Exercise Set #2

1. (7 pts) Consider an asexual (or 1-locus haploid) population with genotypes A_1 and A_2 and respective fitnesses $w_1 = 1$ and $w_2 = 1 - s$. Let p = freq(A_1), q = freq(A_2); prime (‘) indicates next generation.

a. Let $\Delta p = p' - p$ and $\Delta q = q' - q$. Use the fact that $q = 1 - p$ to show $\Delta q = -\Delta p$

b. Using an argument parallel to Gillespie's for a diploid locus with two alleles (e.g., pp. 61–2, 2nd ed/pp. 51–2, 1st ed), derive the recursion $p' = p\frac{1}{\bar{w}}$, where $\bar{w} = 1 - qs$ is the population mean fitness.

c. Using parts a and b, derive the expression $\Delta p = \frac{pqs}{\bar{w}}$

d. Use parts a–c to prove that mean fitness evolves as $\Delta\bar{w} = \bar{w}' - \bar{w} = \frac{pqs^2}{\bar{w}}$

e. The variance of any random variable *X* is defined as $var(X) = E[X - E(X)]^2 = E(X^2) - [E(X)]^2$ where $E(\cdot)$ means expectation or average of the argument over the distribution of *X*. Thinking of the fitnesses $w_1 = 1$ and $w_2 = 1 - s$ as values of a random variable *W* sampled with respective probabilities *p* and *q*, show that the variance in fitness is $var(W) = pqs^2$

f. Combine the results of parts d and e to show that mean fitness evolves according to

(*) $\Delta\bar{w} = \frac{var(w)}{\bar{w}}$

g. Equation (*) is a version of "Fisher's fundamental theorem of natural selection" for organisms with asexual reproduction. What biological insight(s) does this equation reveal about the process of adaptive evolution?

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2. (2 pts) Gillespie: Problem 3.4

3. (2 pts) Gillespie: Problem 3.5 (computer program is optional)

4. (2 pts) Show that Equation 3.5 of Gillespie is correct

5. (2 pts) Gillespie: Problem 3.9 (2nd ed)/3.8 (1st ed)

6. (2 pts) Gillespie: Problem 3.10 (2nd ed) /3.9 (1st ed)

7. (3 pts) Gillespie: Problem 4.2 (2nd ed)