Uncertainty in Economic Analysis

CE 215

Risk and Uncertainty

- We've already mentioned that interest rates reflect the risk involved in an investment.
- Risk and uncertainty can affect an investment in a variety of ways.
  - In some situations, it is helpful to distinguish between risk and uncertainty; it won't be necessary in our discussion.
- We will discuss tools that have been developed to help quantify the role of risk and uncertainty in an economic analysis.

Sources of Uncertainty

- Inaccuracy in the estimates used in the study.
  - Income estimates,
  - Operating expense estimates.
    - Uncertainty about both decreases as experience is gained.
- Uncertainty due to the type of business and future health of the economy.
  - Mining operations are risky because metal prices have always been volatile.
  - Construction is very sensitive to interest rates.

Sources of Uncertainty (cont.)

- Type of physical plant and equipment.
  - General-purpose machinery is usually fairly stable in price.
  - Specialized equipment depends on the demand for the particular operation provided by the machine, and will be more uncertain.
- Length of the study period.
  - The length of the study period is usually tied to the life-time of the equipment, which is rarely known in advance.

Methods for Dealing with Uncertainty

- Breakeven analysis
  - Determines the value required for a key parameter (.e.g, M.A.R.R.) in order for the project to show a profit.
- Sensitivity analysis
  - Calculates changes in, say, net present worth due to changes in various parameters.
- Optimistic-pessimistic estimation
- Risk-Adjusted M.A.R.R.
- Monte-Carlo simulation (and other probabilistic methods.)

Monte Carlo Simulation

- Monte Carlo simulation involves.
  - Randomly selecting values for the various parameters based on the probabilities assigned to them.
    - Incomes, operating expenses, interest rates, etc.
  - Determining the outcome of the study for that particular combination of parameters.
  - Repeating the process for a very large number of iterations.
  - Calculating the likelihood of a particular outcome.
    - E.g., what proportion of the cases produced a profit?
    - What was the expected profit?

Advantages and Disadvantages

- Monte Carlo simulation can provide a great deal of insight into very complicated situations involving uncertainty and probability.
- It is relatively easy to put together a Monte Carlo simulation with Mathcad or other specialized software.
- But the results are meaningless unless the probabilities are based on sound information.

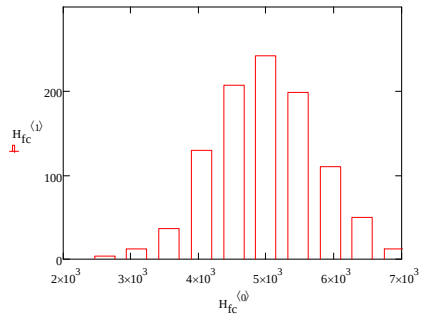
Probability

- The probability of an event occurring is described by a number between zero and one.
  - Zero: Complete impossibility
  - One: Absolute certainty.
- There is also a rigorous set of rules for the manipulation of probabilities.
- However, the *definition* of a probability is the subject of some debate.

Frequentist

- A probability describes the frequency with which an event has occurred in the past.
- For example, concrete manufacturers are required to take several samples of every batch of concrete they make and cure and test those samples for strength. As a result they have hundreds of data points for the strength their standard mixes.

Histogram



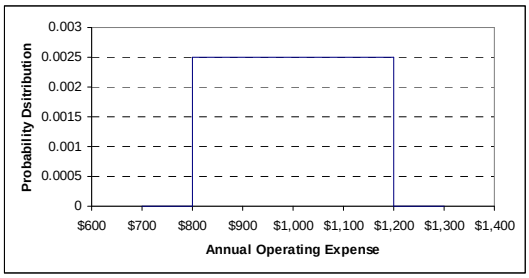
Bayesian

- A probability can describe one's a priori knowledge about the likelihood of an event.
  - Sometimes this is an "intuitive" estimate of the likelihood.

Economic Example

- The operating expenses for Structure N are equally likely to be anywhere between $800 and $1200/year.

Uniform Probability Distribution

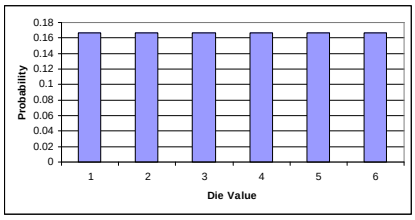


Probability Distributions

- Roughly speaking, a probability distribution describes the likelihood that a random variable will take a certain value.
- In general, probability distributions can be classified as:
  - Discrete distributions, or
  - Continuous distributions.

Discrete Probability Distributions

- The probability that a fair die will land on any of its six faces is one-in-six.



Continuous Probability Distributions

- Continuous distributions assign a probability rate to a continuous range of values.

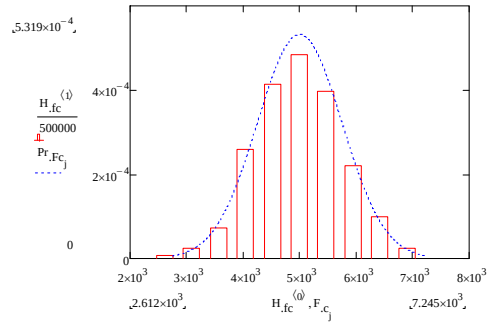
Normal Probability Distribution

- The normal probability distribution is the so-called "bell curve."
- The center or peak of the bell curve is known as the mean, average, or expected value of the distribution.
- The spread or scatter of the bell around the mean is described by the standard deviation σ.
  - 95.45% of the time, the value of the random variable will be within $\pm 2\sigma$ of the mean.

Example

- The expected value for the concrete strength in Slide 10 is 5000 psi.
- The standard deviation 750 psi
- We are 95% confident that it will be between 3500 psi and 6500 psi.
  - I.e., $2\sigma = 1500$ psi; therefore between, 5000 ± 1500

Normal Distribution



Previous Example

- We could assume the following random variables in the previous economic comparison problem to perform a Monte Carlo simulation.

Structure M	Parameters	
First Cost	$12,000	deterministic
Interest rate	μ = 15%	σ = 2.5%
Useful Life	μ = 10 years	σ = 1.5 years
Annual O&M	uniform between	$1,900 and $2500

Structure N

- For Structure N, we could assume

Structure N	Parameters	
First Cost	$40,000	deterministic
Interest rate	μ = 15%	σ = 2.5%
Salvage Value	μ = $10,000	σ = $4000
Useful Life	μ = 25 years	σ = 3.75 years
Annual O&M	uniform between	$900 and $1100

- Since the life spans of both alternatives are unknown, we must use the annual cost method to ensure the same study period for both alternatives.

Comparison

- Based on these parameters, a Monte Carlo simulation was run with 10,000 iterations. The average annual cost for each alternative was
  - Structure M: $4627/year
  - Structure N: $7178/year
- Which favors Structure M.
- Of the 10,000 simulations only three iterations showed Structure N to be preferred over M.
  - The probability that Structure M is preferred is 0.9997.

Effects of Uncertainty

- Another simulation was run assuming the mean values of all the random variables remained the same but Structure M had more scatter in its random variables.

Structure M	Parameters	
First Cost	$12,000	deterministic
Interest rate	μ = 15%	σ = 2.5%
Useful Life	μ = 10 years	σ = 3.0 years
Annual O&M	uniform between	$1,400 and $3000

Results

- A Monte Carlo simulation was run with 10,000 iterations. The average annual cost for each alternative was
  - Structure M: $4776/year
  - Structure N: $7178/year
- The mean cost for Structure M has increased slightly (Structure N was not changed), but
- Structure M is still the preferred alternative.

Results (cont.)

- Of the 10,000 simulations now 189 iterations showed Structure N to be preferred over M.
  - The probability that Structure M is preferred has decreased to 0.9811.
- Increasing the uncertainty increases the probability that Structure M will cost more than Structure N.

Sensitivity to Interest Rate

- Returning to the original parameters for Structure M but increasing the uncertainty on the interest rate by letting $\sigma = 5\%$ leads to average annual costs for each alternative of
  - Structure M: $4642/year
  - Structure N: $7222/year
    - Which are still nearly the same,
- But the probability that Structure M is preferred has decreased to 0.9654.
  - Indicating that the analysis is sensitive to the interest rate.

Summary

- There are many sources of uncertainty in the economic predictions we make.
- The uncertainty can be accounted for in a variety of different ways.
- Monte Carlo simulations account for the uncertainty about many of the parameters.
- The quality of the results depends on the quality of the probabilistic modeling.