Problem 1 (2.5 points)
Let $S_3$ be the symmetric group of degree 3, i.e., the group of permutations on \{1, 2, 3\}.

(i) Find the order of each element of $S_3$.

(ii) List all cyclic subgroups of $S_3$.

Problem 2 (2.5 points)
Use trigonometric identities and induction to prove that
\[
\left( \begin{array}{cc}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta 
\end{array} \right)^n = \left( \begin{array}{cc}
\cos n\theta & -\sin n\theta \\
\sin n\theta & \cos n\theta 
\end{array} \right)
\]
for all $n \in \mathbb{N}$. Show that for a constant $\theta$
\[
H = \left\{ \left( \begin{array}{cc}
\cos n\theta & -\sin n\theta \\
\sin n\theta & \cos n\theta 
\end{array} \right) \mid n \in \mathbb{Z} \right\}
\]
is a cyclic subgroup of $GL(n, \mathbb{R})$. Do you think $H$ is finite?

Problem 3 (2.5 points)
Let $a$ be an element of a group $G$ and let $|a| = 15$. Compute the orders of the following elements of $G$:

(i) $a^3$, $a^6$, $a^9$ and $a^{12}$.

(ii) $a^5$ and $a^{10}$.

(iii) $a^2$, $a^4$, $a^8$ and $a^{14}$.

Problem 4 (2.5 points)
Let $G$ be a group with respect to addition and let $a \in G$. Prove that $|a| = |-a|$.