## Stochastic Population Growth

## Stochastic Population

 Growth- Dennis, Munholland, Scott (1991)
- Stable but variable mean rate of change
- Constant long term trend



## Trend

$$
\text { - Let } r_{\mathrm{i}}=\ln \left(N_{\mathrm{i}} / N_{\mathrm{i}-1}\right)
$$

- $\quad \tau_{\mathrm{i}}=t_{\mathrm{i}}-t_{\mathrm{i}-1}$
- Trend = Mean (change / time)
- Estimate of trend $=\left[\right.$ Mean $r_{\mathrm{i}}$ ]
- $\mu=\Sigma r_{\mathrm{i}} / \Sigma \tau_{\mathrm{i}}=\ln \left(n_{\mathrm{q}} / n_{0}\right) / t_{\mathrm{q}}-t_{0}$
- $=\ln (838 / 39,012) / 27$
- $=-0.1422$

Variance of rate of change

- $\sigma^{2}=\Sigma\left\{\left(1 / \tau_{i}\right)\left[r_{\mathrm{i}}-\mu \tau_{\mathrm{i}}\right]^{2}\right\} /\{q-1\}$
- $\quad=0.645$


## Predicting Future

 Population- We cannot predict exactly what the population size will be.
- But future population sizes are not all equally likely.
- We must describe them with a distribution.


## Predicting Future Population



Predicting Future Population

- Mean $=N_{0} \exp \left(\mu+0.5 \sigma^{2}\right) t$
- Median $=N_{0} e^{\mu t}$

Predicting Future Population
$\mathrm{N}_{\mathrm{o}}$

Extinction


- 1.0, for $\mu<0.0$
- $p=$
- $\exp \left(-2 \mu x_{\mathrm{d}} / \sigma^{2}\right)$, for $\mu>0.0$
- where $x_{\mathrm{d}}=\ln \left(N_{\mathrm{q}} / N_{\mathrm{e}}\right)$,
- $\quad N_{\mathrm{e}}=$ Extinction Threshold
- $\quad x_{d}=\ln (840 / 30)$


## Time to extinction

- Conditional mean time to extinction $=$

■ $T=x_{d} /|\mu|$

- $=\ln (840 / 30) / 0.142=23.4 \mathrm{yrs}$

Is there no hope?
$-p=\left\{\begin{array}{l}1.0, \text { for } \mu<0.0 \\ -\exp \left(-2 \mu x_{\mathrm{d}} / \sigma^{2}\right), \text { for } \mu>0.0\end{array}\right.$ cooperated so that trend was flat ( $\mu=0.001$ )

- then $p_{\text {extinction }}=0.99$
- for $\mu=0.03 \quad p_{\text {exinction }}=0.75$
- for $\mu=0.10 \quad p_{\text {extinction }}=0.0002$


## Will these predictions be

 correct?- Assumptions:
- Long-term trend continues (constant $\mu$ )
- Variance around that trend is normally distributed.
- No density-dependence


## Estimation

- You can calculate the values for this model of stochastic population growth on a calculator by calculating the natural log of lambda at each time period and the interval between surveys. Or you can use a program by Oz Garton called STOCHMVP.

■ STOCHMVP will calculate the probability of persistence for a metapopulation consisting of a specified number of
identical populations too.

- Or you can simulate the population using a program such as RAMAS Metapopw.exe

Minimum Viable Population

- Definition: "A minimum viable population for any given species in any given habitat is the smallest isolated population having a 99\% chance of remaining extant for 1000 years despite the forseeable effects of demographic, environmental and genetic stochasticity and natural catastrophes." (Shaffer 1981)


## Genetic Stochasticity

- Deleterious genetic effects of very small numbers of breeders
- Inbreeding depression, founder effect
- Loss of heterogeneity leads to accumulation of lethal genes \& effects
- $N_{\mathrm{e}}$ - Effective population size used to calculate this

Demographic Stochasticity

- Random effects of small numbers of individuals in the population.
- Example: 10 animals with 50\% survival
- On average 5 will be alive next year
- but with only 10 individuals its like flipping a coin.
- Could be 6 or 4, or 7 or 3, or even 10 or 0 next year.

Environmental Stochasticity

- Random changes in birth and death rates produce random fluctuations in $r_{t}$ or $\lambda_{t}$
- Random fluctuations in physical factors such as weather (particularly temperatures and rainfall/snow at, key seasons) oner random impacts onlb and d due to parasites, predators, disease,...


## Natural Catastrophes

- Extreme cases of environmental stochasticity ( excessively high mortality)
- Examples:
- Puerto Rican Parrot populations were decimated by Hurrican Hugo a decade ago
- Black-footed Ferret almost wiped out by parvovirus


## Estimating MVP

- Dennis et al (1991) showed that Probability of reaching extinction threshold ( $N$ ) within time $t$ is equal to probability of reaching threshold times conditional probability of reaching it within time $t$ :
- $=\operatorname{Prob}\left(N<N_{\mathrm{e}}\right) \operatorname{Prob}\left(T<t \mid N<N_{\mathrm{e}}\right)$
- = p cdf (cumulative distribution function of cond. time to extinction)


## Persistence and MVP

- Probability of persistence to time $t$
- = 1 - $p$ cdf
- Calculate the Minimum Viable Population size by finding the population size which produces the desired probability of persistence using STOCHMVP.

Results for Lemhi Spring Chinook

- Using Shaffer's original definition of a minimum viable population as one which provides a $99 \%$ chance of persistence for 1000 years and assuming persistence means staying above 30 spawners,
- then if the population were stabilized ( $\mu=0.0001$ )
- MVP $=2.4 \times 10^{23}$

Results for Endangered Species

## - Species MVP

- Yellowstone Grizzly Bear 3800
- Palila $6.1 x$ $10^{14}$
- Laysan Finch
$3.4 x$ $10^{18}$
- Whooping Crane 20,800
- Kirtland's Warbler 34,150
- California Condor $2.9 \times 10^{10}$
- Puerto Rican Parrot 14,600


## What's wrong?

- These are impossible.
- Species may never have been this abundant.
- But these species probably existed as a series of populations constituting one or more metapopulations.
- Assuming completely independent populations in terms of environmental stochasticity:
- For s populations the probability of at least one population persisting
- = 1 - ( $p c d f)^{\text {s }}$

```
        Spring Chinook
    Minimum Viable
    - UndM\mp@code{evaipotipGunatpolnf 95%}
```



```
    -Number &fPopuratons
    - -hv/Metal
    - 59,400,000
    -499,000 3
    -40,700 4
    -9,500 5
    - 3,670 10
        426
```


## Complications

- Normal Distribution of $x$ 's $\left(r_{\mathrm{t}}{ }^{\prime}\right) \mathrm{s}$
- Density Dependence
- Increased birth rate or lowered death rate at low population sizes (Ricker type model) increases probability of persistence
- Allee effects (the opposite of above) decreases probablity of persistence
- Spawners are only one age-class
- Holmes et al 2001 showed 5-year running sum is a better index to total population size.


## Spring Chinook

 Minimum Viable - Appminetataoop (2flationroach appilea to sy y moung-sums:- ש゙Tictep4tar less stringent goal of 95\% probability of persistence for
- APPrnber of Populations

Complications: Independence of Populations

■ Palila: There are 2 subpopulations for this species and there is no correlation between populations in these 2 areas $\quad(r=0.035, n=15$, $P=.902$ )

■ Spring Chinook on Salmon River: Lemhi and South Fork are the 2 largest subpopulations and their rates of change are moderately correlated ( $r=0.480, n=27, P=0.011$ )

Stochastic Population Growth:


