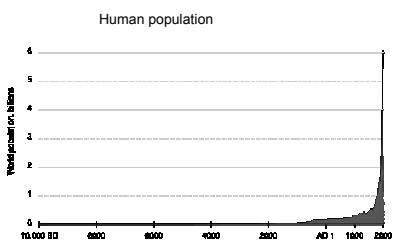
Population Models

- So far we've focused on:
  - Estimation techniques
  - Characteristics of populations
- Introduction to population growth models
  - Unlimited resources, density indpendent growth
  - Limited resources, density dependent growth

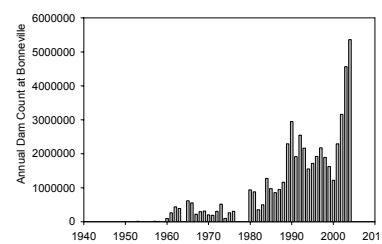
Population Models

- Forecast future conditions
  - Sustainable yield
  - Population viability analysis
  - Trajectory of population size for invasive species
- Hindcast to explore potential mechanisms

Population Growth Examples



American shad, Columbia River



Population Growth: a simple case

- Constant environment
- Unlimited resources
- All animals are the same

Population Growth: a simple case

- Change in numbers = ΔN
- ΔN = (Births – Deaths) + (Immigrants-Emigrants)
- Ignore immigrants and emigrants
  - Assume closed population or
  - assume I = E
  - or combine B + I and D + E
- $\Delta N = B - D$

Population Growth: a simple case

- $\Delta N = N_1 - N_0$
- $N_1 - N_0$ = Births – Deaths
- $N_1 - N_0 = R_B N_0 - R_D N_0$

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- $N_1 = N_0 (1 + R_B - R_D)$

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- $\lambda = (1 + R_B - R_D)$
- $N_1 = N_0 \lambda$

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- $\lambda = (1 + R_B - R_D)$
- $N_1 = N_0 \lambda$
- Note: $\lambda = N_1 / N_0$

Population Growth: a simple case

- Let's project:
- $N_2 = N_1 \lambda$

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Population Growth: a simple case

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- $N_2 = N_1 \lambda$
- $N_2 = (N_0 \lambda) \lambda$
- $N_2 = N_0 \lambda^2$
- $N_3 = N_0 \lambda \lambda \lambda$
- $N_4 = N_0 \lambda \lambda \lambda \lambda$, etc…
- In general, $N_t = N_0 \lambda^t$

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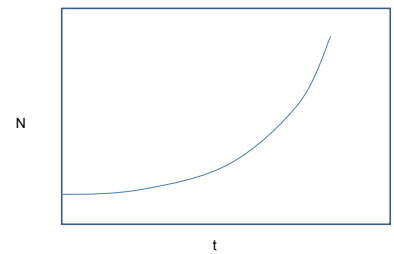

Population Growth: a simple case

- This treatment of growth rate (λ) is very simple and intuitive: λ = 1.06 = 6% increase per year

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Assumptions

- Birth rate is constant
- Death rate is constant
- It treats all members of the population as equal or it assumes a stable age distribution
  - Reasonable for some populations
    - Non-overlapping generations (insects, annual plants)
- λ can also be estimated from age-specific birth and death rates

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- Note: only describes the population size per time interval
  - What if species does not have seasonal reproduction?
  - What if we want compare species with different intervals of population change (the tortoise vs. the hare)
  - λ is mathematically cumbersome in some instances. Use calculus-based continuous-time analog

$$dN/dt = r N$$

- $N_t = N_0 \lambda^t$
- $\ln(N_t) = \ln(N_0) + (\ln \lambda)\, t$
- $\ln(N_t) = \ln(N_0) + r\, t$
- $y = a + b\, x$

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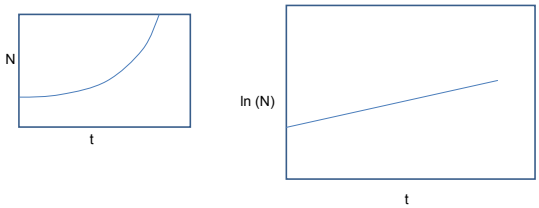

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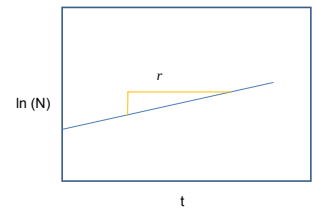
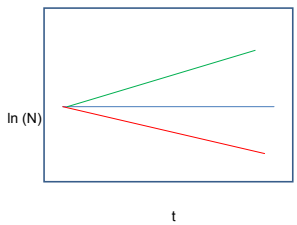


- $r = \ln(\lambda)$
- $\lambda = e^r$

r	lambda
-0.51083	0.6
-0.22314	0.8
-0.05129	0.95
0	1
0.04879	1.05
0.182322	1.2
0.336472	1.4
0.693147	2
2.302585	10



Finite and Instantaneous Rates

$\lambda = 1.13$ / year

What is the daily rate?

Finite and Instantaneous Rates

$\lambda = 1.13$ / year

What is the daily rate?

- Instantaneous rates can easily be subdivided, but finite rates can't:

$\lambda = 1.13 / 12 = 0.094$ / month = 90.6% decrease / month

Finite and Instantaneous Rates

- Instantaneous rates can easily be subdivided, but finite rates can't:

$\lambda = 1.13$ / year

$r = 0.12$ / year

$= 0.01$ / month

$= 0.000329$ / day

$\lambda = e^r = 1.000329$ / day = 0.03% /day

Finite and Instantaneous Rates

- Finite survival rate:

  $S = N_t / N_0$
- Instantaneous mortality rate:

  $z = - \ln (S)$

  $S = e^{-z}$

Finite and Instantaneous Rates

- Finite survival rates are multiplicative
- Instantaneous mortality rates are additive:

  $z_{week} = 7\ z_{daily}$

Unlimited Growth Assumptions

- b – d is constant, implies constant environment and unlimited resources
- All members of the population are equal or population has a stable age distribution
  - Reasonable for some populations
    - Non-overlapping generations (insects, annual plants)
- **Nonetheless, simple exponential growth models provide good predictions in many cases, e.g., collared dove in England**

Changing environment

- Population change is in the real world is dynamic, b and d change
- Observed change is caused by:
  - Real changes (Process error)
    - deterministic causes
    - stochastic process error
      - factors we don't know about
      - true randomness, e.g. demographic stochasticity
  - Sampling (observation) error
- We can incorporate into models

Stochastic population growth

- Mills 2007 Figure 5.5

Stochastic population growth

t	lambda	N
0		100
1	1.2	120
2	1.2	144
3	1.2	173
4	1.2	207
5	1.2	249
6	1.2	299
7	1.2	358
8	1.2	430

Stochastic population growth

t	lambda	N	Lambda	N
0		100	-	100
1	1.2	120	1.2	120
2	1.2	144	1.4	168
3	1.2	173	1	168
4	1.2	207	1.1	185
5	1.2	249	1.3	240
6	1.2	299	1.1	264
7	1.2	358	1.3	344
8	1.2	430	1.2	412
average	1.2		1.2	
Std Dev	0.000		0.131	

Stochastic population growth

t	lambda	N	Lambda	N	Lambda	N
0		100	-	100	-	100
1	1.2	120	1.2	120	1.2	120
2	1.2	144	1.4	168	1.6	192
3	1.2	173	1	168	0.8	154
4	1.2	207	1.1	185	0.9	138
5	1.2	249	1.3	240	1.5	207
6	1.2	299	1.1	264	0.9	187
7	1.2	358	1.3	344	1.5	280
8	1.2	430	1.2	412	1.2	336
average	1.2		1.2		1.2	
Std Dev	0.000		0.131		0.312	

Stochastic population growth

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8	1.2	430	1.2	412	1.2	336
Arithmetic Mean	1.2		1.2		1.2	
Std Dev	0.000		0.131		0.312	
Geometric Mean	1.200		1.194		1.164	

$$\overline{\lambda}_G = (\lambda_1 * \lambda_2 * \lambda_3 * \ldots * \lambda_t)^{1/t}$$

Unlimited growth summary

- Unrealistic (long-term) assumptions
- Finite and instantaneous forms each have advantages
- Stochasticity affects ability to accurately predict future conditions
- Provides accurate predictions in many cases
  - short time intervals
  - invading/colonizing populations
  - post-disturbance dynamics