1 The Percentile, Residual, and BCA methods for Bootstrap confidence intervals

1.1 The Percentile method

The Percentile method is probably the most intuitively clear method for creating a bootstrap confidence interval. It involves creating many (≥ 5000) bootstrap samples, and calculating a $\widehat{\theta}_b$ value for each bootstrap sample. Then a 95% interval for θ, for example, would be:

$$\widehat{\theta}_{b,.025} < \theta \leq \widehat{\theta}_{b,.975}$$

1.2 The Residual method

The Residual method is somewhat similar to the Percentile method, except it is based on a bootstrap distribution of residuals from the original estimate $\widehat{\theta}$. For this method, many (≥ 5000) bootstrap samples are created and $e_b = \widehat{\theta}_b - \widehat{\theta}$ is calculated for each bootstrap sample. A 95% interval for θ, for example, would then be:

$$\widehat{\theta} - e_{b,.975} \leq \theta < \widehat{\theta} - e_{b,.025}$$

1.3 The BCA method

The BCA method creates an interval similar to the percentile interval, except that instead of having the interval be $\widehat{\theta}_{b,.025} < \theta \leq \widehat{\theta}_{b,.975}$, it is like $\widehat{\theta}_{b,\alpha_L} < \theta \leq \widehat{\theta}_{b,\alpha_U}$,where L and U are chosen to make the interval median unbiased and adjusted for skewness. The values α_L and α_U are chosen to have the same cumulative probability as z_L and z_U, defined as:

$$z_L = \frac{z_0 - z_{1-\alpha/2}}{1 - a(z_0 - z_{1-\alpha/2})} + z_0 \text{ and } z_U = \frac{z_0 + z_{1-\alpha/2}}{1 - a(z_0 + z_{1-\alpha/2})} + z_0.$$

The value z_0 measures median unbiasedness and is defined via: $P(Z \leq z_0) = p_0$, where p_0 is the proportion of $\widehat{\theta}_b \leq \widehat{\theta}$. The value a measures skewness of the data and is given by:

$$a = \frac{\sum(\widehat{\theta}_{(\,)} - \widehat{\theta}_{-i})^3}{6\left[\sum(\widehat{\theta}_{(\,)} - \widehat{\theta}_{-i})^2\right]^{3/2}},$$

where $\widehat{\theta}_{-i}$ is the estimate of θ computed without the i^{th} observation, X_i, and $\widehat{\theta}_{(\,)}$ is the mean of the $\widehat{\theta}_{-i}$ values. Note that if $a = 0$ and $z_0 = 0$, then the BCA method is the same as the percentile method.