Problem Set 21

Problem 1. *Let L_1, L_2, L_3, L_4 be 4 lines in $\mathbb{P}^3$. Let $C(L_i)$ denote the Chow form of L_i. Determine the number of lines which intersect all 4 lines by computing the degree of the variety in $\mathbb{P}^5$ determined by $\mathbb{G}(1,3) \cap C(L_1) \cap \cdots \cap C(L_4)$.*

Problem 2. *In general, you can determine the number of lines which intersect 4 curves, $D_1, \ldots, D_4$ in $\mathbb{P}^3$ by computing the Chow form of each curve and intersecting all of the Chow forms with $\mathbb{G}(1,3)$. Since $\mathbb{G}(1,3)$ has degree 2, the expected number of lines will be $2\Pi_{i=1}^{4} deg(C(D_i))$. How many lines intersect 4 random twisted cubics in $\mathbb{P}^3$?*

Problem 3. *Pick a cubic surfaces in $\mathbb{P}^3$. Use the algorithm given in class to determine the number of lines which lie on the surface. (If the surface is general enough then the answer should be 27).*

Problem 4. *Let D be the curve given as the image of the map $\phi : \mathbb{P}^1 \to \mathbb{P}^4$ where $\phi([s:t]) = [s5 : s^4t : s^2t^2 : st^4 : t^5]$. Determine the osculating plane to D at $[1:2:4:16:32]$.*