1. (# 5, Section 2.8) Show that on any sequence space $X$ we can define a linear functional $f$ by setting $f(x) = \xi_n \ (n \text{ fixed})$, where $x = (\xi_j)$. Is $f$ bounded if $X = l^\infty$?

2. (# 8, Section 2.8) The null space $N(M^*)$ of a set $M^* \subset X^*$ is defined to be the set of all $x \in X$ such that $f(x) = 0$ for all $f \in M^*$. Show that $N(M^*)$ is a vector space.

3. (# 3, Section 2.9) Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{e_1, e_2, e_3\}$ for $\mathbb{R}^3$, where $e_1 = (1, 1, 1)$, $e_2 = (1, 1, -1)$, $e_3 = (1, -1, -1)$. Find $f_1(x)$, $f_2(x)$, $f_3(x)$, where $x = (1, 0, 0)$.

4. (# 7, Section 2.9) Find a basis for the null space of the functional $f$ defined on $\mathbb{R}^3$ by $f(x) = \alpha_1 \xi_1 + \alpha_2 \xi_2 + \alpha_3 \xi_3$, where $\alpha_1 \neq 0$ and $x = (\xi_1, \xi_2, \xi_3)$.

5. (# 11, Section 2.9) If $x$ and $y$ are different vectors in a finite dimensional vector space $X$, show that there is a linear functional $f$ on $X$ such that $f(x) \neq f(y)$.

6. (# 4, Section 2.10) Let $X$ and $Y$ be normed spaces and $T_n : X \to Y \ (n = 1, 2, \ldots)$ bounded linear operators. Show that convergence $T_n \to T$ implies that for every $\epsilon > 0$ there is an $N$ such that for all $n > N$ and all $x$ in any given closed ball we have $\|T_nx - Tx\| < \epsilon$.

7. (# 8, Section 2.10) Show that the dual space of the space $c_0$ is $l^1$. (Cf. Prob. 1 in Section 2.3.)