Math 395/CS 395 - Analysis of Algorithms - Spring 2022

Homework 2
Due: Friday, April 1, 2022

• Include a problem sheet.
• Read sections 4.1 - 4.5.

PROBLEMS:

1. Using Figure 2.4 from the textbook by Cormen as a model, illustrate the operation of merge sort on the array \( A = \langle 3, 41, 52, 26, 38, 57, 9, 49 \rangle \).

2. Rewrite the \texttt{Merge} procedure so that it does not use sentinels, instead stopping once either array \( L \) or \( R \) has had all its elements copied back to \( A \) and then copying the remainder of the other array back into \( A \).

3. Use mathematical induction to show that when \( n \) is an exact power of 2, the solution of the recurrence

\[
T(n) = \begin{cases} 
2 & \text{if } n = 2, \\
2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 
\end{cases}
\]

is \( T(n) = n \lg n \).

4. We can express insertion sort as a recursive procedure as follows. In order to sort \( A[1..n] \), we recursively sort \( A[1..n-1] \) and then insert \( A[n] \) into the sorted array \( A[1..n-1] \). Write a recurrence for the worst-case running time of this recursive version of insertion sort.

5. Referring back to the searching problem (see problem 2 from HW 1), observe that if the sequence \( A \) is sorted, we can check the midpoint of the sequence against \( \nu \) and eliminate half of the sequence from further consideration. The \textbf{binary search} algorithm repeats this procedure, halving the size of the remaining portion of the sequence each time. Write pseudocode, either iterative or recursive, for binary search. Argue that the worst-case running time of binary search is \( \Theta(\lg n) \).