Discussion of Chi Square Goodness of Fit Test from:

MEASURING & MONITORING *Plant Populations*

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To test statistically which of these three years is different, we can compare each of the pairs of means using two-sided *t* tests. However, we must modify the *P* value used for the ANOVA for each *t* test performed, by dividing the *P* value used for the overall ANOVA by the number of *t* tests to be performed. In this case, our overall *P* value is 0.05. If we want to compare all three mean values (mean 1 with mean 2, mean 2 with mean 3, and mean 1 with mean 3), we divide the overall *P* value by 3. Our new threshold *P* value for each of these tests is thus 0.05/3 = 0.0167.

When we do these pairwise *t* tests we come up with the following statistics:

Years Compared	DF	t-value	P
1989 vs. 1991	58	1.8771	0.0655
1989 vs. 1993	58	2.6234	0.0111
1991 vs. 1993	58	0.7340	0.4659

Only the *P* value of 0.0111 for the years 1989 vs. 1993 is less than our threshold of 0.0167. We therefore conclude that there has been a significant change between those two years (but not between any of the other pairs of years). This procedure is called the Bonferroni *t* test and works reasonably well when the number of comparisons are few (Glantz 1992). As the number of comparisons increases above 8 to 10, however, the value of *t* required to conclude a difference exists becomes much larger than it needs to be, and the method becomes overly conservative (Glantz 1992). Other multiple comparison tests are less conservative and preferable in these cases. Three such tests are the Student-Neuman-Keuls test, the Scheffe test, and the Tukey test, some or all of which are performed by many microcomputer statistical packages. There is debate over which of these is the preferable test; see Zar (1996:218) for a discussion of this. Another such test, the Duncan multiple-range test, is not conservative enough and should be avoided (Day and Quinn 1989).

3. Testing the difference between two proportions (independent samples): the chi-square test

The chi-square test is used to analyze frequency data when individual quadrats are the sampling units and point cover data when individual points are the sampling units. (Even though cover is expressed as a percentage, cover data are appropriately analyzed by calculating mean values, except when individual points are the sampling units.) If the frequency data are collected on more than one species, each species is usually analyzed separately. Another alternative is to lump species into functional groups, such as annual graminoids, and analyze each of the groups.

a. 2 x 2 contingency table to compare two years

To estimate the frequency of a plant species in two separate years, we've taken two independent random samples of 400 quadrats each. In each of these quadrats the species is either present or absent. For analysis we put these data into a 2 x 2 contingency table, as follows:

	1990	1994	Totals
Present	123 (0.31)	157 (0.39)	280 (0.35)
Absent	277 (0.69)	243 (0.61)	520 (0.65)
Totals	400 (1.00)	400 (1.00)	800 (1.00)

The numbers in parentheses are frequencies of occurrence in 1990 and 1994, and, in the last column, for both years combined. The chi-square test is conducted on actual numbers of quadrats, *not* percentages. The chi-square test is not appropriately applied to percentage data.

Just as for the *t* test and ANOVA, we must formulate a null hypothesis. Our null hypothesis states that the true proportion of the target plant species (the proportion we would get if we placed all of the quadrats of our particular size that could be placed in the sampled area) is the same in both years. This is equivalent to saying there has been no change in the proportion of the key species from 1990 to 1994.

Before we can calculate the chi-square statistic we must determine the values that would be expected in the event there was no difference between years. The total frequencies in the right hand column are used for this purpose. Thus, in both 1990 and 1994, 0.35 x 400 quadrats, or 140 quadrats, would be expected to contain the species, and in both 1990 and 1994, 0.65 x 400 quadrats, or 260 quadrats, would be expected to not contain the species. The following table shows these expected values:

	1990	1994	Totals
Present	140	140	280
Absent	260	260	520
Totals	400	400	800

Now we can compute the chi square statistic as follows:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where: χ^2 is the chi square statistic.
Σ = summation symbol.
O = Number observed.
E = Number expected.

Applying this formula to our example we get:

$$\chi^2 = \frac{(123\text{-}140)^2}{140} + \frac{(277\text{-}260)^2}{260} + \frac{(157\text{-}140)^2}{140} + \frac{(243\text{-}260)^2}{260}$$

$$= 2.06 + 1.11 + 2.06 + 1.11 = 6.34$$

We then compare the chi-square value of 6.34 to a table of critical values of the chi-square statistic (see table in Appendix 5) to see if our chi-square value is sufficiently large to be significant.[4] The *P* value we have selected for our threshold before sampling began is 0.10. Now we need to determine the number of degrees of freedom. For a contingency table, the number of degrees of freedom, *v*, is given by:

$$v = (r - 1)(c - 1)$$

Where: r = number of rows in the contingency table.
c = number of columns in the contingency table.

[4] If we've sampled more than 5% of the population we should apply the finite population correction factor to the chi-square test. This increases the chi-square statistic and gives us greater power to detect change. See Section F of this chapter for instructions on how to do this.

For a 2 x 2 table v = (2-1)(2-1) = 1. Therefore, we enter the table at degrees of freedom = 1, and the *P* threshold of 0.10. The critical chi-square value from the table is 2.706. Since our value of 6.34 is larger than the critical value, we reject the null hypothesis of no difference in frequency of the plant species and conclude there has been an increase in its frequency. We would also report our calculated *P* value, which we could interpolate from the chi-square table, but could obtain more easily through a statistics program. For this example, the *P* value is 0.012.

Statistics texts differ on whether to use the chi-square statistic as calculated above in the special case of a 2 x 2 contingency table. Some authors (e.g., Zar 1996) state this value over-estimates the chi-square statistic and recommend that the Yates correction for continuity be applied to the formula as follows:

$$\chi^2 = \sum \frac{(|O - E| - \frac{1}{2})^2}{E}$$

Other authors (e.g., Steel and Torrie 1980; Sokal and Rohlf 1981) point out that the Yates correction is overly conservative and recommend against its use. Salzer (unpub. data) has shown through repeated sampling of simulated frequency data sets that the Yates correction is not needed. Munro and Page (1993) point out that the Yates correction is required only when the expected frequency of one of the cells in the table is less than 5. With the proper selection of quadrat size (see Chapters 7 and 8) this should rarely occur in plant frequency monitoring studies. Accordingly, we recommend calculating χ^2 without the Yates correction.

Statistical packages for personal computers calculate the chi-square statistic and give exact *P* values. For 2 x 2 tables, however, you should be aware of whether the program applies the Yates correction factor. Some programs, such as SYSTAT, give both the uncorrected and corrected chi-square values. Other programs such as STATMOST give only the corrected chi-square value. Because you want the uncorrected chi-square value, this presents a problem for 2 x 2 tables; no program applies the correction to larger tables.

b. Larger contingency tables for more than two years

When you have more than two years of data to compare, you can increase the size of the contingency table accordingly. For three years of data, you would use a 2 x 3 table; for four years, a 2 x 4 table; and so on. The chi-square statistic is computed according to the directions given above for a 2 x 2 table. Also, when using a table of critical values you need to calculate the degrees of freedom according to the directions given above. Because there will never be more than two rows (present and absent), the number of degrees of freedom will always be 1 fewer than the number of years. Thus, for a 2 x 3 table there, are 2 degrees of freedom; for a 2 x 4 table, there are 3 degrees of freedom; and so on.

It is important to realize that, just as for an ANOVA, a significant result in a chi-square table larger than 2 x 2 is an indication only that the frequency in at least one year is significantly different than expected. Which year(s) are different cannot be determined without further testing. This can be done by subdividing the larger contingency table into smaller 2 x 2 tables. Because this involves making multiple comparisons on the same set of data, however, the Bonferroni adjustment to the *P* value must be made before running these tests (directions on the use of the Bonferroni adjustment are given under Section D.2, above).

c. Contingency tables for analysis of point cover data

If you've collected cover data using a point intercept method and if the sampling units are the individual points (as opposed to transects or point frames), the data can be arrayed into a contingency table and analyzed using the chi-square statistic. The procedure is the same as for the frequency data described above (except you may wish to change "present" and "absent" to "hits" and "misses"). Just as for frequency data, analysis is done on a species-by-species basis or on functional groups of species. Total plant cover or any other type of cover (e.g., litter or bare ground) can also be analyzed this way.

E. Permanent Quadrats, Transects, and Points: the Use of Paired-Sample Significance Tests

1. Independent vs. paired samples

Thus far we've discussed significance tests for independent samples. Independent samples are ones in which different sets of sampling units are selected randomly (or systematically with random starts) in each year of measurement. Now we'll consider the case in which sampling units are randomly selected only in the first year of measurement. The sampling units are then permanently marked, and the same (or at least approximately the same) sampling units are measured in the subsequent monitoring year.

Because the two samples are no longer independent (the second sample is dependent upon the first), the use of the independent-sample significance tests discussed previously is not appropriate. Instead, a paired-sample significance test is used.

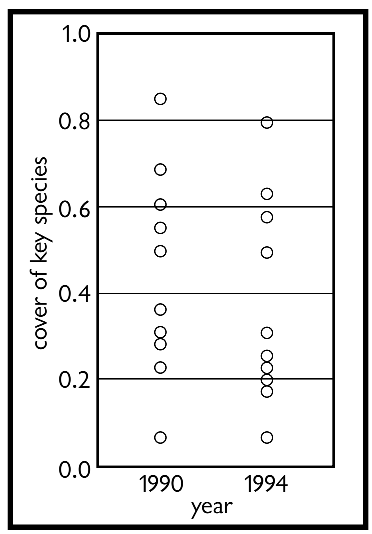

FIGURE 11.10. Cover estimates (in percent) for 1990 and 1994. Data from 10 permanent transects of 50 points each.

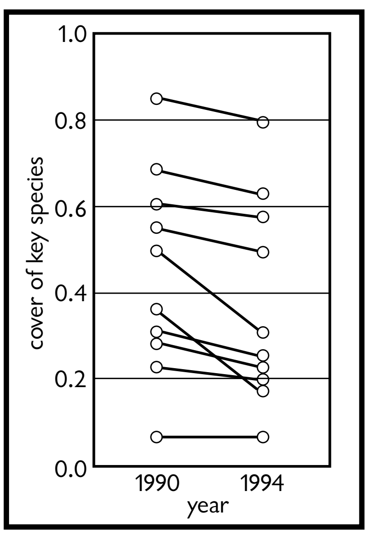

FIGURE 11.11. Cover estimates (in percent) for 1990 and 1994. Same data as in Figure 11.10 but by focusing on changes in each permanent transect, you can detect a change that was masked by the variability between transects obvious in Figure 11.10.

2. Paired *t* test: use it when you can

The appropriate significance test for two paired samples is the paired *t* test (unless the samples are proportions, in which case McNemar's test, discussed below, is the test to use). There is often a great advantage to testing change using a paired *t* test rather than an independent-sample *t* test. This is because the paired *t* test is often much more powerful in detecting change. To see why this is so, let's examine Figures 11.10 and 11.11 (adapted from Glantz 1992).

The data depicted in Figure 11.10 are cover estimates (in percent) for 10 transects in 1990 and 1994. The estimates were derived by placing 50 points at systematic intervals along a line (transect), recording whether the target plant species was present or absent, and reporting a total cover for the species on the transect. For

Critical Values of the Chi-Square Distribution

ν	α = 0.999	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005	0.001
1	0.000	0.000	0.000	0.001	0.004	0.016	0.102	0.455	1.323	2.706	3.841	5.024	6.635	7.879	10.828
2	0.002	0.010	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210	10.597	13.816
3	0.024	0.072	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	9.348	11.345	12.838	16.266
4	0.091	0.207	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.143	13.277	14.860	18.467
5	0.210	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.070	12.833	15.086	16.750	20.515
6	0.381	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.645	12.592	14.449	16.812	18.548	22.458
7	0.599	0.989	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.017	14.067	16.013	18.475	20.278	24.322
8	0.857	1.344	1.646	2.180	2.733	3.490	5.071	7.344	10.219	13.362	15.507	17.535	20.090	21.955	26.124
9	1.152	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.389	14.684	16.919	19.023	21.666	23.589	27.877
10	1.479	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.549	15.987	18.307	20.483	23.209	25.188	29.588
11	1.834	2.603	3.053	3.816	4.575	5.578	7.584	10.341	13.701	17.275	19.675	21.920	24.725	26.757	31.264
12	2.214	3.074	3.571	4.404	5.226	6.304	8.438	11.340	14.845	18.549	21.026	23.337	26.217	28.300	32.909
13	2.617	3.565	4.107	5.009	5.892	7.042	9.299	12.340	15.984	19.812	22.362	24.736	27.688	29.819	34.528
14	3.041	4.075	4.660	5.629	6.571	7.790	10.165	13.339	17.117	21.064	23.685	26.119	29.141	31.319	36.123
15	3.483	4.601	5.229	6.262	7.261	8.547	11.037	14.339	18.245	22.307	24.996	27.488	30.578	32.801	37.697
16	3.942	5.142	5.812	6.908	7.962	9.312	11.912	15.338	19.369	23.542	26.296	28.845	32.000	34.267	39.252
17	4.416	5.697	6.408	7.564	8.672	10.085	12.792	16.338	20.489	24.769	27.587	30.191	33.409	35.718	40.790
18	4.905	6.265	7.015	8.231	9.390	10.865	13.675	17.338	21.605	25.989	28.869	31.526	34.805	37.156	42.312
19	5.407	6.844	7.633	8.907	10.117	11.651	14.562	18.338	22.718	27.204	30.144	32.852	36.191	38.582	43.820
20	5.921	7.434	8.260	9.591	10.851	12.443	15.452	19.337	23.828	28.412	31.410	34.170	37.566	39.997	45.315
21	6.447	8.034	8.897	10.283	11.591	13.240	16.344	20.337	24.935	29.615	32.671	35.479	38.932	41.401	46.797
22	6.983	8.643	9.542	10.982	12.338	14.041	17.240	21.337	26.039	30.813	33.924	36.781	40.289	42.796	48.268
23	7.529	9.260	10.196	11.689	13.091	14.848	18.137	22.337	27.141	32.007	35.172	38.076	41.638	44.181	49.728
24	8.085	9.886	10.856	12.401	13.848	15.659	19.037	23.337	28.241	33.196	36.415	39.364	42.980	45.559	51.179
25	8.649	10.520	11.524	13.120	14.611	16.473	19.939	24.337	29.339	34.382	37.652	40.646	44.314	46.928	52.620
26	9.222	11.160	12.198	13.844	15.379	17.292	20.843	25.336	30.435	35.563	38.885	41.923	45.642	48.290	54.052
27	9.803	11.808	12.879	14.573	16.151	18.114	21.749	26.336	31.528	36.741	40.113	43.195	46.963	49.645	55.476
28	10.391	12.461	13.565	15.308	16.928	18.939	22.657	27.336	32.620	37.916	41.337	44.461	48.278	50.993	56.892
29	10.986	13.121	14.256	16.047	17.708	19.768	23.567	28.336	33.711	39.087	42.557	45.722	49.588	52.336	58.301
30	11.588	13.787	14.953	16.791	18.493	20.599	24.478	29.336	34.800	40.256	43.773	46.979	50.892	53.672	59.703
31	12.196	14.458	15.655	17.539	19.281	21.434	25.390	30.336	35.887	41.422	44.985	48.232	52.191	55.003	61.098
32	12.811	15.134	16.362	18.291	20.072	22.271	26.304	31.336	36.973	42.585	46.194	49.480	53.486	56.328	62.487
33	13.431	15.815	17.074	19.047	20.867	23.110	27.219	32.336	38.058	43.745	47.400	50.725	54.776	57.648	63.870
34	14.057	16.501	17.789	19.806	21.664	23.952	28.136	33.336	39.141	44.903	48.602	51.966	56.061	58.964	65.247
35	14.688	17.192	18.509	20.569	22.465	24.797	29.054	34.336	40.223	46.059	49.802	53.203	57.342	60.275	66.619
36	15.324	17.887	19.233	21.336	23.269	25.643	29.973	35.336	41.304	47.212	50.998	54.437	58.619	61.581	67.985
37	15.965	18.586	19.960	22.106	24.075	26.492	30.893	36.336	42.383	48.363	52.192	55.668	59.833	62.883	69.346
38	16.611	19.289	20.691	22.878	24.884	27.343	31.815	37.335	43.462	49.513	53.384	56.896	61.162	64.181	70.703
39	17.262	19.996	21.426	23.654	25.695	28.196	32.737	38.335	44.539	50.660	54.572	58.120	62.428	65.476	72.055
40	17.916	20.707	22.164	24.433	26.509	29.051	33.660	39.335	45.616	51.805	55.758	59.342	63.691	66.766	73.402
41	18.576	21.421	22.906	25.215	27.326	29.907	34.585	40.335	46.692	52.949	56.942	60.561	64.950	68.053	74.745
42	19.239	22.138	23.650	25.999	28.144	30.765	35.510	41.335	47.766	54.090	58.124	61.777	66.206	69.336	76.084
43	19.906	22.859	24.398	26.785	28.965	31.625	36.436	42.335	48.840	55.230	59.304	62.990	67.459	70.616	77.419
44	20.576	23.584	25.148	27.575	29.787	32.487	37.363	43.335	49.913	56.369	60.481	64.201	68.710	71.893	78.750
45	21.251	24.311	25.901	28.366	30.612	33.350	38.291	44.335	50.985	57.505	61.656	65.410	69.957	73.166	80.077
46	21.929	25.041	26.657	29.160	31.439	34.215	39.220	45.335	52.056	58.641	62.830	66.617	71.201	74.437	81.400
47	22.610	25.775	27.416	29.956	32.268	35.081	40.149	46.335	53.127	59.774	64.001	67.821	72.443	75.704	82.720
48	23.295	26.511	28.177	30.755	33.098	35.949	41.079	47.335	54.196	60.907	65.171	69.023	73.683	76.969	84.037
49	23.983	27.249	28.941	31.555	33.930	36.818	42.010	48.335	55.265	62.038	66.339	70.222	74.919	78.231	85.351
50	24.674	27.991	29.707	32.357	34.764	37.689	42.942	49.335	56.334	63.167	67.505	71.420	76.154	79.490	86.661

Critical Values of the Chi-Square Distribution **(cont.)**

ν | α =	0.999	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005	0.001
51	25.368	28.735	30.475	33.162	35.600	38.560	43.874	50.335	57.401	64.295	68.669	72.616	77.386	80.747	87.968
52	26.065	29.481	31.246	33.968	36.437	39.433	44.808	51.335	58.468	65.422	69.832	73.810	78.616	82.001	89.272
53	26.765	30.230	32.019	34.776	37.276	40.308	45.741	52.335	59.534	66.548	70.393	75.002	79.843	83.253	90.573
54	27.468	30.981	32.793	35.586	38.116	41.183	46.676	53.335	60.600	67.673	72.153	76.192	81.069	84.502	91.872
55	28.173	31.735	33.570	36.398	38.958	42.060	47.610	54.335	61.665	68.796	73.311	77.380	82.292	85.749	93.168
56	28.881	32.491	34.350	37.212	39.801	42.937	48.546	55.335	62.729	69.919	74.468	78.567	83.513	86.994	94.461
57	29.592	33.248	35.131	38.027	40.646	43.816	49.482	56.335	63.793	71.040	75.624	79.752	84.733	88.236	95.751
58	30.305	34.008	35.913	38.844	41.492	44.696	50.419	57.335	64.857	72.160	76.778	80.936	85.950	89.477	97.039
59	31.021	34.770	36.698	39.662	42.339	45.577	51.356	58.335	65.919	73.279	77.931	82.117	87.166	90.715	98.324
60	31.738	35.535	37.485	40.482	43.188	46.459	52.294	59.335	66.981	74.397	79.082	83.298	88.379	91.952	99.607
61	32.459	36.301	38.273	41.303	44.038	47.342	53.232	60.335	68.043	75.514	80.232	84.476	89.591	93.186	100.888
62	33.181	37.068	39.063	42.126	44.889	48.226	54.171	61.335	69.104	76.630	81.381	85.654	90.802	94.419	102.166
63	33.906	37.838	39.855	42.950	45.741	49.111	55.110	62.335	70.165	77.745	82.529	86.830	92.010	95.649	103.442
64	34.633	38.610	40.649	43.776	46.595	49.996	56.050	63.335	71.225	78.860	83.675	88.004	93.217	96.878	104.716
65	35.362	39.383	41.444	44.603	47.450	50.883	56.990	64.335	72.285	79.973	84.821	89.177	94.422	98.105	105.988
66	36.093	40.158	42.240	45.431	48.305	51.770	57.931	65.335	73.344	81.085	85.965	90.349	95.626	99.330	107.258
67	36.826	40.935	43.038	46.261	49.162	52.659	58.872	66.335	74.403	82.197	87.108	91.519	96.828	100.554	108.526
68	37.561	41.713	43.838	47.092	50.020	53.548	59.814	67.335	75.461	83.308	88.250	92.689	98.028	101.776	109.791
69	38.298	42.494	44.639	47.924	50.879	54.438	60.756	68.334	76.519	84.418	89.391	93.856	99.228	102.996	111.055
70	39.036	43.275	45.442	48.758	51.739	55.329	61.698	69.334	77.577	85.527	90.531	95.023	100.425	104.215	112.317
71	39.777	44.058	46.246	49.592	52.600	56.221	62.641	70.334	78.634	86.635	91.670	96.189	101.621	105.432	113.577
72	40.520	44.843	47.051	50.428	53.462	57.113	63.585	71.334	79.690	87.743	92.808	97.353	102.816	106.648	114.835
73	41.264	45.629	47.858	51.265	54.325	58.006	64.528	72.334	80.747	88.850	93.945	98.516	104.010	107.862	116.092
74	42.010	46.417	48.666	52.103	55.189	58.900	65.472	73.334	81.803	89.956	95.081	99.678	105.202	109.074	117.346
75	42.757	47.206	49.475	52.942	56.054	59.795	66.417	74.334	82.858	91.061	96.217	100.839	106.393	110.286	118.599
76	43.507	47.997	50.286	53.782	56.920	60.690	67.362	75.334	83.913	92.166	97.351	101.999	107.583	111.495	119.850
77	44.258	48.788	51.097	54.623	57.786	61.586	68.307	76.334	84.968	93.270	98.484	103.158	108.771	112.704	121.100
78	45.010	49.582	51.910	55.466	58.654	62.483	69.252	77.334	86.022	94.374	99.617	104.316	109.958	113.911	122.348
79	45.764	50.376	52.725	56.309	59.522	63.380	70.198	78.334	87.077	95.476	100.749	105.473	111.144	115.117	123.594
80	46.520	51.172	53.540	57.153	60.391	64.278	71.145	79.334	88.130	96.578	101.879	106.629	112.329	116.321	124.839
81	47.277	51.969	54.357	57.998	61.261	65.176	72.091	80.334	89.184	97.680	103.010	107.783	113.512	117.524	126.083
82	48.036	52.767	55.174	58.845	62.132	66.076	73.038	81.334	90.237	98.780	104.139	108.937	114.695	118.726	127.324
83	48.796	53.567	55.993	59.692	63.004	66.976	73.985	82.334	91.289	99.880	105.267	110.090	115.876	119.927	128.565
84	49.557	54.368	56.813	60.540	63.876	67.876	74.933	83.334	92.342	100.980	106.395	111.242	117.057	121.126	129.804
85	50.320	55.170	57.634	61.389	64.749	68.777	75.881	84.334	93.394	102.079	107.522	112.393	118.236	122.325	131.041
86	51.085	55.973	58.456	62.239	65.623	69.679	76.829	85.334	94.446	103.177	108.648	113.544	119.414	123.522	132.277
87	51.850	56.777	59.279	63.089	66.498	70.581	77.777	86.334	95.497	104.275	109.773	114.693	120.591	124.718	133.512
88	52.617	57.582	60.103	63.941	67.373	71.484	78.726	87.334	96.548	105.372	110.898	115.841	121.767	125.913	134.745
89	53.386	58.389	60.928	64.793	68.249	72.387	79.675	88.334	97.599	106.469	112.022	116.989	122.942	127.106	135.978
90	54.155	59.196	61.754	65.647	69.126	73.291	80.625	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208
91	54.926	60.005	62.581	66.501	70.003	74.196	81.574	90.334	99.700	108.661	114.268	119.282	125.289	129.491	138.438
92	55.698	60.815	63.409	67.356	70.882	75.100	82.524	91.334	100.750	109.756	115.390	120.427	126.462	130.681	139.666
93	56.472	61.625	64.238	68.211	71.760	76.006	83.474	92.334	101.800	110.850	116.511	121.571	127.633	131.871	140.893
94	57.246	62.437	65.068	69.068	72.640	76.912	84.425	93.334	102.850	111.944	117.632	122.715	128.803	133.059	142.119
95	58.022	63.250	65.898	69.925	73.520	77.818	85.376	94.334	103.899	113.038	118.752	123.858	129.973	134.247	143.344
96	58.799	64.063	66.730	70.783	74.401	78.725	86.327	95.334	104.948	114.131	119.871	125.000	131.141	135.433	144.567
97	59.577	64.878	67.562	71.642	75.282	79.633	87.278	96.334	105.997	115.223	120.930	126.141	132.309	136.619	145.789
98	60.356	65.694	68.396	72.501	76.164	80.541	88.229	97.334	107.045	116.315	122.108	127.282	133.476	137.803	147.010
99	61.137	66.510	69.230	73.361	77.046	81.449	89.181	98.334	108.093	117.407	123.225	128.422	134.642	138.987	148.230
100	61.918	67.328	70.065	74.222	77.929	82.358	90.133	99.334	109.141	118.498	124.342	129.561	135.807	140.169	149.449

Critical Values of the Chi-Square Distribution **(cont.)**

ν | α =	0.999	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005	0.001
101	62.701	68.146	70.901	75.083	78.813	83.267	91.085	100.334	110.189	119.589	125.458	130.700	136.971	141.351	150.667
102	63.484	68.965	71.737	75.946	79.697	84.177	92.038	101.334	111.236	120.679	126.574	131.838	138.134	142.532	151.884
103	64.269	69.785	72.575	76.809	80.582	85.088	92.991	102.334	112.284	121.769	127.689	132.975	139.297	143.712	153.099
104	65.055	70.606	73.413	77.672	81.468	85.998	93.944	103.334	113.331	122.858	128.804	134.111	140.459	144.891	154.314
105	65.841	71.428	74.252	78.536	82.354	86.909	94.897	104.334	114.378	123.947	129.918	135.247	141.620	146.070	155.528
106	66.629	72.251	75.092	79.401	83.240	87.821	95.850	105.334	115.424	125.035	131.031	136.382	142.780	147.247	156.740
107	67.418	73.075	75.932	80.267	84.127	88.733	96.804	106.334	116.471	126.123	132.144	137.517	143.940	148.424	157.952
108	68.207	73.899	76.774	81.133	85.015	89.645	97.758	107.334	117.517	127.211	133.257	138.651	145.099	149.593	159.162
109	68.998	74.724	77.616	82.000	85.903	90.558	98.712	108.334	118.563	128.298	134.369	139.784	146.257	150.774	160.372
110	69.790	75.550	78.458	82.867	86.792	91.471	99.666	109.334	119.608	129.385	135.480	140.917	147.414	151.948	161.581
111	70.582	76.377	79.302	83.735	87.681	92.385	100.620	110.334	120.654	130.472	136.591	142.049	148.571	153.122	162.788
112	71.376	77.204	80.146	84.604	88.570	93.299	101.575	111.334	121.699	131.558	137.701	143.180	149.727	154.294	163.995
113	72.170	78.033	80.991	85.473	89.461	94.213	102.530	112.334	122.744	132.643	138.811	144.311	150.882	155.466	165.201
114	72.965	78.862	81.836	86.342	90.351	95.128	103.485	113.334	123.789	133.729	139.921	145.441	152.037	156.637	166.406
115	73.761	79.692	82.682	87.213	91.242	96.043	104.440	114.334	124.834	134.813	141.030	146.571	153.191	157.808	167.610
116	74.558	80.522	83.529	88.084	92.134	96.958	105.396	115.334	125.878	135.898	142.138	147.700	154.344	158.977	168.813
117	75.356	81.353	84.377	88.955	93.026	97.874	106.352	116.334	126.923	136.982	143.246	148.829	155.496	160.146	170.016
118	76.155	82.185	85.225	89.827	93.918	98.790	107.307	117.334	127.967	138.066	144.354	149.957	156.648	161.314	171.217
119	76.955	83.018	86.074	90.700	94.811	99.707	108.263	118.334	129.011	139.149	145.461	151.084	157.800	162.481	172.418
120	77.755	83.852	86.923	91.573	95.705	100.624	109.220	119.334	130.055	140.233	146.567	152.211	158.950	163.648	173.617
121	78.557	84.686	87.773	92.446	96.598	101.541	110.176	120.334	131.098	141.315	147.674	153.338	160.100	164.814	174.816
122	79.359	85.521	88.624	93.320	97.493	102.458	111.133	121.334	132.142	142.398	148.779	154.464	161.250	165.980	176.014
123	80.162	86.356	89.475	94.195	98.387	103.376	112.089	122.334	133.185	143.480	149.885	155.589	162.398	167.144	177.212
124	80.965	87.192	90.327	95.070	99.283	104.295	113.046	123.334	134.228	144.562	150.989	156.714	163.546	168.308	178.408
125	81.770	88.029	91.180	95.946	100.178	105.213	114.004	124.334	135.271	145.643	152.094	157.839	164.694	169.471	179.604
126	82.575	88.866	92.033	96.822	101.074	106.132	114.961	125.334	136.313	146.724	153.198	158.962	165.841	170.634	180.799
127	83.381	89.704	92.887	97.698	101.971	107.051	115.918	126.334	137.356	147.805	154.302	160.086	166.987	171.796	181.993
128	84.188	90.543	93.741	98.576	102.867	107.971	116.876	127.334	138.398	148.885	155.405	161.209	168.133	172.957	183.186
129	84.996	91.383	94.596	99.453	103.765	108.891	117.834	128.334	139.440	149.965	156.508	162.331	169.278	174.118	184.379
130	85.804	92.223	95.451	100.331	104.662	109.811	118.792	129.334	140.482	151.045	157.610	163.453	170.423	175.278	185.571
131	86.613	93.063	96.307	101.210	105.560	110.732	119.750	130.334	141.524	152.125	158.712	164.575	171.567	176.438	186.762
132	87.423	93.904	97.163	102.089	106.459	111.652	120.708	131.334	142.566	153.204	159.814	165.696	172.711	177.597	187.953
133	88.233	94.746	98.021	102.968	107.357	112.573	121.667	132.334	143.608	154.283	160.915	166.816	173.854	178.755	189.142
134	89.044	95.588	98.878	103.848	108.257	113.495	122.625	133.334	144.649	155.361	162.016	167.936	174.996	179.913	190.331
135	89.856	96.431	99.736	104.729	109.156	114.417	123.584	134.334	145.690	156.440	163.116	169.056	176.138	181.070	191.520
136	90.669	97.275	100.595	105.609	110.056	115.338	124.543	135.334	146.731	157.518	164.216	170.175	177.280	182.226	192.707
137	91.482	98.119	101.454	106.491	110.956	116.261	125.502	136.334	147.772	158.595	165.316	171.294	178.421	183.382	193.894
138	92.296	98.964	102.314	107.372	111.857	117.183	126.461	137.334	148.813	159.673	166.415	172.412	179.561	184.538	195.080
139	93.111	99.809	103.174	108.254	112.758	118.106	127.421	138.334	149.854	160.750	167.514	173.530	180.701	185.693	196.266
140	93.926	100.655	104.034	109.137	113.659	119.029	128.380	139.334	150.894	161.827	168.613	174.648	181.840	186.847	197.451