1 Randomized Complete Block (RCB) Design

We use blocking whenever possible in an experiment to increase power and precision by reducing error variance. The model for the RCB design is:

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij},$$

where α_i is the treatment effect (usually a fixed effect) and β_j is the block effect (usually a random effect). Thus the analysis of an RCB design is similar to a CRD with 2 factors and no replication, and we use the treatment-by-block interaction as the error term to test for the treatment effect. If the treatments and blocks do interact, this lowers the power of the test for the treatment, and complicates interpretation of the data. We can use our standard residual-by-predicted plot to try to detect some forms of treatment-block interaction. We can attempt to remove a treatment-by-block interaction by transforming the response, either via an optimal Box-Cox transformation or using Tukey's 1 degree of freedom method. See the SAS code for this lecture for an example. As discussed in the text, we do not formally test for a difference among blocks, even though an F test will be calculated by most computer programs. Rather than test for a block difference, we can quantify the effectiveness of blocking using the concept of relative efficiency.

1.1 Relative Efficiency

One way to measure the effectiveness of the blocking is by comparing the variance estimate from our RCB design to an estimate of what the variance would have been with a CR design. As shown in the text, the formula for this relative efficiency is:

$$E_{RCB:CRD} = \frac{(\nu_{rcb}+1)(\nu_{crd}+3)}{(\nu_{rcb}+3)(\nu_{crd}+1)} \frac{\sigma^2_{crd}}{\sigma^2_{rcb}},$$

where σ^2_{rcb} is estimated by the variance estimate from the RCB analysis, and σ^2_{crd} is estimated using mean squares from the RCB analysis, as shown in the text. The larger the relative efficiency, the more effective the blocking has been.

1.2 Related Randomized Block Designs

Variants of the basic RCB design above are often encountered. For one example, suppose we had randomized our flavored milk subjects (from the example discussed in lecture) also to a type of cookie. Then we would have a factorial treatment structure in each block, this is often called a **randomized block factorial design**.

For a different type of variation on the RCB, suppose that we divide people into age strata (15-25, 26-35, 36-45, ...). Suppose that we obtain subjects for each stratum, and randomize them so that each subject drinks one of the three types of flavored milk, and an equal number of subjects in each age stratum drinks each type of flavored milk. Now we are blocking on age, but we have replication of the treatments in each block. This type of design is called a **randomized block design with replication, or a generalized randomized block (GRCB) design**. The analysis of a GRCB design proceeds like the analysis of a CRD with factorial treatment structure with replication, including a test for interaction between blocks and treatments.