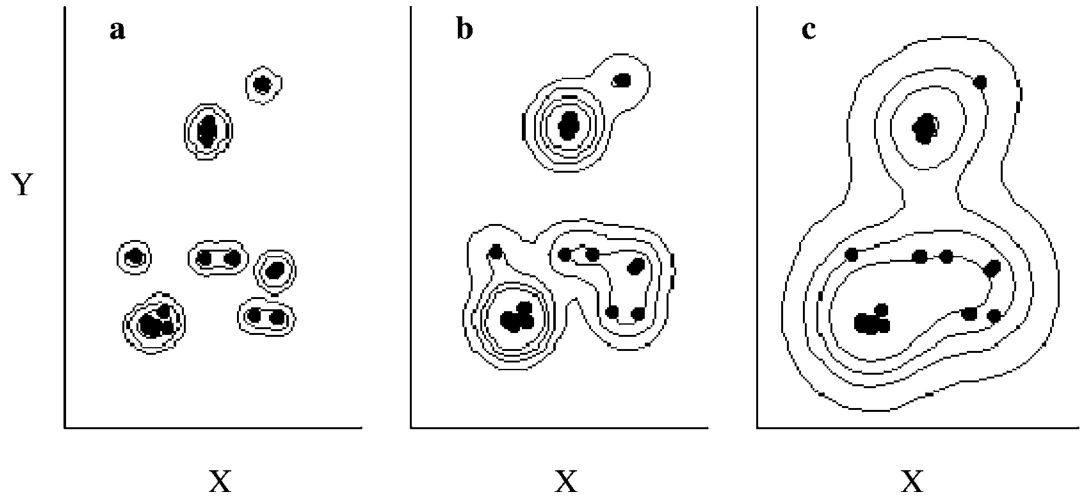

Figure 8. Fixed kernel density contours for an adult female ocelot (*Felis pardalis*) studied in 1996 using (a) LSCVh, (b) CVh, and (c) reference methods for choosing the smoothing parameter.

to our simulation results, the kernel estimate using LSCVh appeared to undersmooth both example data sets (i.e., Figs. 7, 8), resulting in many disjunct contours.

Because wildlife researchers must estimate the home range from a single sample of locations, a highly variable method would likely be more problematic than a method that is slightly biased. Many authors consider LSCVh to be an extreme case in the bias versus variance tradeoff producing unbiased but highly variable estimates (Park and Marron 1990, Jones et al. 1996, Kernohan et al. 2000).

Computational Considerations

Values for LSCVh and CVh must be found by minimizing their respective functions numerically (Silverman 1986, Worton 1995). When a numerical technique finds a minima, it can either be global (i.e., the true lowest point in the function) or local (i.e., the lowest point only within a certain neighborhood of the function). The simplex method we used as well as the other methods described in Press et al. (1986) are not guaranteed to find the global minima. Therefore, methods for choosing the smoothing parameter producing frequent local minima would present a problem for home-range analyses. Least squares cross-validation is known to produce local minima (Park and Marron 1990), and we found multiple minima in both CVh (1 of 10) and LSCVh (2 of 10) functions. Because of the small number of functions we graphed, we were unable to determine whether LSCVh is more likely to produce local minima than CVh.

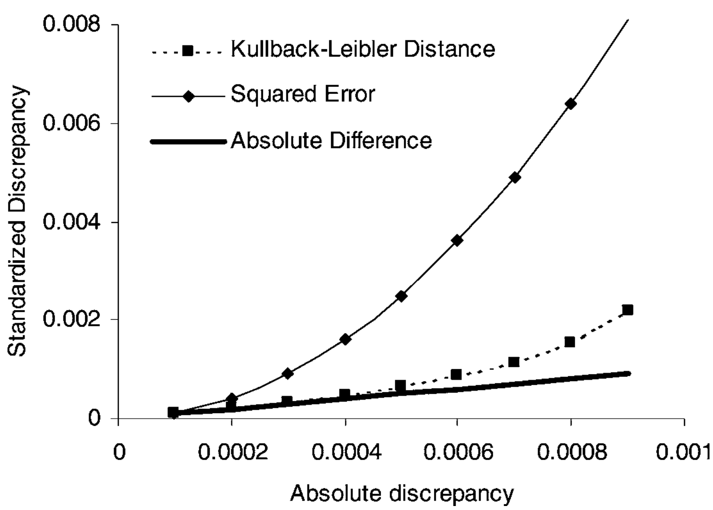

Figure 9. Relationship between squared difference and log difference to absolute difference used to illustrate the weighting of outliers for squared error and Kullback-Leibler distance relative to the absolute difference.

We were surprised to find such a large discrepancy in the estimated home range computed by the Animal Movement program when LSCVh smoothing was selected. The 90% contour calculated by Animal Movement was 1.4 times larger than that computed by our program and program HOME RANGE. Consistency among methods for analyzing data is imperative for making comparisons among different studies. This consistency can be corrupted if different home-range programs supposedly using the same method for choosing the smoothing parameter are in fact not using the same algorithm.

Management Implications

It is unlikely that any method for selecting the smoothing parameter for kernel home-range analysis will be a panacea for inadequate sample size. However, due to animal mortality, lost contact, and/or logistics, studies are often limited in the number of locations that can be collected and the question arises, "for a given sample size, how should I choose the smoothing parameter?" Regardless of research objectives, if sample sizes are $<\sim 50$, we recommend using CVh. We also recommend using CVh if researchers are more concerned with obtaining good estimates in high use areas or if researchers seek a conservative approach to dealing with lack of independence among locations or mitigating location acquisition bias. We recommend using LSCVh if researchers are interested in estimates of the tails of the utilization distribution but only if sample size is sufficiently large. Future research should determine the frequency of local minima in the CVh or LSCVh functions using data from actual field studies. Until then, we encourage researchers to plot the LSCVh and/or CVh functions over a range of potential values for the smoothing parameter.

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