Math 432 - Numerical Linear Algebra - Fall 2013

Homework 1
Assigned: Friday, August 30, 2013
Due: Friday, September 6, 2013

• Include a cover page and a problem sheet.

0. Give a brief description of your academic background and research interests. If you work in a lab or research group, give your supervisor’s name and describe your project. One paragraph is fine.

1. Given matrices 
\[ A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ -3 & -1 & 5 \\ 1 & 3 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & -1 & 4 \\ 0 & 2 & -2 \\ 0 & 0 & 3 \end{pmatrix} \]

compute the indicated matrices. If an operation cannot be performed, indicate why not.

(a) \(2A + C^T\) and \(C - 3B\)
(b) \(CA\) and \(AC\)
(c) \(C^TD\) and \(BA^T\)
(d) \(\det(D)\) and \(\det(A)\)

2. Let \(A\) be a nonsingular matrix.

(a) Show that \(A^{-1}\) is unique.
(b) Show that \(A^{-1}\) is nonsingular and \((A^{-1})^{-1} = A\).
(c) Show that \(A^T\) is nonsingular and \((A^T)^{-1} = (A^{-1})^T\).
(d) If \(B\) is nonsingular, show that \(AB\) is nonsingular and \((AB)^{-1} = B^{-1}A^{-1}\).

3. Calculate the determinant of the matrix 
\[
\begin{pmatrix}
1 & 0 & 4 & 1 \\
-2 & 1 & -3 & 2 \\
0 & 0 & 0 & 2 \\
3 & 2 & 1 & -1
\end{pmatrix}
\]

by first expanding along the second column.

4. Let \(D = \text{diag}[d_{11}, d_{22}, \ldots, d_{nn}]\) be an \(n\times n\) diagonal matrix. Show that \(\det(D) = d_{11}d_{22} \ldots d_{nn}\).
5. Let $\alpha$ be a real number and let

$$A = \begin{pmatrix} \alpha & 4 \\ 1 & \alpha \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & \alpha & 0 \\ -3 & -1 & 5 \\ 1 & 3 & \alpha \end{pmatrix}.$$  

(a) For what value(s) of $\alpha$ is $A$ singular?
(b) For what value(s) of $\alpha$ is $B$ singular?

6. Compute the spectrum of matrix

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

Recall that the spectrum of a matrix is the set of all its eigenvalues.