1. Consider the matrix
\[
\begin{pmatrix}
a & -1 & 0 \\
-1 & 4 & 1 \\
0 & 1 & 5 \\
\end{pmatrix}
\]
(a) For what values of \( a \) will this matrix be positive definite?
(b) For what values of \( a \) will this matrix be strictly diagonally dominant?

2. Compute the Cholesky factorization \( A = HH^T \) of
\[
A = \begin{pmatrix}
1 & 1 & 1 \\
1 & 1.001 & 1.001 \\
1 & 1 & 2 \\
\end{pmatrix}
\]
Solve the system \( Ax = b \), where
\[
b = \begin{pmatrix}
3 \\
3.0020 \\
4.0010 \\
\end{pmatrix}
\]
using the obtained Cholesky factorization. Verify your answer using the MATLAB program \texttt{CHOLES} or Matlab program \texttt{chol}. Note that Matlab program \texttt{chol}(\( A \)) computes the Cholesky factor \( R \) such that \( A = R^T R \), where \( R \) is upper triangular.

3. Let \( H = I - \frac{2uu^T}{u^Tu} \) be a Householder matrix. Then prove that
(a) \( Hu = -u \)
(b) \( Hv = v \) if \( v^Tu = 0 \).

4. Given vector \( x = (1 \ 2 \ 3 \ 4)^T \), compute a Householder matrix \( H = I - \frac{2uu^T}{u^Tu} \) such that \( Hx \) has zeros in the positions 2 through 4. Compute \( Hx \).
5. Find QR factorization of

\[
A = \begin{pmatrix}
10 & 1 & 1 & 1 \\
2 & 10 & 1 & 1 \\
1 & 1 & 10 & 1 \\
1 & 1 & 1 & 10 \\
\end{pmatrix}
\]

using the Householder algorithm. Verify your answer using the MATCOM program \texttt{HOUSEQRN} or Matlab program \texttt{qr} in the form \([Q, R] = \texttt{qr}(A)\).